In several chapters, Pavlov discusses a large number of experiments dealing with functionally induced pathological states of the nervous system, and suggests, also, some therapeutic measures. He concludes: 'This . . and other observations suggest that a gradual development of internal inhibition in the cortex should be used for re-establishment of the balance of normal conditions in cases of an unbalanced nervous system . I do not know whether similar therapeutic measures . . are applied in human neurotherapy'[4]

The above remark is vitally important, it is not only a result of a lifetime of scientific work, but it expresses a principle which is used, *without being formulated explicitly,* all through psychotherapy In the present volume, this principle is not only formulated in physiological terms, but is also made the foundation of a physiological method for its semantic application This method is found in the training and development of the consciousness of abstracting (see Part VII), which, when applied, not only restores nervous balance as empirically shown, but also gives powerful *preventive semantic means* if used in early education

Further consequences and conclusions are given in Part VII At this point, we shall merely state that the above explanations also show why a theory of *universal agreement,* in the broadest sense; namely, agreement with one's self, eliminating internal 'conflict', and with others, eliminating family, social, and international conflicts is neurologically not only possible, but also a necessary semantic consequence of using the human nervous system in its structurally appropriate way

It is well known that the use of the terms 'positive' and 'negative' is optional, but the opposing character of the issues involved is not optional, because these are experimental and structural In former days, we not only made our selection, and called some issues positive and some negative, but we naturally had and have some semantic responses connected with them Thus something 'positive' implied certainty, 'reality', 'truth', 'absolute' , something 'negative' implied the negation of these.

In 1933, it appears likely that we shall have to revise *in toto* these semantic orientations, which obviously we have been practicing since the days of savagery

What are the facts? Curiously enough:

1) The electricity which lights our lamps or runs our dynamos, we call, in the old language, 'negative' electricity

2) The numbers which are the foundation of the most important complex numbers in mathematics are formally based on negative numbers.

3) The foundation of so-called 'human mentality' is the 'negative' reaction

4) Because words are never the things we speak about, the sole link between languages and the objective world being structural, the only 'positive' facts about this world are of the old 'negative' character

5) Finally, the main difficulty of the *A*-system can be found in the positive 'is' of identity, involving us in false to fact evaluation and semantic disturbances, on the other hand, a $\bar{A}$-system is based on the complete elimination of identity formulated as a negative premise of the 'this *is not* this' type (see Part VII).

At present, only in *technical* mathematics can people behave semantically like 'gentlemen' They analyse and agree, no quarrels are possible Linguistic and semantic researches show that the structure of all languages can, and *must,* be made similar to empirical structures, and then, also, the rest of humans can, and probably will, behave in a less silly and futile way than they have done in the past and are doing in the present

BOOK II

A GENERAL INTRODUCTION TO NON-ARISTOTELIAN SYSTEMS AND GENERAL SEMANTICS

Of all men, Aristotle is the one of whom his followers have worshipped his defects as well as his excellencies which is what he himself never did to any man living or dead, indeed, he has been accused of the contrary fault (354) AUGUSTUS DE MORGAN

There is one very important fact on which we must be in no doubt, and that is that for any given deductive theory there is not *any one* system of fundamental notions nor *any one* system of fundamental propositions; there are generally several equally possible, *i e* from which it is equally possible to deduce correctly all the theorems . This fact is very important, because it shows that there are in themselves no *undefinable* notions nor *indemonstrable* propositions, they are only so relatively to a certain adopted order, and they cease (at any rate partly) to be such if another order is adopted This destroys the traditional conception of *fundamental ideas* an *fundamental truths*, fundamental, that is to say, absolutely and essentially (120) LOUIS COUTURAT

In this direction finality is not sought, for it is apparently unattainable All that we can say is, in the words of a leading analyst, "sufficient unto the day is the rigor thereof " (23) E T BELL

In mathematics it is new ways of looking at old things which seem to be the most prolific sources of far-reaching discoveries (23) E T. BELL

The first will show us how to change the language suffices to reveal generalizations not before suspected (417) H POINCARÉ

In sum, *all the scientist creates in a fact is the language in which he enunciates it* (417) H POINCARÉ

This long discussion brings us to the final conclusion that the concrete facts of nature are events exhibiting a certain structure in their mutual relations and certain characters of their own The aim of science is to express the relations between their characters in terms of the mutual structural relations between the events thus characterised (573) A N WHITEHEAD

We cease to seek resemblances, we devote ourselves above all to the differences, and among the differences are chosen first the most accentuated, not only because they are the most striking, but because they will be the most instructive (417) H POINCARÉ

The materialistic theory has all the completeness of the thought of the middle ages, which had a complete answer to everything, be it in heaven or in hell or in nature There is trimness about it, with its instantaneous present, its vanished past, its non-existent future, and its inert matter. This trimness is very medieval and ill accords with brute fact (573)

A N WHITEHEAD

The existence of analogies between central features of various theories implies the existence of a general theory which underlies the particular theories and unifies them with respect to those central features *

E H MOORE

Neither the authority of man alone nor the authority of fact alone is sufficient. The universe, as known to us, is a joint phenomenon of the observer and the observed, and every process of discovery in natural science or in other branches of human knowledge will acquire its best excellence when it is in accordance with this fundamental principle. (82)

R D CARMICHAEL

It is evident that if we adopt this point of view toward concepts, namely that the proper definition of a concept is not in terms of its properties but in terms of actual operations, we need run no danger of having to revise our attitude toward nature (55)

P. W BRIDGMAN

To say the facts are incomprehensible is a rationalization of individual ignorance

Ignorance, however, may be no fault. It becomes so only when the individual permits himself to rationalize it, *i e*, give it a disguise, which effectually blocks him in the utilization of his intelligence, which might otherwise solve the problem in hand (241)

SMITH ELY JELLIFFE

The symbol *A* is not the counterpart of anything in familiar life To the child the letter *A* would seem horribly abstract, so we give him a familiar conception along with it "*A* was an Archer who shot at a frog" This tides over his immediate difficulty, but he cannot make serious progress with word-building so long as Archers, Butchers, Captains, dance round the letters. The letters are abstract, and sooner or later he has to realise it In physics we have outgrown archer and apple-pie definitions of the fundamental symbols To a request to explain what an electron really is supposed to be we can only answer, "It is part of the A B C of physics" (149)

A S EDDINGTON

No previous existing system of thought had properly formed a working hypothesis to explain why for this or that individual it was necessary to "go up three steps or else be constipated," "or to take pills in multiples of three," or other analogous symptoms which will occur to the reader and which are found in bewildering profusion in all pathological cases, be they hysterias, or compulsion neuroses, phobias, schizophrenias, or what not. (241)

SMITH ELY JELLIFFE

The Dormouse . went on "—that begins with an M, such as mouse-traps, and the moon, and memory, and muchness—you know you say things are 'much of a muchness'—did you ever see such a thing as a drawing of a muchness!"

"Really, now you ask me," said Alice, very much confused, "I don't think—"

"Then you shouldn't talk," said the Hatter.**

LEWIS CARROLL

4.1212 What *can* be shown *cannot* be said (590)

L. WITTGENSTEIN

**Introduction to a Form of General Analysis* Yale Univ Press

***Alice in Wonderland*

PART VII

ON THE MECHANISM OF TIME-BINDING

There should be no theoretical objection to the hypothesis of the formation of new physiological paths and new connections within the cerebral hemispheres (394) I. P. PAVLOV

It seems desirable in this place to clearly emphasize the fact that in the use of psychoanalysis we are dealing solely with a method for gaining data One occasionally hears the statement that psychoanalysis is nonsense A method, or a tool, is not nonsense (241) SMITH ELY JELLIFFE

It is by means of internal inhibition that the signalizing activity of the hemispheres is constantly corrected and perfected (394) I P PAVLOV

We are dealing here with types of associative reaction peculiar to the cortical system, correctly opposed to the unqualified affective reactivity of the thalamus and usefully analysed by Head (411) HENRI PIÉRON

This example and other observations suggest that a gradual development of internal inhibition in the cortex should be used for re-establishment of the balance of normal conditions in cases of an unbalanced nervous system (394) I P PAVLOV

A self-satisfied rationalism is in effect a form of anti-rationalism It means an arbitrary halt at a particular set of abstractions (575) A N WHITEHEAD

. . the 'fallacy of misplaced concreteness' . . consists in neglecting the degree of abstraction involved when an actual entity is considered merely so far as it exemplifies certain categories of thought (578) A. N. WHITEHEAD

In the Garden of Eden Adam saw the animals before he named them in the traditional system, children named the animals before they saw them (575) A. N WHITEHEAD

The negative judgment is the peak of mentality (578) A. N. WHITEHEAD

CHAPTER XXIV

ON ABSTRACTING

. . . to be an abstraction does not mean that an entity is nothing It merely means that its existence is only one factor of a more concrete element of nature (573) A N WHITEHEAD

Aristotle, in building his theories, had at his disposal, besides his personal gifts, a good education according to his day and the science current in 400-300 B C Even in those days, the Greek language was a very elaborate affair Aristotle and his followers simply took this language for granted The problems of the structure of language and its effect on *s r* had not yet arisen To them, the language they used was *the* (unique) language When I use the expression '*the* language', I do not mean anything connected with the language, as *Greek,* I mean only the structure of it, which was much similar in the other national languages of this group The language Aristotle inherited was of great antiquity, and originated in periods when knowledge was still more scanty Being a keen observer, and scientifically and methodologically inclined, he took this language for granted and systematized the modes of speaking This systematization was called 'logic' The primitive structural metaphysics underlying this inherited language, and expressed in its structure, became also the 'philosophical' background of this system. The subject-predicate form, the 'is' of identity, and the elementalism of the *A*-system are perhaps the main semantic factors in need of revision, as they are found to be the foundation of the insufficiency of this system and represent the mechanism of semantic disturbances, making general adjustment and sanity impossible These doctrines have come down to us, and through the mechanism of language the semantic disturbing factors are forced upon our children A whole procedure of training in delusional values was thus started for future generations

As the work of Aristotle was, at his date, the most advanced and 'scientific', quite naturally its influence was wide-spread In those days, no one spoke of this influence as 'linguistic', involving *s r* Aristotle's work was, and still is, spoken of as 'philosophy', and we speak mostly of the influence of *A* 'philosophy' rather than of the *A* structure of language, and its semantic influence

As we have already seen, when we make any proposition whatsoever we involve creeds, or metaphysics, which are embodied silently as structural assumptions and in our undefined terms The use of terms not definable in simpler terms at a given date is inherent and seemingly unavoidable

When our primitive ancestors were building their language, quite naturally they started with the lowest orders of abstractions, which are the most immediately connected with the outside world They established a language of 'sensations'. Like infants, they identified their feelings with the outside world and personified most of the outside events

This primitive semantic tendency resulted in the building of a language in which the 'is' of identity was fundamental If we saw an animal and called it 'dog' and saw another animal roughly resembling the first, we said, quite happily, 'it *is* a dog', forgetting or not knowing that the objective level is un-speakable and that we deal only with absolute individuals, each one different from the other Thus the mechanism of identification or confusion of orders of abstractions, natural at a very primitive stage of human development, became systematized and structurally embodied in this most important tool of daily use called 'language' Having to deal with many *objects,* they had to have names for objects These names were 'substantives' They built 'substantives', grammatically speaking, for other feelings which were not 'substantives', ('colour', 'heat', 'soul',) Judging by the lower order abstractions, they built adjectives and made a completely anthropomorphised world-picture Speaking about speaking, let us be perfectly aware from the beginning that, when we make the simplest statement of any sort, this statement already presupposes some kind of structural metaphysics The early vague feelings and savage speculations about the structure of this world, based on primitive insufficient scientific data, was influencing the building of the language Once the language was built, and, particularly, systematized, these primitive structural metaphysics and *s r* had to be projected or reflected on the outside world—a procedure which became habitual and automatic

Was such a language structurally reliable and safe? If we investigate, we can easily become convinced that it was not Let us take three pails of water, the first at the temperature of 10° centigrade, the second at 30°, and the third at 50°. Let us put the left hand in the first pail and the right in the third If we presently withdraw the left hand from the first pail and put it in the second, we feel how nicely *warm* the water in the second pail is But, if we withdraw the right hand from the third pail and put it in the second, we notice how *cold* the water is The temperature of the water in the second pail was practically not different in the two cases, yet our feelings registered a marked difference The difference in the 'feel' depended on the former conditions to which our hands had been subjected Thus, we see that a language of 'senses' is not a very reliable language, and that we cannot depend on it for general purposes of evaluation.

How about the term 'dog'? The number of individuals with which any one is directly acquainted is, by necessity, limited, and usually is small Let us imagine that someone had dealt only with good-natured 'dogs', and had never been bitten by any of them Next he sees some animal, he says, 'This *is* a dog', his associations (relations) do not suggest a bite, he approaches the animal and begins to play with him, and is bitten Was the statement 'this *is* a dog' a safe statement? Obviously not He approached the animal with semantic expectations and *evaluation* of his verbal definition, but was bitten by the non-verbal, un-speakable objective level, which has different characteristics

Judging by present standards, knowledge in the days of Aristotle was very meagre It was comparatively easy 2300 years ago to summarize the few facts known, and so to build generalizations which would cover those few facts

If we attempt to build a $\bar{A}$-system, 1933, can we escape the difficulties which beset Aristotle? The answer is that some difficulties are avoidable, but that some are inherent in the structure of human knowledge, and so cannot be entirely evaded. We can, however, invent new methods by which the harmful semantic effect of these limitations can be successfully eliminated

There is no escape from the fact that we must start with undefined terms which express silent, structural creeds or metaphysics If we state our undefined terms explicitly, we, at least, make our metaphysics conscious and public, and so we facilitate criticism, co-operation, The modern undefined scientific terms, such as 'order', for instance, underlie the exact sciences and our wider world-outlook We must start with these undefined terms as well as the modern structural world-outlook as given by science, 1933 That settles the important semantic point of our structural metaphysics. It need hardly be emphasized that in a human class of life, where creeds are characterized by having dates, they *should* always be labelled with this date For sanity, the creeds utilized in 1933 should be of the issue of 1933

Now as to the *structure* of our language What structure shall we give to our language? Shall we keep the old structure, with all its primitive implications and corresponding *s r*, or shall we deliberately build a language of new structure which will carry new modern implications and *s r*? There seems to be only one reasonable choice For a $\bar{A}$-system, we must build a new language We must abandon the 'is' of identity, to say the least We have already seen that we have an excellent substitute in an actional, behaviouristic, operational, functional language This type of language involves modern asymmetrical implications of 'order', and eliminates the 'is' of identity, which always introduces false evaluation

To these fundamental starting points, we must add the principle that our language should be of *non-el* structure With these *minimum* semantic requirements, we are ready to proceed

Let us take any object of ordinary experience, let us say the one we usually call a 'pencil', and let us briefly analyse our nervous relationship to it We can see it, touch it, smell it, taste it , and use it in different ways Is any of the relationships just mentioned an 'all-embracing' one, or is our acquaintance through any of them only *partial?* Obviously, each of these means provides an acquaintance with this object which is not only *partial,* but is also *specific* for the nerve centres which are engaged Thus, when we look at the object, we do not get odor or taste stimuli, but only visual stimuli ,.

If the object we call 'pencil' were lying on the surface of this paper and we were to look at it along the surface of the paper in a perpendicular direction to its length, it would generally be seen as an elongated object, pointed at one end But, if we were to observe it along the plane of the paper at right angles to our former direction, it would be seen as a disk This illustration is rough, but serves to show that the acquaintance derived through any specific means (e g , vision) is also *partial* in another sense, it varies with the position , of any specified observer, Smith, or a camera

Furthermore, any given means provides, for *different* observers, different acquaintances Thus, vision shows the pencil to one observer, Smith, as a pointed rod, and to another observer, Jones, as a disk Feeling, through other receptors, is just as dependent upon many conditions; and different observers receive different impressions This is well illustrated by the familiar tale of the five blind men and the elephant

Because of differences in sensitivity in the receptors of Smiths and Browns (partial colour-blindness, astigmatism, far-sightedness ,), any given means of acquaintance (e g , vision) gives to different observers different reports of the one object The acquaintance is thus personal and individual

Again, the reports received through particular channels are influenced by the kind of reports that have already come through that channel To one who has not seen trees frequently, a spruce and a balsam are not seen to be different They are just 'evergreens' With better educated seeing, this individual later differentiates, perhaps, four kinds of spruce

Because of this factor of experience, the *response* of each individual to similar external stimuli is individual We can only *agree* on colours, shapes, distances , by ignoring the fact that the effect of the 'same' stimulus is different in different individuals Besides that, we have no accurate means of comparing our impressions

The 'time' factor enters, in that we cannot become acquainted with our pencil *on all sides at once* Nor can we observe the outer form and the inner structure at the 'same time' We may even neglect to examine the inner structure entirely. Even more important is the fact that all our means together give us only a *partial* and personal acquaintance with the 'pencil'. Continually we invent extra-neural means which reveal new characteristics and finer detail Nor is this process ever completed No one can ever acquire a 'complete' acquaintance with even so simple an object as a pencil The chemistry, the physics, the uses of the varieties , offer fields of acquaintance that can be extended indefinitely Nature is inexhaustible; the events have infinite numbers of characteristics, and this accounts for the wealth and infinite numbers of possibilities in nature

I used the word 'acquaintance' deliberately, because it seems vague, and, as yet, *el* gambling on words have not spoiled this term I had to avoid the *el* terms 'senses' and 'mind' as much as possible in this analysis If we recall the example of paper roses in the case of hay fever, we shall realize that the terms 'senses' and 'mind' are not reliable, particularly in humans As a further instance, we have but to remember the experiment with newspaper headlines, also cited earlier

We become better acquainted with the object by exploring it in manifold ways, and building for ourselves different pictures, all partial, and supplied by direct or indirect contact with different nerve centres In these explorations, different nerve centres supply their *specific* responses to the different stimuli Other higher nerve centres summarize them, eliminate weaker details, and so, gradually, our acquaintance becomes fuller while yet remaining *specific* and *partial,* and the semantic problems of *evaluation, meanings,* begin to be important

If we try to select a term which would describe structurally the processes which are essential for our acquaintance with the object, we should select a term which implies 'non-allness' and the specificity of the response to the stimuli

If we pass from such a primitive level to a level of 1933, and enquire what we actually know about an object and the structure of its material, we find that in 1933 we know positively that the internal structure of materials is very *different* from what we gather by our rough 'senses' on the macroscopic level It appears of a dynamic character and of an extremely fine structure, which neither light, nor the nerve centres affected by light, can register

What we see is structurally only a specific *statistical mass-effect* of happenings on a much finer grained level We *see* what we see because we *miss* all the finer details For our purpose, it is usually enough to deal only with sight; this simplifies writing, and the comments made apply to all other 'senses', though perhaps in different degrees

In 1933, in our human economy, we have to take into account at least three levels The one is the sub-microscopic level of science, what science 'knows' *about* 'it' The second is the gross macroscopic, daily experience level of rough objects The third is the verbal level

We must also evaluate an important semantic issue, namely, the relative importance of these three levels We know already that to become acquainted with an object, we must not only explore it from all possible points of view and put it in contact with as many nerve centres as we can, as this is an essential condition of 'knowing', but we must also not forget that our nerve centres must summarize the different partial, abstracted, specific pictures In the human class of life, we find a new factor, non-existent in any other form of life, namely, that we have a capacity to collect all known experiences of different individuals Such a capacity increases enormously the number of observations a single individual can handle, and so our acquaintance with the world around, and in, us becomes much more refined and exact This capacity, which I call the time-binding capacity, is only possible because, in distinction from the animals, we have evolved, or perfected, extra-neural means by which, without altering our nervous system, we can refine its operation and expand its scope Our scientific instruments record what ordinarily we cannot see, hear, Our neural verbal centres allow us to exchange and accumulate experiences, although no one could live through all of them, and they would be soon forgotten if we had no neural and extra-neural means to record them

Again the organism works as-a-whole. All forms of human activities are interconnected It is impossible to select a special characteristic and treat it in a delusional *el* 'isolation' as the most important Science becomes an extra-neural extension of the *human* nervous system We might expect the structure of the nervous system to throw some light on the structure of science, and, vice versa, the structure of science might elucidate the working of the human nervous system

This fact is very important, semantically, and usually is not sufficiently emphasized or analysed enough When we take these undeniable facts into account, we find the results already reached to be quite natural and necessary, and we understand better why an individual cannot be considered entirely sane if he is wholly ignorant of *scientific method* and structure, and so retains primitive *s r*

For a theory of sanity, all three levels are important Our 'senses' react as they do because they are united as-a-whole in one living structure, which has potentialities or capacities for language and science

If we enquire what we *do* in science, we find that we 'observe' silently and then record our observations *verbally* From a neurological point of view, we abstract whatever we and the instruments can, then we summarize, and, finally, we generalize, by which we mean the processes of abstracting carried further

In our 'acquaintance' with daily objects, we do substantially a similar thing. We abstract whatever we can, and, according to the degree of intelligence and information we have, we summarize and generalize From the psychophysiological point of view, the ignorant is neurologically deficient But to 'know' or to 'believe' something which is false to facts is still more dangerous and akin to delusions, as psychiatry and daily experience teach us[1] It is a neurological fallacy to treat science in 'isolation' and disregard its psychophysiological role

In the building of our language, a similar neurological process becomes evident. If we were to see a series of different individuals, whom we might call Smith, Brown, Jones , we could, by a process of abstracting the characteristics, segregate the individuals by sizes or colours , then, by concentration on one characteristic and disregarding the others, we could build classes or higher abstractions, such as 'whites', 'blacks', Abstracting again, with rejection of the colour difference , we would finally reach the term 'man' This procedure is general

Anthropological studies show clearly how the degree of 'culture' among primitive peoples can be measured by the orders of the abstractions they have produced Primitive languages are characterized particularly by an enormous number of names for individual objects Some savage races have names for a pine or an oak , but have no 'tree', which is a higher abstraction from 'pines', 'oaks', Some other tribes have the term 'tree', but do not have a still higher abstraction 'woods' It does not need much emphasis to see that higher abstractions are extremely *expedient* devices There is an enormous economy which facilitates mutual understanding in being able to be brief in a statement and yet cover wider subjects

Let us consider a primitive statement 'I have seen tree$_1$', followed by a description of the individual characteristics 'I have seen tree$_2$', with minute individual description , where tree$_1$, tree$_2$, stand for names of the individual trees If an event of interest had happened in a place where there were a hundred trees, it would take a long while to observe fairly well the individual trees and still longer to give an approximate description of them Such a method is non-expedient, *fundamentally endless,* the mechanism is cumbersome, involves many *irrelevant* characteristics, and it is impossible to express in a few words much that might be *important* Progress must be slow, the general level of development of a given race or individual must be low It should be noticed that the problem of *evaluation* enters, at once implying many most important psycho-logical and semantic processes Similar remarks apply to the abstracting of infants, 'mentally' deficient grown-ups, and some 'mentally' ill

Indeed, as the readers of my *Manhood of Humanity* already know, the 'human class of life' is chiefly differentiated from 'animals' by its rapid rate of progress through the rapid rate of accumulation of past experiences This is possible only when expedient means of communication are established, that is, when higher and higher orders of abstractions are worked out

All scientific 'laws', and other generalizations of higher order (even single words), are precisely such methods of expediency, and represent abstractions of very high order They are uniquely important because they accelerate progress and help the further summarizing and abstracting of results achieved by others Naturally, this process of abstracting has also unique practical consequences When chemical 'elements' were 'permanent' and 'immutable', our physics and chemistry were much undeveloped With the advent of higher abstractions, such as the monistic and general dynamic theories of all 'matter' and 'electricity', unitary field theories , the creative freedom of science and the control over 'nature' have increased enormously and will increase still more

Psychiatry also seems to give data indicating that 'mental' illnesses are connected either with arrested development or with regression to phylogenetically older and more primitive levels, all of which, of course, involves lower order abstractions From the point of view of a theory of sanity, a sharp differentiation between 'man' and 'animal' becomes imperative For with 'man', the lack of knowledge of this difference may lead to the copying of animals, which would involve semantic *regression* and ultimately become a 'mental' illness

Although organisms have had acquaintance with objects for many hundreds or thousands of millions of years, the higher abstractions which characterize 'man' are only a few hundreds of thousands of years old As a result, the nervous currents have a natural tendency to select the older, more travelled, nervous paths Education should counteract this tendency which, from a *human* point of view, represents regression or under-development

By now we know how important it is for a $\bar{A}$-system to abandon the older implications and adopt an actional, behaviouristic, operational, or functional language On the neurological level, what the nervous system *does* is abstracting, of which the summarization, integration , are only special aspects Hence, I select the term *abstracting* as fundamental

The standard meaning of 'abstract', 'abstracting' implies 'selecting', 'picking out', 'separating', 'summarizing', 'deducting', 'removing', 'omitting', 'disengaging', 'taking away', 'stripping', and, as an adjective, not 'concrete' We see that the term 'abstracting' implies structurally and semantically the activities characteristic of the nervous system, and so serves as an excellent *functional physiological* term

There are other reasons for making the term 'abstracting' fundamental, which, from a *practical* point of view, are important A bad habit cannot be easily eliminated except by forming a new semantic counter-reaction All of us have some undesirable but thoroughly established *linguistic habits* and *s r* which have become almost automatic, overloaded with unconscious 'emotional' evaluation This is the reason why new 'non-systems' are, in the beginning, so extremely difficult to acquire We have to break down the old structural habits before we can acquire the new *s r* The $\bar{E}$ geometries or the $\bar{N}$ systems are not any more difficult than the older systems were Perhaps they are even simpler The main semantic difficulty, for those accustomed to the old, consists in breaking the old structural linguistic habits, in becoming once more flexible and receptive in feelings, and in acquiring new *s r* Similar remarks apply in a more marked degree to a $\bar{A}$-system The majority of us have very little to do *directly* with $\bar{E}$ or $\bar{N}$ systems (although indirectly we all have a good deal to do with them) But all of us live our immediate lives in a human world still desperately A Hence a $\bar{A}$-system, no matter what benefits it may give, is much handicapped by the old semantic blockages

In building such a system, this natural resistance or persistence of the old *s r* must be taken into consideration and, if possible, counteracted One of the most pernicious bad habits which we have acquired 'emotionally' from the old language is the feeling of 'allness', of 'concreteness', in connection with the 'is' of identity and elementalism One of the main points in the present $\bar{A}$-system is first to remove entirely from our *s r* this allness' and 'concreteness', both of which are structurally unjustified and lead to identification, absolutism, dogmatism, and other semantic disturbances Usually, the term 'abstract' is contrasted with 'concrete', which is connected with some vague feeling of 'allness' By making the functional term *abstracting* fundamental, we establish a most efficient semantic counter-reaction to replace the older terms which had such vicious structural implications Indeed, it is comparatively easy to accept the term 'abstractions of different orders', and any one who does so will see how much clarity and how much semantic balance he will automatically acquire

From a *non-el* point of view, the term 'abstracting' is also very satisfactory The structure of the nervous system is in ordered levels, and all levels go through the process of abstracting from the other levels.

The term implies a general activity, not only of the nervous system as-a-whole, but even of all living protoplasm, as already explained The characteristic activities of the nervous system, such as summarizing, integrating , are also included by implication

If we wish to use our terms in the strictly *non-el* way, we must abandon the older division of 'physiological abstractions', which implies 'body', and of 'mental abstractions', which, in turn, implies 'mind', both taken in an *el* way We can easily do that by postulating abstractions of different orders We should notice that the above use of the term 'abstracting' differs from the old usage The semantic difference is in uniting all the abstractions our nervous system performs under the one term, and in distinguishing between different abstractions by the order of them, which is functionally, as well as structurally, justified

The term 'first order abstractions' or 'abstractions of lower order' does not distinguish between 'body' and 'mind'. *Practically,* it corresponds roughly to 'senses' or immediate feelings, except that by implication it *does not eliminate 'mind'* Neither does the term 'abstractions of higher orders' eliminate 'body' or 'senses', although it corresponds roughly to 'mental' processes

From the point of view of 'order', the term 'abstracting' has a great deal in its favor We have seen what serious structural and semantic importance the term 'order' has, and how the activity of the nervous system has to be spoken of in terms of order If we establish the term 'abstracting' as fundamental for its *general* semantic implications, we can easily make the meanings more definite and specific in each case by having 'abstractions of different orders'

We have seen also that the terms we select should involve environment by implication: it is not difficult to see that the term 'abstracting' implies 'abstracting from something' and so involves the environment as an implication.

The term 'abstractions of different orders' is, in this work, as fundamental as the term 'time-binding' was in the author's earlier *Manhood of Humanity* Hence, it is impossible to be comprehensive about it at this stage, more will be forthcoming as we proceed

But we have already come to some important semantic results We have selected our structural metaphysics, and decided that in 1933 we should accept the metaphysics of 1933, which is given *exclusively* by science. We have decided to abandon the false to facts 'is' of identity and to use, instead, the best available language, namely, an actional, behaviouristic, functional, operational language, based on 'order' And, finally, we have found a term which is functionally satisfactory and has the correct structural and neural implications, and which represents a *non-el* term, and of which the meanings can be expanded and refined indefinitely by assigning to them different orders

In passing on to the general scientific outlook, similar structural remarks upon a *non-el* point of view apply, and are semantically of importance Because of the *non-el* character of the work of the writers on the Einstein and new quantum theories, much use is made of this material in the present work There is a marked structural, methodological, *and semantic* parallelism between all modern *non-el* strivings, which are extremely effective psycho-logically More material on this subject is given in Parts IX and X

Now, returning to the analysis of the object which we called 'pencil', we observe that, in spite of all 'similarities', this object is unique, is different from anything else, and has a *unique* relationship to the rest of the world Hence, we should give the object a *unique name* Fortunately, we have already become acquainted with the way mathematicians manufacture an endless array of individual names without unduly expanding the vocabulary If we call the given object 'pencil_1', we could call another similar object 'pencil_2', In this way, we produce individual names, and so cover the *differences* By keeping the main root word 'pencil', we keep the implications of daily life, and also of *similarities* The habitual use of such a device is structurally and semantically of extreme importance It has already been emphasized repeatedly that our abstracting from physical objects or situations proceeds by missing, neglecting, or forgetting, and that those disregarded characteristics usually produce errors in evaluation, resulting in the disasters of life If we acquire this extensional mathematical habit of using special names for unique individuals, we become conscious, not only of the similarities, but also of the differences, which consciousness is one of the mechanisms for helping the proper evaluation and so preventing or eliminating semantic disturbances.

So we now have before us a unique object which we call by a unique name 'pencil$_1$'. If we enquire what science 1933 has to say about this object, we find that this object represents structurally an extremely complex, dynamic process. For our purpose, which is *intuitive,* it is of little importance whether we accept the object as made up of atoms and the atom as made up of whirling electrons, or whether we accept the newer quantum theory, as given in Part X, according to which the atom is formulated in terms of 'electrons' but the 'electron' is the region where some waves reinforce each other, instead of being a 'bit' of something. It is of no importance from our point of view whether the atoms are of a finite size or whether they extend indefinitely and are noticeable to us only in the regions of reinforcement of the waves. Naturally, this last hypothesis has a strong semantic appeal, since it would account, when worked out, for many other facts, such as 'fulness', in a *non-el* language, but probably it would necessitate a postulation of some sub-electronic structures.

What is important for our *s.r.* is that we realize the fact that the gross macroscopic materials with which we are familiar are *not* simply what we see, feel, but consist of dynamic processes of some extremely fine structure, and that we realize further that our 'senses' are not adapted to register these processes without the help of extra-neural means and higher order abstractions.

Let us recall, in this connection, the familiar example of a rotary fan, which is made up of separate radial blades, but which, when rotating with a certain velocity, gives the impression of a *solid disk*. In this case the 'disk' is not 'reality', but a nervous integration, or abstraction from the rotating blades. We not only see the 'disk' (*b*) where there is no disk, but, if the blades rotate fast enough, we could not throw sand through them, as the sand would be too slow to get through before being struck by one of the blades.

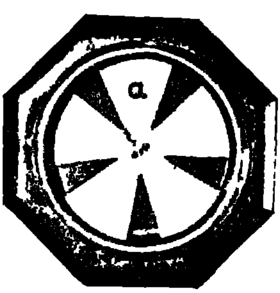


FIG 1a

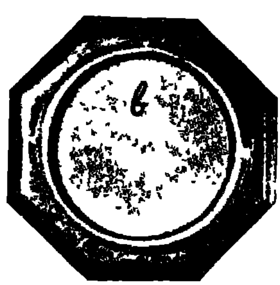


FIG 1b

The 'disk' represents a *joint phenomenon* of the rotating blades (*a*) and of the abstracting power of our nervous system, which registers only the gross macroscopic aspects and slow velocities, but *not* the finer activities on subtler levels We cannot blame 'the finite mind' for the failure to register the separate blades, because physical instruments may behave similarly For instance, the illustrations (*a*) and (*b*) are photographs of a small fan which I use in lectures, and the photographic camera also missed the rotating blades and registered only a 'disk', in Fig 1b

Something roughly similar may be assumed for our purpose as going on in what we usually call 'materials' These are composed of some dynamic, fine-grained processes, not unlike the 'rotating blades' of our example, and what we register is the 'disk', be it a table or a chair or ourselves

For a similar reason, we may assume that we cannot put our finger through a table, as our finger is too thick and too slow, and that, for some materials, it takes X-rays to be agile enough to penetrate.

The above analogies are helpful for our purpose only, but are oversimplified and should not be taken as a scientific explanation

This neural process seems to be very general, and in all our daily experiences the dynamic fine structures are lost to our rough 'senses'. We register 'disks', although investigation discovers not 'disks', but rotating 'blades' Our gross macroscopic experience is only a nervous abstraction of some definite order.

As we need to speak about such problems, we must select the best language at our disposal This ought to be *non-el* and, structurally, the closest to facts Such a language has been built, and is to be found in the differential and four-dimensional language of space-time, and in the new quantum mechanics In practice, it is simple to ascribe to every 'point of space' a date, but it takes some training to get this *s r* The language of space-time is *non-el* To the new notion of a 'point' in 'space-time' such a 'point', always having a date associated with it and hence never identical with any other point, the name of 'point-event', or simply 'event', has been given

How to pass from point-events to extended macroscopic events is a problem in mathematical 'logic' Several quite satisfactory schemes have been given, into the details of which we do not need to enter here As the *non-el* structure of the language of space-time appears different from the older *el* language of 'space' *and* 'time', quite obviously the old term 'matter', which belonged to the descriptive apparatus of 'space' *and* 'time', should be abandoned also, and the 'bits' of materials we dealt with should be referred to by structurally new terms. In fact, we know that the old term 'matter' can be displaced by some other term connected with the 'curvature' of 'space-time'

There is on record a striking example of what the structure of a form of representation means In a paper printed in the Proceedings of the National Academy of Science, February, 1926, Professor G Y. Rainich, the mathematician, tried to introduce 'mass' into space-time, the terms belonging to forms of representation of different structure He succeeded, but at the price of splitting space-time into the original space *and* time This is, as far as my knowledge goes, the first proof of how intimately a form of representation is inwardly and structurally interconnected This fact is of extraordinary semantic importance for psycho-logicians and psychiatrists, who always study symbolism of some sort It would be of great interest to have such problems worked out by them

As abstracting in many orders seems to be a general process found in all forms of life, but particularly in humans, it is of importance to be clear on this subject and to select a language of proper structure As we know already, we use *one* term, say 'apple', for at least *four* entirely different entities, namely, (1) the event, or scientific object, or the sub-microscopic physico-chemical processes, (2) the ordinary object manufactured from the event by our lower nervous centres, (3) the psychological picture probably manufactured by the higher centres, and (4) the verbal definition of the term If we use a language of adjectives and subject-predicate forms pertaining to 'sense' impressions, we are using a language which deals with entities *inside our skin* and characteristics entirely non-existent in the outside world Thus the events outside our skin are neither cold nor warm, green nor red, sweet nor bitter , but these characteristics are manufactured by our nervous system inside our skins, as responses only to different energy manifestations, physico-chemical processes , When we use such terms, we are dealing with characteristics which are absent in the external world, and build up an anthropomorphic and delusional world non-similar in structure to the world around us Not so if we use a language of order, relations, or structure, which can be applied to sub-microscopic events, to objective levels, to semantic levels, and which can also be expressed in words In using such language, we deal with characteristics found or discovered on all levels which give us *structural* data uniquely important for knowledge. The ordering on semantic levels in the meantime abolishes identification It is of extreme importance to realize that the relational , attitude is optional and can be applied everywhere and always, once the above-mentioned benefits are realized Thus, any object can be considered as a set of relations of its parts , any 'sense' perception may be considered as a response to a stimulus , which again introduces relations , As relations are found in the scientific sub-microscopic world, the objective world, and also in the psycho-logical and verbal worlds, it is beneficial to use such a language because it is *similar in structure* to the external world and our nervous system , and it is applicable to all levels The use of such a language leads to the discovery of invariant relations usually called 'laws of nature', gives us structural data which make the only possible content of 'knowledge', and eliminates also anthropomorphic, primitive, and delusional speculations, identifications, and harmful ***s.r.***

CHAPTER XXV

ON THE STRUCTURAL DIFFERENTIAL

You cannot recognise an event, because when it is gone, it is gone . . But a character of an event can be recognised . Things which we thus recognise I call objects (573) A N WHITEHEAD

When there is a judgment of identity or difference, it is because a particular associative reaction of the second order is occurring, conditioned by the primary reaction, whether the same or different, this is a gain in perceptive knowledge (411) HENRI PIÉRON

To some extent, the practice of thinking, deciding, feeling, appreciating, and sympathizing molds the personality of the thinker Presumably, the stable patterns of cortical association are changed by the performance of these acts just as on a lower plane muscles are changed by systematic exercises (222) C JUDSON HERRICK

Experimental analysis of the memory of forms insusceptible of symbolic schematization has convinced me of the great importance of ocular kinaesthesia and the small part played by visualization in nearly all individuals, with the general illusion of really visual representations, a very strong illusion, especially when symbolic and verbal schematization is possible. Ideas which are substituted for visual representation, and play the same part, are easily mistaken for it (411) HENRI PIÉRON

The eyes of the dog give to him sometimes a more intelligent expression than that of his master, and there is no doubt that he uses them to very good advantage, but they are not our eyes (221) C JUDSON HERRICK

Before I recapitulate, in the form of a structural diagram, what has been said in the previous chapter, I must explain briefly the use of the term 'event' The introduction of new terms in a language always represents initial difficulties to the student It is always advisable, if only possible, to introduce terms which are structurally close to our daily experience At present, in physics, we have a dual language, one of 'space-time', in which 'matter' is connected somehow with its 'curvature', the other of the quanta The structure of both languages is quite different, and at present scientists have not succeeded in translating one language into the other Einstein, in his latest unified field theory, has succeeded, by the introduction of new notions, in amalgamating the electromagnetic phenomena with the general theory of relativity, but even this new language does not include the quantum theory For my purpose, it is important to amalgamate both languages as an intuitive pictorial device, which, from a technical point of view, still awaits formulation As the 'space-time' continuum is the closest to our daily experience, I accept the language of 'events' as fundamental and add only a few pictorial notions taken from the quantum theory There is no doubt that the day is not far off when the unified field theory will be extended to include the new quantum theory, and so this anticipation does not appear illegitimate

If we take something, anything, let us say the object already referred to, called 'pencil', and enquire what it represents, according to science 1933, we find that the 'scientific object' represents an 'event', a mad dance of 'electrons', which is different every instant, which never repeats itself, which is known to consist of extremely complex dynamic processes of very fine structure, acted upon by, and reacting upon, the rest of the universe, inextricably connected with everything else and dependent on everything else If we enquire *how many characteristics* (*m o*) we should ascribe to such an event, the only possible answer is that we should ascribe to an event infinite numbers of characteristics, as it represents a process which never stops in one form or another, neither, to the best of our knowledge, does it repeat itself

In our diagram, Fig 1, we indicate this by a parabola (A), which is supposed to extend indefinitely, which extension we indicate by a broken off line (B) We symbolize the characteristics by small circles (C), the number of which is obviously indefinitely great

Underneath, we symbolize the 'object' by the circle (O), which has a finite size The characteristics of the object we also denote by similar little circles (C′) The number of characteristics which an object has is large but *finite,* and is denoted by the finite number of the small circles (C′)

Then we attach a label to the object, its name, let us say 'pencil$_1$', which we indicate in our diagram by the label (L) We ascribe, also, characteristics to the labels, and we indicate these characteristics by the little circles (C″)

The number of characteristics which we ascribe by *definition* to the label is still smaller than the number of characteristics the object has To the label 'pencil$_1$' we would ascribe, perhaps, its length, thickness, shape, colour, hardness, But we would mostly *disregard* the accidental characteristics, such as a scratch on its surface, or the kind of glue by which the two wooden parts of the objective 'pencil' are held together, If we want an objective 'pencil' and come to a shop to purchase one, we say so and specify verbally only these characteristics which are of particular *immediate interest* to us

It is clear that the object is often of interest to us for some special characteristics of immediate usefulness or value If we enquire as to the neurological processes involved in registering the object, we find that the

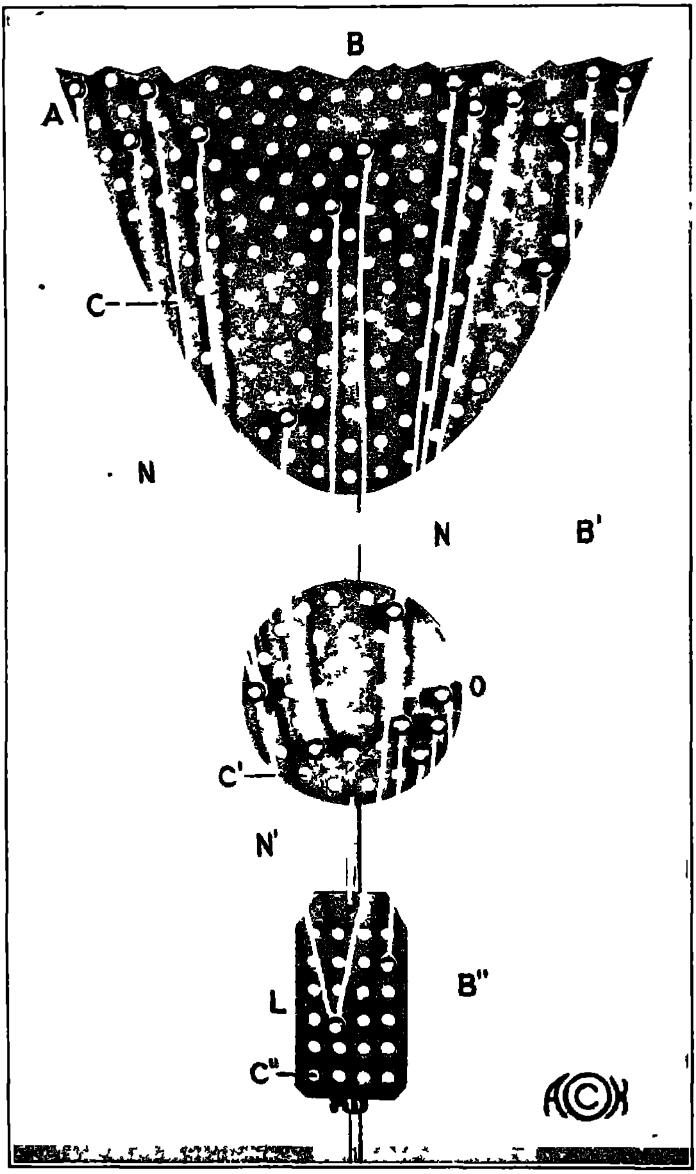


Fig 1

nervous system has *abstracted*, from the infinite numbers of sub-microscopic characteristics of the event, a large but finite number of macroscopic characteristics In purchasing a 'pencil' we usually are not interested in its smell or taste But if we were interested in these abstractions, we would have to find the smell and the taste of our object by experiment

But this is not all The object represents in this language a gross macroscopic abstraction, for our nervous system is not adapted for abstracting directly the infinite numbers of characteristics which the endlessly complex dynamic fine structure of the event represents We must consider the object as a 'first abstraction' (with a finite number of characteristics) from the infinite numbers of characteristics an event has The above considerations are in perfect accord not only with the functioning of the nervous system but also with its structure Our nervous system registers objects with its lower centres first, and each of these lower specific abstractions we call an object If we were to define an object, we should have to say that an object represents a first abstraction with a finite number of *m o* characteristics from the infinite numbers of *m o* characteristics an event has

Obviously, if our inspection of the object is through the lower nervous centres, the number of characteristics which the object has is larger (taste, smell., of our pencil,) than the number of characteristics which we need to ascribe to the label The label, the *importance* of which lies in its *meanings to us,* represents a still higher abstraction from the event, and usually labels, also, a *semantic reaction*

We have come to some quite obvious and most important structural conclusions of evaluation of the *non-el* type We see that the object *is not* the event but an abstraction from it, and that the label *is not* the object nor the event, but a still further abstraction The nervous process of abstracting we represent by the lines (N), (N') The characteristics *left out,* or not abstracted, are indicated by the lines (B'), (B")

For our semantic purpose, the distinction between lower and higher abstractions seems fundamental, but, of course, we could call the object simply the first order abstraction, and the label, with its meanings, the second order abstraction, as indicated in the diagram

If we were to enquire how this problem of abstracting in different orders appears as a limiting case among animals, we should select a definite individual with which to carry on the analysis For our analysis, which is deliberately of an extensional character, we select an animal with a definite, proper name, corresponding to 'Smith' among us. Such an animal suggests itself at once on purely verbal grounds

It is the one we call 'Fido'. Practically all English speaking people are acquainted with the name 'Fido'. Besides, most of us like dogs and are aware of how 'intelligent' they are.

Investigations and experimenting have shown that the nervous system of a Fido presents, in structure and function, marked similarities to that of a Smith. Accordingly, we may assume that, in a general way, it functions similarly. We have already spoken of the event in terms of recognition, namely, that we can never recognize an event, as it changes continually. Whitehead points out the fundamental difference between an event and an object in terms of *recognition*, namely, that an event cannot be recognized, and that an object can be recognized. He defines the object as the recognizable part of the event. The use of this definition helps us to test whether Fido has 'objects'. Since experiments show that Fido can recognize, we have to ascribe to Fido objects by definition. If we enquire what the objects of Fido represent, the structure and function of his nervous system, which are very similar to ours, would suggest that Fido's objects represent, also, *abstractions* of some low order, from the events. Would his objects appear the 'same' as ours? No. First of all, the abstractions from events which we call objects are not the 'same', even when abstracted by different individuals among humans. An extreme example of this can be given in that limited form of colour-blindness which is called Daltonism, when an object which appears green to most persons appears red to the certain few who suffer from this disease. There is, at present, no doubt that the nervous abstractions of all organisms are individual, not only with each individual, but at different 'times' with one individual, and differ, also, for these higher groups (abstractions) which we call species. We can infer how the world appears to a particular organism only if its nervous structure is quite similar to our own. With species widely separated neurologically, such inferences are entirely unjustified. So, on general grounds, the 'objects' of Fido are not the 'same' as ours; on neurological grounds, they appear only similar. In daily experience, we know that we should have difficulty in recognizing our own glove among a thousand, but Fido could perform this detection for us much better. So the 'same' glove must have been registered in the nervous system of Fido differently from the way it has been in ours.

We indicate this similarity of the human object (O_h) and the animal object (O_a) by making the circle (O_a) smaller, and emphasize the difference between the objects by differently spacing the holes representing the characteristics. Whether we call the objects (O_h) and (O_a) 'first order' abstractions or '100th order' abstractions, or simply 'lower

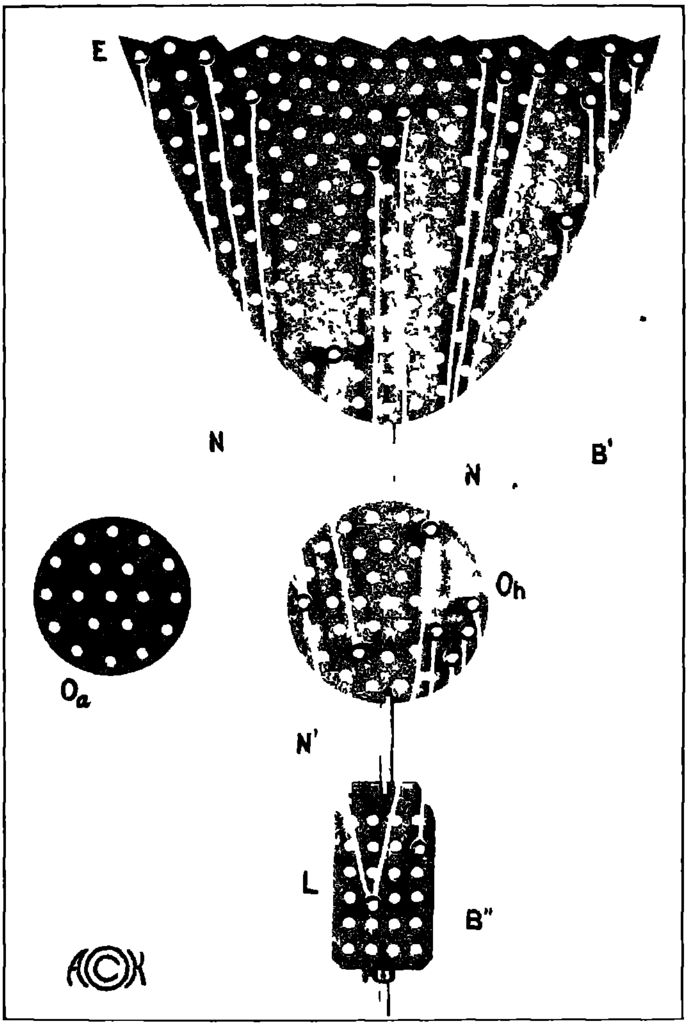


FIG 2

order' abstractions, is mainly optional There is no neurological doubt that all 'objects' represent *low order abstractions* and the use of a number to indicate the order is simply a matter of convention and convenience If we were to start with the simplest living cell, we might ascribe to its abstractions the term 'first order' abstractions If we were to survey in this way all known forms of life, we might ascribe to Fido and Smith very large numbers as their orders of abstractions But this is unnecessary, as we shall presently see

We note that Fido does abstract from events, at any rate, in lower orders, 'has objects' (O_a) which he can recognize The question is, does he abstract in higher orders? We might answer that he does within certain limits Or, we might prefer to take the limits of his abstracting capacities for granted and to include them all as lower order abstractions For the sake of convenience and simplicity, we select the last method and say that he does not abstract in higher orders In our schematic representation, we shall discover some very important differences between the abstracting capacities of humans and animals, and so we introduce here only as much complexity as we need As animals have no speech, in the human sense, and as we have called the verbal labelling* of the object 'second order abstraction' we say that animals do not abstract in higher orders

If we compare our diagram and what it represents with the well-known facts of daily life, we see that Smith's abstracting capacities are not limited to two orders, or to any '*n*' orders of abstractions

In our diagrams, the label (L) stands for the *name* which we assigned to the object But we can also consider the level of the first label (L) as a *descriptive* level or statement We know very well that Smith can always say something *about* a statement (L), on record Neurologically considered, this *next* statement (L_1) about a statement (L) would be the nervous response to the former statement (L) which he has seen or heard or even produced by himself inside his skin So his statement (L_1), *about* the former statement (L), is a *new abstraction* from the former abstraction In my language, I call it an abstraction of a higher order In this case, we shall be helped by the use of numbers If we call the level (L) an abstraction of *second order,* we must call an *abstraction from this abstraction* an abstraction of *third order,* (L_1) Once an abstraction of third order has been produced, it becomes, in turn, a fact on record, potentially a stimulus, and can be abstracted further and a statement made about it, which becomes an abstraction of the fourth order (L_2) This process has no definite limits, for, whenever statements of any order are made, we can always make a statement about them and so produce an abstraction of still higher order. This capacity is practically universal among organisms which we call 'humans' Here we reach a fundamental difference between 'Smith'

*In the present system the terms 'label', 'labelling' , are always connected with their meanings, and so, for simplicity, from now on the reference to meanings will be omitted.

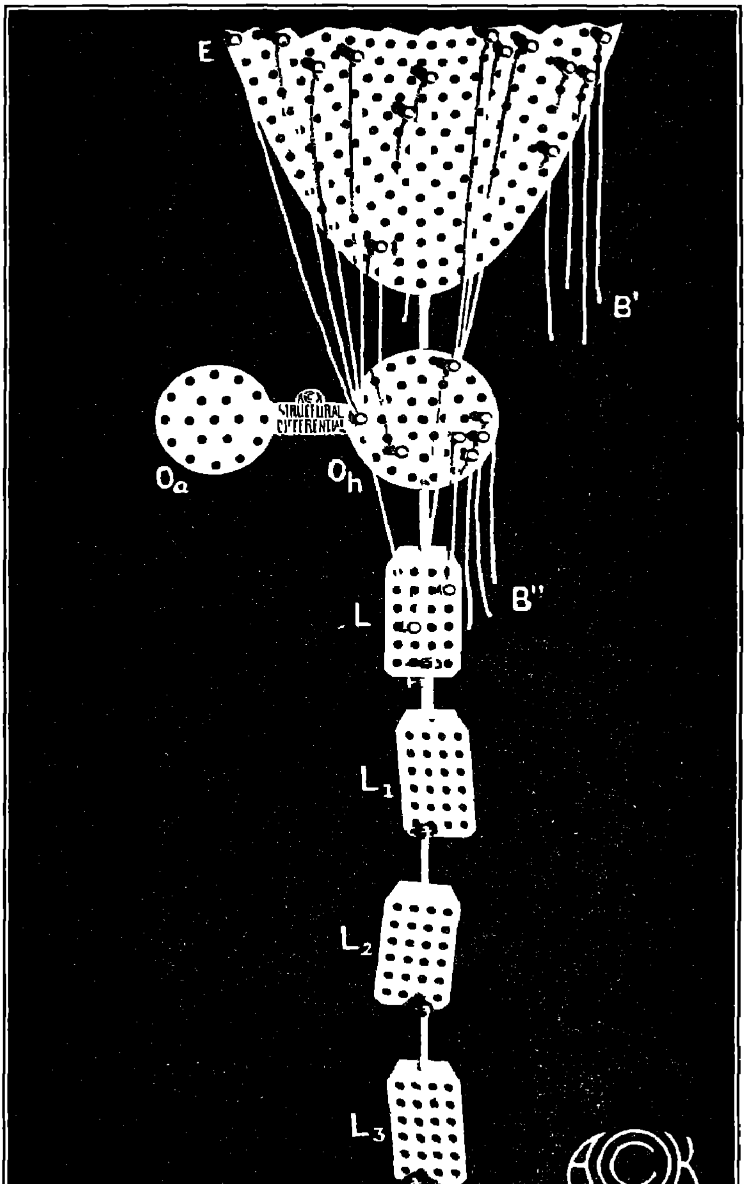


FIG 3
THE STRUCTURAL DIFFERENTIAL

and 'Fido'. Fido's *power of abstracting stops somewhere,* although it may include a few orders. Not so with 'Smith', his power of abstracting has no known limit (see Part VI).

Perhaps the reader is semantically perplexed by the unfamiliarity of the language of this analysis. It must be granted that the introduction of any new language is generally perplexing, and it is justified *only* if the new language accomplishes something structurally and semantically which the old languages did *not* accomplish. In this case, it has brought us to a new *sharp* distinction between 'man' and 'animal'. The number of orders of abstractions an 'animal' can produce is *limited*. The number of orders of abstractions a 'man' can produce is, in principle, *unlimited*.

Here is found the fundamental mechanism of the 'time-binding' power which characterizes man, and which allows him, in principle, to gather the *experiences* of all past generations. A higher order abstraction, let us say, of the $n+1$ order, is made as a response to the stimulus of abstractions of the nth order. Among 'humans' the abstractions of high orders produced by others, as well as those produced by oneself, are stimuli to abstracting in still higher orders. Thus, in principle, we start where the former generation left off. It should be noticed that, in the present analysis, we have abandoned the structurally *el* methods and language, and the whole analysis becomes simple, although non-familiar because it involves new *non-el s.r.*

The preceding explanation justifies my former statement that the ascribing of absolute numbers to the orders of abstractions of 'animal' and of 'man' is unnecessary. In our diagram we could ascribe as many orders of abstractions to the animal as we please, yet we should have to admit, for the structural correctness of description of experimental facts, that the 'animal's' power of abstracting has limits, while the number of orders of abstractions a 'man' can produce has no known limits.

From an epistemological and semantic point of view, there is an important benefit in this method. In this language, we have discovered *sharp* verbal and analytical methods, in terms of the *non-el* 'orders of abstractions', by which these two 'classes of life', or these two high abstractions, can be differentiated. The terms 'animal' and 'man' each represent a name for an abstraction of very high order, and not a name for an objective individual. To formulate the difference between these 'classes' becomes a problem of *verbal structural ingenuity and methods,* as in life we deal only with absolute individuals on the un-speakable, objective levels. In our diagram, we could hang on the 'animal' object as many levels of labels, which stand for higher order abstractions, as we please, yet somewhere we would have to stop, but with 'man' we could continue indefinitely.

This *sharp* difference between 'man' and 'animal' may be called the *'horizontal difference'*. The habitual use of *our hands* in showing these different horizontal levels is extremely useful in studying this work, and it facilitates greatly the acquiring of the structurally new language and corresponding *s.r.* The solution of the majority of human semantic difficulties (evaluation), and the elimination of pathological identification, lie precisely in the maintenance, without confusion, of the sharp differentiation between these horizontal levels of orders of abstractions.

Let us now investigate the possibility of a sharp *'vertical difference'*. We have already come to the conclusion that Fido abstracts objects from events, and that, if his nervous system is similar to ours, his lower order abstractions are similar to ours. Here we may ask the question: Does Fido 'know', or can he 'know', that he abstracts? It seems undeniable that Fido does not 'know' and *cannot 'know'* that he abstracts, *because it takes science to 'know'* that we abstract, and Fido has no science. It is semantically important that we should be entirely convinced on this point. We do not argue about the kind of 'knowledge' animals may have or about the relative value of this 'knowledge' as compared with ours. Science was made possible by the human nervous system and the invention of extra-neural means for investigation and recording, which animals lack entirely. Whoever claims that animals have science should, to say the least, show libraries and scientific laboratories and instruments produced by animals.

We see that, although Fido has abstracted, he not only does not 'know' but *cannot* 'know' that he abstracts, as this last 'knowledge' is given exclusively by science, which animals do not have. *In this consciousness of abstracting, we find a most important 'vertical difference' between Smith and Fido.* The difference is sharp again.

If, in our diagram, Fig. 4, we ascribe to Fido more horizontal orders of abstractions, let us say two, (H_1) and (H_2), nevertheless, the 'animal' stops somewhere. This extended diagram illustrates that 'man' is capable of abstracting in higher and higher orders indefinitely. In this diagram, we symbolize the fact that Fido does not and cannot 'know' that he abstracts, by not connecting the characteristics of his object (O_a) by lines (A_n) with the event (E). *Without science, we have no event, Fido's gross macroscopic object* (O_a) represents '*all that he 'knows' or cares about*'. We see that the *vertical difference* (V_1) formulated as consciousness of abstracting for Smith appears sharp, and completely differentiates Fido from Smith. In it, we find the semantic

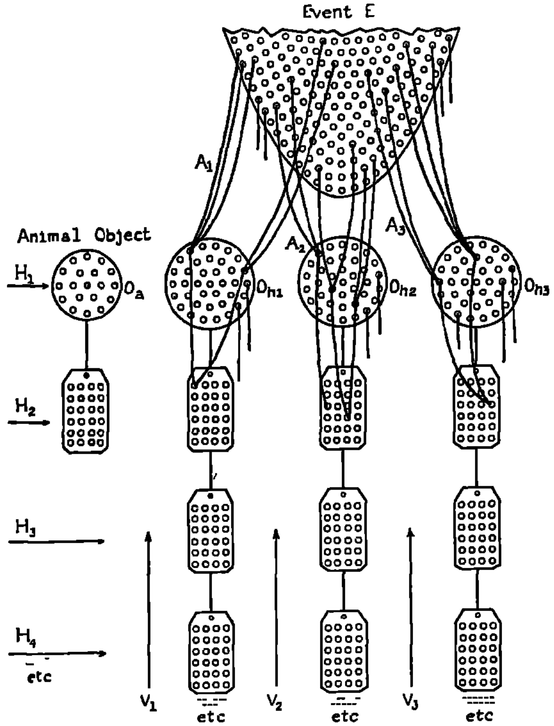


FIG 4

mechanism of all proper *evaluation,* based on *non-identification* or the differentiation between orders of abstractions, impossible with animals

In this diagram we have introduced more objects, because each individual abstracts, in general, from an event *different* objects, in the sense that they are *not identical* in every respect We must be aware continuously that in life on the un-speakable objective level we deal only with absolute individuals, be they objects, situations, or *s r* The vertical stratification not only gives us representation for the sharp difference between 'man' and 'animal', but also allows us to train our *s r* in the absolute individuality of our objects and those of different observers, and for the differences between their individual abstractions What has been said here applies equally to all first order effects on the objective level, such as immediate feelings,

The present theory can only be fully beneficial when the reader acquires in *his system* the habitual feeling of both the vertical and the horizontal stratifications with which identification becomes impossible.

In the experiments of Doctor Philip S Graven with the 'mentally' ill, training in the realization of this stratification has either resulted in complete recovery or has markedly improved the conditions of the patient

The diagram is used in *two* distinct ways One is by showing the abstracting from the event to the object, and the applying of a name to the object The other is by illustrating the level of statements which can be made about statements If we have different objects, and label them with different names, say, A_1, A_2, A_3 A_n, we still have *no* proposition. To make a proposition, we have to accept some undefined relational term, by which we relate one object to the other The use of this diagram to illustrate the *levels or orders of statements* implies that we have selected some metaphysics as expressed in our undefined relational terms We should be fully aware of the difference between these *two* uses of the one diagram for the structural illustration of two aspects of one process.

If we enquire: What do the characteristics of the event represent? We find that they are given only by science and represent at each date the highest, most verified, most reliable abstractions 'Smith' has produced

Theory and practice have shown that the points illustrated by the above structural diagrams have a crucial semantic significance, as, without using them, it is practically impossible to train ourselves or others and to accomplish the psychophysiological re-education For this reason, the diagrams have been produced for home and school use, separately, in the simplified form illustrated in Fig 5 This structural diagram is called the 'Anthropometer' or the 'Structural Differential', as it illustrates the fundamental structural difference between the world, *and so the environment,* of the animal and man If we live in such a very complex *human* world, but our *s r*, owing to wrong *evaluation,* are adjusted only to the simpler animal world, free, to say the least, from man-made complications, then adjustment and sanity for *humans* is impossible Our *s r* are bound to follow the simpler animalistic patterns, *pathological for man.* All human experience, scientific or otherwise, shows that we still copy animals in our nervous reactions, trying to adjust ourselves to a world of fictitious, simple *animal structure,* while *actually* we live in a world of very complex *human structure* which is quite different Naturally, under such conditions, which, ultimately, turn out to be delusional, *human* adjustment is impossible and results in false evaluations, animalistic *s r*, and the general state of un-sanity

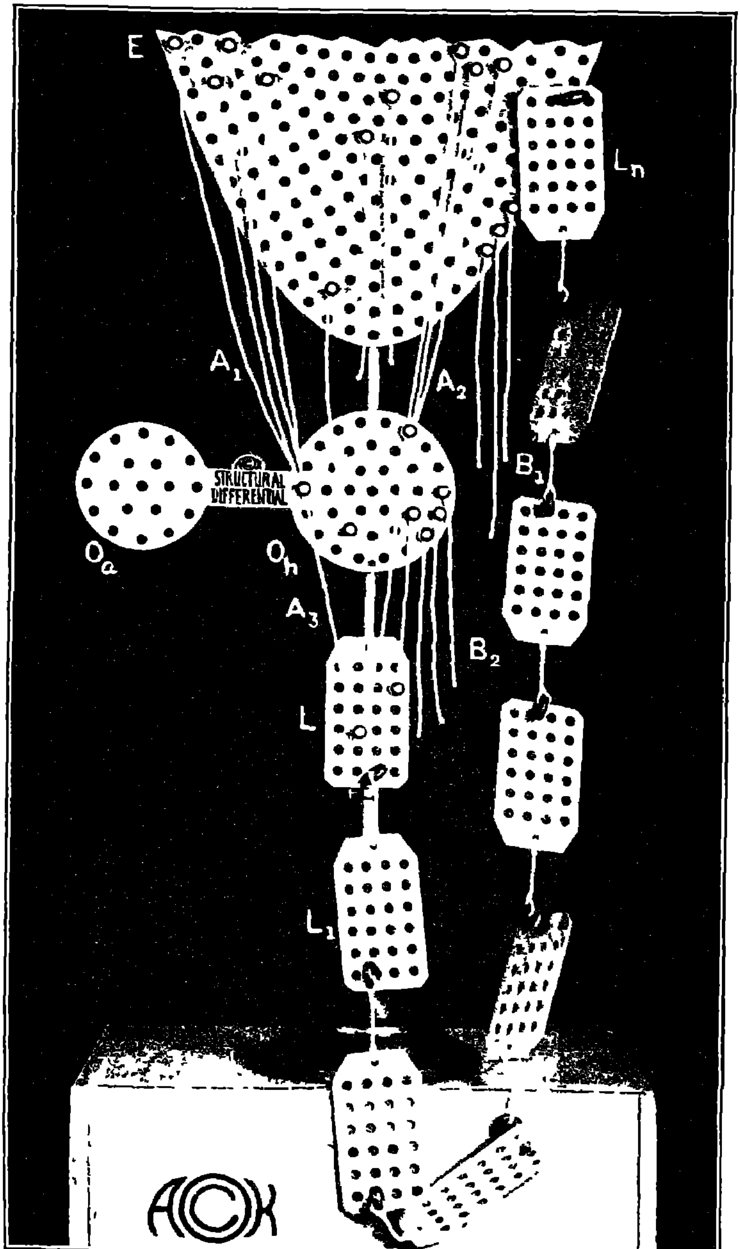


FIG 5
THE STRUCTURAL DIFFERENTIAL

Any one who will work out the present analysis with the aid of the Differential will find clearly that the majority of human difficulties, the preventable or curable 'mental' or semantic disturbances included, are due to this fatal *structural* error, resulting in false evaluation due to identification or lack of differentiation

The Structural Differentials are manufactured in two forms (1) in a printed map-like scroll for hanging on the walls or black-board, (2) in relief form with detachable labels As the main problem is to train and re-educate the *semantic psychophysiological reactions* in non-identity, the relief form is the most effective because of the freely hanging strings, detachable labels , which give means to engage more nerve centres in the training I shall describe the latter type in some detail

For the event we have a parabola in relief (E), broken off to indicate its limitless extension The disk (O_h) symbolizes the human object, the disk (O_a) represents the animal object The label (L) represents the higher abstraction called a name (with its meaning given by a definition) The lines (A_n) in the relief diagram are hanging strings which are tied to pegs They indicate the process of abstracting The free hanging strings (B_n) indicate the most important characteristics *left out,* neglected, or forgotten in the abstracting The Structural Differentials are provided with a number of separate labels attached to pegs These are hung, one to the other, in a series, and the last one may be attached by a long peg to the event, to indicate that the characteristics of the event represent the highest abstractions we have produced at each date *The objective level is not words, and cannot be reached by words alone We must point our finger and be silent, or we shall never reach this level* Our personal feelings, also, *are not* words, and belong to the objective level.

The whole of the present theory can be illustrated on the Structural Differential by the childishly simple operation of the teacher pointing a finger to the event and then to the object, saying 'This *is not* this' and insisting on silence on the pupil's part One should continue by showing with the finger the object and the label, saying again 'This *is not* this', *insisting on silence* on the objective level, then, showing the first and the second label, saying again 'This *is not* this',

In a more complex language, one would say that the object *is not* the event, that the label *is not* the un-speakable object, and that a statement about a statement *is not* the 'same' statement, nor on one level We see and are made to visualize that the $\bar{A}$-system is based *on the denial of the 'is' of identity,* which necessitates the differentiation of orders of abstractions

The little word 'to be' appears as a very peculiar word and is, perhaps, responsible for many human semantic difficulties. If the anthropologists are correct, only a few of the primitive peoples have this verb. The majority do not have it and do not need it, because all their *s.r.* and languages are practically based on, and involve, literal *identification*.[1] In passing from the primitive stage of human society to the present slightly higher stage, which might be called the infantile stage, or infantile period, too crude identification was no longer possible. Languages were built, based on slightly modified or limited identification, and, for flexibility, the 'is' of identity was introduced explicitly. Although very little has been done in the *structural* analysis of languages in general, and of those of primitive peoples in particular, we know that in the Indo-European languages the verb 'to be', among others, is used as an *auxiliary verb* and also for the purpose of positing false to facts identity. With the primitive prevalent lack of consciousness of abstracting, and the primitive belief in the magic of words, the *s.r.* were such that words were identified with the objective levels. Perhaps it is not too much to say that the primitive 'psychology' peculiarly required such a fundamental identity. Identity may be defined as 'absolute sameness in all respects' which, in a world of ever-changing processes and a human world of indefinitely many orders of abstractions, appears as a *structural* impossibility. Identity appears, then, as a primitive 'over-emotional' generalization of similarity, equality, equivalence, equipollence.., and, in no case, does it appear in fact as 'absolute sameness in all respects'. As soon as the structurally *delusional* character of identity is pointed out, it becomes imperative for sanity to eliminate such delusional factors from our languages and *s.r.* With the advent of 'civilization', the use of this word was enlarged, but some of the fundamental primitive implications and psycho-logical semantic effects were preserved. If we use the 'is' at all, and it is extremely difficult to avoid entirely this auxiliary verb when using languages which, to a large extent, depend on it, we must be particularly careful not to use 'is' as an identity term.

In 1933, the amount of knowledge we have about the primitive peoples is considerable. Anthropologists have gathered an enormous number of descriptive facts, on which they practically all agree, but the several schools of anthropology differ widely as to the interpretation of these facts. Roughly speaking, the British school tries to interpret the facts from the point of view of ascribing to the primitives the deficient 'psychology' and 'logic' of the white man. The French and Polish schools avoid these unjustified tendencies, and attempt to reconstruct the original primitive 'psychologies' and 'logics' which could be responsible for the developments, or the lack of developments, of the primitive peoples. All schools accept, as yet, the existing *el* 'psychologies' and two-valued *A* 'logic' as the standard, normal, and, perhaps, even as the final disciplines for an adult human civilization. No school suspects that an *A* stage of civilization appears to be built, to a large extent, on the slightly refined *primitive identifications* which produced only an *infantile period* of human development. They do not suspect that a future $\bar{A}$ society may differ as greatly from the present *A* society as the latter differs from the primitive society.

In my work, I prefer to follow the French and Polish schools of anthropology, as it seems to me that these schools are freer from semantic identification and aristotelianism than the others.

In 1933, it seems, beyond doubt, that *if* any single *semantic* characteristic could be selected to account for the primitive state of the individuals and their societies, we could say, without making too great a mistake, that it would be found in *identification,* understood in the more general sense as it is used in the present work. There is very little doubt, at present, that different physico-chemical factors, environment, climate, kind of food, colloidal behaviour, endocrine secretions . . . , are fundamental factors which condition the potentialities, as well as the behaviour, of an organism. It is equally certain that, as an end-result, these physico-chemical factors are connected with definite types of *s.r.* It is known that the reverse is also true, namely, that *s.r* affect colloidal behaviour, endocrine secretions, and metabolism. The exact type of dependence is not known, because too little experimenting on humans has been made. The present analysis is conducted from the semantic point of view, and its results, no matter how far-reaching, are limited to this special aspect.

Simple analysis shows that identification is a necessary condition which underlies the reactions of animals, of infants, and of primitives. If found in 'civilized' grown-ups, it equally indicates some remains of earlier periods of development, and can always be found in the analysis of any private or public difficulties which prevent any satisfactory solution. Identification in a slightly modified form represents, also, the very foundation of the *A*-system and those institutions which are founded on this system.

Mathematics gives us practically the only linguistic system free from pathological identifications, although mathematicians use this term uncritically. The more identification is eliminated from other sciences, the more the mathematical functional semantics and method are applied, and the further a given science progresses.

The best we know in 1933 is that the general structure of the world was not different in prehistoric times from what we find it today We have no doubt that the materials in great antiquity consisted of molecules, molecules of atoms, and atoms of electrons and protons , or whatever else we shall be able to discover some day We have no doubt that blood was circulating in the higher animals and humans, that vitamins exhibited very similar characteristics as today, that different forms of radiant energy influenced colloidal behaviour , , regardless of whether or not the given animal, primitive man or infant 'knew' or 'knows' about them.

How about the primitive *physical needs and wants* of an animal, a primitive man, and an infant? Besides all mystical and mythological reasons for identification, the structural facts of life *necessitated identification* on this level of development *Without* modern knowledge, what a hungry animal, primitive man, or an infant 'wants' 'is' an 'object', say, called an 'apple' He would 'define' his 'apple' the best he could as to shape, colour, smell, taste, Was this what his organism needed? Obviously not We could, at present, produce an undigestible synthetic apple which would satisfy his eventual objective definitions; he might eat it, many such 'apples', and eventually die of hunger Is an abundant and pleasant diet free from unsuspected and unseen 'vitamins' satisfactory for survival? Again, no! Thus, we see clearly that what the organism needed for survival were the physico-chemical processes, not found in the 'ordinary object', but exclusively in the 'scientific object', or the event Here we find the age old and necessary, on this early level, identification of the ordinary object with the scientific object This form of identification is extremely common even in 1933, and, to a large extent, responsible for our low development, because, no matter what we 'think' or feel about an object, an object represents *only* an abstraction of low order, only a *general symbol* for the *scientific object,* which remains the only possible survival concern of the organism. But, obviously, such identification, being false to facts, can never be entirely reliable. If any one fancies that he deals with 'ultimate reality', yet that *in o* reality represents only a shadow cast by the scientific object, he begins, with experience, to distrust the object and populates his world with delusional mysticism and mythologies to account for the mysteries of the shadow

As any organism represents an *abstracting* in *different orders* process, which, again, the animal, the primitive man, and the infant cannot know, they, by necessity, identify different orders of abstractions Thus, names are identified with the un-speakable objects, names for action with the un-speakable action itself, names for a feeling with the un-speakable feelings themselves. By confusing descriptions with inferences and descriptive words with inferential words, the 'judgements', 'opinions', 'beliefs', and similar *s.r.*, which represent mostly, if not exclusively, inferential semantic end-products, are projected with varying pathological intensity on the outside world. By this method pre-'logical' primitive semantic attitudes were built. Mere similarities were evaluated as identities, primitive syllogisms were built of the type: 'stags run fast, some Indians run fast, some Indians are stags'. It is common to find among primitive peoples a kind of 'logic' based on the *post hoc, ergo propter hoc* (after this and, therefore, because of this) fallacy which obviously represents an identification of an ordinal description with an inference. The 'question begging epithets', which exercise a tremendous semantic influence on primitive and immature peoples and represent a semantic factor in many primitive as well as modern taboos, are also based on such confusions of orders of abstractions.

Identification is one of the primitive characteristics which cannot be eliminated from the animal or the infant, because we have no means to communicate with them properly. It cannot be eliminated from primitive peoples as long as they preserve their languages and environments. Identification is extremely wide-spread among ourselves, embodied strongly in the structure of our inherited language and systems. To change that primitive state of affairs, we need special simple means, such as a $\bar{A}$-system may offer, to combat effectively this serious menace to our *s.r.* It should never be forgotten that identification is practically never dangerous in the animal world, because unaided nature plays no tricks on animals and the elimination by non-survival is very sharp. It is dangerous in the primitive stage of man, however, as it prevents the primitive man to become more civilized, but under his primitive conditions of life his dangers are not so acute. It becomes only very dangerous to the infant if not taken care of, and to the modern white man in the midst of a very far advanced industrial system which affects all phases of his life, when his *s.r.* are left unchanged from the ages gone by, and still remain on the infantile level.

The present $\bar{A}$-system is not only based on the complete rejection of the 'is' of identity, but every important term which has been introduced here, as well as the Structural Differential, is aimed at the elimination of these relics of the animal, the primitive man, and the infant in us.

Thus, the primitive 'mentality' does *not differentiate relations* enough; to counteract this, I introduce the *Structural Differential*. The primitive identifies; I introduce a system based on the denial of the 'is' of identity all through. The primitive man pays most attention to what is conveyed to him through the eye and the ear, I introduce the Structural Differential which indicates to the eye the stratification of human knowledge, which represents to the eye the verbal denial of the 'is' of identity. If we identify, we do not differentiate. If we differentiate, we cannot identify; hence, the Structural Differential.

The terms used also convey similar processes. Once we have *order,* we differentiate and have orders of abstractions Once we abstract, we eliminate 'allness', the semantic foundation for identification. Once we abstract, we abstract in different orders, and so we *order,* abolishing fanciful infinities Once we differentiate, differentiation becomes the denial of identity Once we discriminate between the objective and verbal levels, we learn 'silence' on the un-speakable objective levels, and so introduce a most beneficial neurological 'delay'—engage the cortex to perform its natural function. Once we discriminate between the objective and verbal levels, structure becomes the only link between the two worlds This results in search for similarity of structure and relations, which introduces the aggregate feeling, and the individual becomes a *social being* Once we differentiate, we discriminate between descriptions and inferences Once we discriminate, we consider descriptions separately and so are led to *observe* the facts, and only from description of facts do we tentatively form inferences,. Finally, the consciousness of abstracting introduces the general and permanent differentiation between orders of abstractions, introduces the ordering, and so stratifications, and abolishes for good the primitive or infantile identifications The semantic passing from the primitive man or infantile state to the adult period becomes a semantic, accomplished fact It should be noticed that these results are accomplished by starting with primitive means, the use of the simplest terms, such as 'this *is not* this', and by the direct appeal to the primitive main receptors—the eye and the ear.

The elimination of the 'is' of identity appears as a serious task, because the *A*-system and 'logic' by which we regulate our lives, and the influence of which has been eliminated only partially from science, represent only a very scholarly formulation of the restricted primitive identification Thus, we usually assume, following *A* disciplines, that the 'is' of identity is fundamental for the 'laws of thought', which have been formulated as follows:

1) The Law of Identity: whatever is, is

2) The Law of Contradiction nothing can both be and not be.

3) The Law of Excluded Middle everything must either be or not be

It is impossible, short of a volume, to revise this 'logic' and to formulate a $\bar{A}$, ∞-valued, *non-elementalistic* semantics which would be structurally similar to the world and our nervous system, but it must be mentioned, even here, that the 'law of identity' is never applicable to processes The 'law of excluded middle', or 'excluded third', as it is sometimes called, which gives the two-valued character to A 'logic', establishes, as a general principle, what represents only a limiting case and so, *as a general principle,* must be unsatisfactory As on the objective, un-speakable levels, we deal exclusively with absolute individuals and individual situations, in the sense that they are not identical, all statements which, by necessity, represent higher order abstractions must only represent *probable* statements Thus, we are led to ∞-valued semantics of probability, which introduces an inherent and general principle of uncertainty.

It is true that the above given 'laws of thought' can and have been expressed in other terms with many scholarly interpretations, but fundamentally the semantic state of affairs has not been altered.

From a *non-el* point of view, it is more expedient to treat the A-system on a similar footing with the E-system, namely, to consider the above 'laws of thought' as postulates which underlie that system and which express the 'laws of thought' of a given epoch and, eventually, of a race We know other systems among the primitive peoples which follow other 'laws', in which identity plays a still more integral part of the system. Such natives reason quite well, their systems are consistent with their postulates, although these are quite incomprehensible to those who try to apply A postulates to them From this point of view, we should not discuss how 'true' or 'false' the A-system appears, but we should simply say that, at a different epoch, other postulates seem structurally closer to our experience and appear more expedient Such an attitude would not retard so greatly the appearance of new systems which will supersede the present $\bar{A}$-system

In the present system, 'identification' represents a label for the semantic process of inappropriate evaluation on the un-speakable levels, or for such 'feelings', 'impulses', 'tendencies', As in human life, we deal with many orders of abstractions, we could say in an ordinal language that identification originates *or* results in the confusion of orders of abstractions This conclusion may assume different forms · one represented by the identification of the scientific object or the event with the ordinary object, which may be called ignorance, pathological to *man;* another, the identification of the objective levels with the verbal levels, which I call objectification, a third, the identification of descriptions with inferences, which I call confusion of higher order abstractions In the latter case, we should notice that inferences involve usually more intense semantic components, such as 'opinions', 'beliefs', 'wishes'., than descriptions These inferences may have a definite, objective, un-speakable character and may represent, then, a semantic state which *is not* words, and so objectifications of higher order may be produced

When we introduce the ordinal language, we should notice that under known conditions we deal with an ordered natural series, namely, events first, object next; object first, label next, description first, inferences next, This order expresses the natural importance, giving us the natural base for evaluation and so for our natural *human s r* If we identify two different orders, by necessity, we evaluate them equally, which always involves errors, resulting potentially in semantic shocks As we deal in life with an established natural order of values which can be expressed, for my purpose, by a series decreasing in value: events or scientific objects, ordinary objects, labels, descriptions, inferences , identification results in a very curious semantic situation

Let us assume that the scientifically established value of any level could be expressed as 100, and the value of the next as 1. With the consciousness of abstracting we could not disregard, nor identify, these values, nor forget that $100 > 1$ If we confuse the orders of abstractions, this can be expressed as the identification in value and we have a *semantic* equation · (1) $100 = 100$, or (2) $1 = 1$, or any other number, say (3) $50 = 50$

As we deal fundamentally with a natural, directed inequality, say, $100 > 1$, and, under some semantic pressure, 'want', 'wishful thinking', or ignorance, or lack of consciousness of abstracting, or 'mental' illness , we identified the two in value, we produce in the first and third cases an *over*-evaluation on the right-hand side, and, in the second and third cases, an *under*-evaluation on the left-hand side. Thus, on the *semantic level,* any identification of *essentially different in value* different orders of abstractions, appears as the *reversal* of the natural order of evaluation, with different degrees of intensity If the *natural* order of scientific evaluation would be $100 > 1$, and we would evaluate through identification as $2 = 2$, or $3 = 3$, $50 = 50$., $100 = 100$, we would be ascribing twice, or three times, or fifty times, or a hundred times , more *delusional* values to the right-hand side and under-evaluate the left-hand side, than the natural order of evaluation would require Nature exhibits, in my language and in this field, an asymmetrical relation of 'more', or 'less' inaccessible to *A* procedure Under the influence of aristotelianism, when, through identification, we ascribe to nature delusional values, adjustment becomes very difficult, particularly under modern complex life-conditions

The above example indicates the degrees of intensity which we find in life in the reversal of the natural order of evaluation through identification, produced by, and resulting in, the lack of consciousness of abstracting Un-sanity, which affects practically all of us, represents the reversal of lesser intensity, the reversal of greater intensity—the more advanced 'mental' ills.

We should realize that *experimentally* we find in this field a fundamental difference in value, which, on semantic levels, can be expressed as an asymmetrical relation of 'more' or 'less', establishing some natural order. If any one should claim a natural 'identity', the burden of proof falls on him. If 'absolute sameness in all respects' cannot be found in this world, then such a notion appears as false to facts, and becomes a structural falsification, preventing sanity and adjustment If he accepts the fundamental, natural differences in value, but prefers to assume a different order of evaluation depending on his metaphysics, be it the *elementalistic* materialism, or equally *elementalistic* idealism, the semantic results are not changed, because identification in the second case would also ascribe delusional identity to essentially different orders of abstractions. It should be noticed that the $\bar{A}$ formulation applies equally to the older different, opposite doctrines and renders them illegitimate on similar grounds

The status of the event, or the scientific object, is slightly more complex, because the event is *described* at each date by very reliable, constantly revised and tested, *hypothetical,* structural, inferential terms, exhibiting the peculiar circularity of human knowledge If we should treat these inferential structures, not as hypothetical, but should identify them semantically with the eventual processes on the level of the submicroscopic event, we would have semantic disturbances of identification

I have selected the above given order, not only for convenience and simplicity, but because of its experimental character When we identify in values, we always exhibit in our *s r* the reversed natural order, introduced here on space-time structural and evaluational grounds

The above analysis represents a very rough outline, but is sufficient for my purpose Any attentive and informed reader can carry it further as far as desired. The main point appears that different orders of abstractions exhibit different characteristics, and so any identification of entities essentially different in one or more aspects must introduce delusional semantic factors I speak mostly about evaluation, because evaluation appears experimentally as an essential factor in all *s r* and can be applied even profitably in those cases of 'mental' illness where no definite evaluation appears, the *absence* of evaluation being a form of evaluation (*m o*) In training, it is of utmost importance to eliminate identification entirely, which invariably appears as a delusional semantic factor To achieve these ends, all and every available means should be employed.

When one studies carefully the older disciplines, one is amazed to learn to what an extent the recorded 'thinkers' rebelled against the limitations and insufficiencies of aristotelianism, which system, naturally, became antiquated a short time after its formulation. One is amazed to find that 'everything has already been said', and that, to a large extent, these important, separated statements were *inoperative*. It is of little importance that some 'wise statements' had been made by some one, somewhere, if they had no influence on the great masses of the race. The reason for this tremendous public waste of private efforts is that aristotelianism, with its further elaborations and its delusional identification, elementalism., represents a co-ordinated *system* which moulded our *s r*, languages, and institutions, and which influenced every phase of our lives Under such conditions, isolated doctrines, no matter how wise, become powerless in the face of such a system, or, more correctly, a system of interlocked systems Only a revision of the system and the tentative formulation of a $\bar{A}$-system can make many older fundamental clarifications workable, which, although known to a few specialists, appear generally unknown to the great masses and unavailable in elementary education, which alone can be generally effective One is also amazed at the power of structurally correct terminology, and feels full of sympathy toward the primitive interpretation as the 'magic of words'! Happy, structural high abstractions really have a strong creative character Since, for instance, the principle of 'least action', or the 'general principle of relativity' (the theory of the absolute) , have been formulated, all of our structural knowledge has been recast, clarified, and we constantly hear of some remarkable applications of the new knowledge Similarly, if it is pointed out that our main private and public difficulties are due to infantilism produced by 'aristotelianism', in general, and, in particular, by identification and elementalism, we at once have practical means for a revision and applications In such a first and novel attempt over-subtlety is impossible and even not desirable It is preferable, as well as expedient, to formulate the general outline and, thereby, draw more men into the work for the details

For thousands of years, millions upon millions of humans have used a great deal of their nervous energy in worrying upon delusional questions, forced upon them by the pernicious 'is' of identity, such as:

'What *is* an object?', 'What *is* life?', 'What *is* hell?', 'What *is* heaven?', 'What *is* space?' 'What *is* time?', and an endless array of such irritants The answer, based on the human discrimination of orders of abstractions and so proper *human evaluation,* is definite, undeniable, simple, and *unique* 'Whatever one might *say* something *"is"*, *it is not'* Whatever we might *say* belongs to the verbal level and *not* to the un-speakable, objective levels.

Let me repeat once more that the 'is' of identity forces us into semantic disturbances of wrong *evaluation* We establish, for instance, the *identity* of the un-speakable objective level with words, which, once stated, becomes obviously false to facts The 'is' of identity, if used as indicating 'identity' (structurally *impossible* on the objective levels), says nothing Thus, the question, 'What *is* an object?', may be answered, 'An object *is* an object'—a statement which says nothing If used in definitions or classifications, such as 'Smith *is* a man', a type of statement used even in the *Principia Mathematica,* or 'A *is* B or not B', as in the formulation of the law of 'excluded third' in the two-valued *A* 'logic', it always establishes an *identity,* false to facts The first statement expresses the *identity* of a proper name with a class name which must lead to the confusion of classes (higher order abstractions) with individuals (lower order abstractions) This confusion leads automatically to disturbed evaluation in life, because the characteristics of a class are *not* the 'same' as, nor identical with, the characteristics of the individual I shall not analyse in detail the 'A *is* B', because, obviously, it *is not.*

How about Fido? Fido has no science and, therefore, no 'event' For him, the object is *not* an abstraction of some order, but *'is all'* he 'knows' and cares about Smith not only abstracts in indefinite numbers of different orders, and does it automatically and habitually, but if he enquires he may also become *conscious of abstracting*—*'is not all'*, and *'this is not this'* Now, Fido can *never* be conscious of abstracting, as his nervous system is incapable of being extended by extra-neural means, and this extension appears to be a necessary condition for the acquiring of *consciousness of abstracting*

Although for Smith, 'This *is not* this', as illustrated on the Structural Differential, for Fido, that diagram would eventually mean 'this *is* this', the structure of his world being represented by the single disk (O_a) Fido cannot be conscious of abstracting, he *must* identify, because he 'knows' nothing of this process, and there is no means of informing him of these relations and structure

If we *are not conscious* of abstracting, we must identify—in other words, whenever we confuse the different orders of abstractions, unavoidable if we use the 'is' of identity, we duplicate or copy the animal way of 'thinking', with similar 'emotional' responses. In the following chapters, this tragedy will be explained in detail, and it will be shown that practically all human difficulties involve this semantic factor of copying animals in our nervous reactions and evaluation as a component.

A theory which not only throws light on this serious problem, but which also gives means of replacing the old harmful s.r by more beneficial ones, may be useful, in spite of various temporary difficulties which are due to the old identity-reactions and the lack of familiarity with the new.

The old identity-reactions are extremely ingrained, particularly with grown-ups. Serious effort and permanent reminders are necessary to overcome them. The Structural Differential represents such a structural visual reminder, which we should keep constantly before our eyes until the pernicious disturbances of evaluation have been overcome. For Smith, the fundamental evaluation can be expressed in simple and quite primitive language—'This *is not* this'.

The above most vital semantic factors of evaluation indispensable for adjustment and sanity are conveyed to him whenever he looks at the stratification indicated on the Differential. The hanging free strings indicating the *non-abstracted* characteristics train his *s.r* to be aware of the non-allness of, and the lack of identity between, his abstractions.

Our old *s.r* were similar to Fido's, we were never *fully* conscious of abstracting. Through wrong evaluation we identified what is inherently different and longed for, or assumed some impossible 'allness' in our 'knowings'.

Practice has shown me, definitely, that to acquire these new reactions of *consciousness of abstracting* is difficult and requires 'time' and effort to accomplish, in spite of the exceptional, nearly primitive, simplicity of the means employed. The 'silence on the objective levels' sounds very innocent, yet it is extremely difficult to acquire, as it involves a complete checking of all semantic disturbances, identifications, confusions of orders of abstractions, habitual 'emotions', 'preconceived ideas', practically impossible without the use of the *objective* Differential to which we can point our finger and be silent, to begin with. In fact, to disregard this point, actually means failure in accomplishing the desired semantic results. At present, as far as experience has gone, the main results were achieved when a given individual had conquered this first, simple, and obvious semantic obstacle If the simple rules and conditions given in the present system for abolishing identification are followed persistently in the training with the Differential, a complete and very beneficial structural and semantic change in the character and 'mental' capacities of a given individual occurs, seemingly all out of proportion with the simplicity of the training But if we consider the content of all knowledge as uniquely *structural*, and if the majority of us are semantically tied up, blocked, with antiquated, animalistic, primitive, infant-like, 'mentally'-ill and *A* structure and identity-reactions, owing to the lack of consciousness of abstracting, which we renounce *in toto* by acquiring the consciousness of abstracting, such remarkable transformation becomes intelligible

The publication of the Structural Differential in separate, conveniently large copies has been forced upon me by experience and by various difficulties found in the re-educating of our *s r*, without which a $\bar{A}$-system, adjustment, sanity, and all the desirable results which depend on them, are impossible

CHAPTER XXVI

ON 'CONSCIOUSNESS' AND CONSCIOUSNESS OF ABSTRACTING

> But a felt 'contrary' is consciousness in germ. . . . Consciousness requires more than the mere entertainment of theory. It is the feeling of the contrast of theory, as *mere* theory, with fact, as *mere* fact This contrast holds whether or no the theory be correct (578) A. N WHITEHEAD

A language, to be most useful, should be similar in its structure to the structure of the events which it is supposed to represent The language of 'abstractions of different orders' appears to be satisfactory in point of structure It is a *non-el* language, since it does not discriminate between 'senses' and 'mind', It is a functional language, since it describes, by implication, what is going on in the nervous system when it reacts to stimuli It is a language which can be made as flexible and as sharp as desired, thus making it possible to establish sharp verbal differences, of both horizontal and vertical type, between the terms 'man' and 'animal'

The last semantic characteristic of potential sharpness is extremely important for a theory of sanity Evidence of 1933 leads us to conclude that, under the influence of external stimuli, the most primitive and simplest forms of life were moulded, transformed, and influenced in the process of survival, and, therefore, of adjustment In this way, more and more complex structures evolved It should be emphasized that organisms represent *functional* units, and that an additive change in structure does not necessarily involve a *simply additive* change in function By physico-chemical, structural, colloidal necessity the organism works as-a-whole Being a relative whole, any additive structural factor becomes a reactive and functional factor which influences the working of the whole This is, perhaps, best illustrated by the boy who was born without a cortex, but with no other obvious defects He was incomparably more helpless and unadjusted than animals who have no cortex, or even no nervous system at all Although we could speak in *additive* terms of the difference between this boy and a normal boy, as one having no cortex and the other 'plus a cortex', yet the functioning was so different as not to be expressible in a 'plus' language.

Similar remarks could be generalized to all life We must be very careful in building sharp distinctions, since the anatomical differences alone are unreliable If we want to have more reliable differences, we should look for *functional* differences.

We have already discovered *functional* differences that are expressed by the horizontal and by the vertical differences between the abstracting capacities of Smith and Fido. The analysis of these differences is the subject of the present chapter.

'Thought' represents a reaction of the organism-as-a-whole, produced by the working of the whole, and influencing the whole From our daily experience, we are familiar with what we usually denote as being 'conscious', in other words, we are aware of something, be it an object, a process, an action, a 'feeling', or an 'idea' A reaction that is very habitual and semi-automatic is not necessarily 'conscious'. The term 'consciousness', taken separately, is not a complete symbol, it lacks content, and one of the characteristics of 'consciousness' is to have some content Usually, the term 'consciousness' is taken as undefined and *undefinable,* because of its immediate character for every one of us Such a situation is not desirable, as it is always semantically useful to try to define a complex term by simpler terms We may limit the general and undefined term 'consciousness' and make it a definite symbol by the deliberate ascribing of some content to this term For this 'consciousness of something' I take 'consciousness of abstracting' as fundamental. Perhaps the only type of meanings the term 'consciousness' has is covered by the functional term 'consciousness of abstracting', which represents a general process going on in our nervous system Even if this is not the only type of meanings, the term 'consciousness of abstracting' appears to be of such crucial semantic importance that its introduction is necessary

The term 'consciousness', because of its hitherto undefined and traditionally *undefinable* character, did not allow us further analysis. Neither did we have any *workable,* educational, semantic means to handle the vast field of psycho-logical processes which this incomplete symbol indicated If we now select the term 'consciousness of abstracting' as fundamental, we not only make the last symbol complete by assigning functional content to it, but we also find means to define it more specifically in *simpler terms* Through understanding of the processes we gain educational means of handling and influencing a large group of semantic psycho-logical reactions.

Let us analyse this new term by aid of the diagram called the Structural Differential referred to in the previous chapter Here the object (O_h) represents a nervous abstraction of a low order In this abstracting, some characteristics of the event were missed or not abstracted, these are indicated by the not connected lines (B′) When we abstracted from our object further, by coining a definition or ascribing 'meanings' to the label (L), again we did not abstract 'all' the characteristics of the object into the definition, but some characteristics were left out, as indicated by the lines (B") In other words, the number of characteristics which we ascribe to the label, by some process of 'knowing', or 'wanting', or 'needing', or 'interest', does *not* cover the number of characteristics the object has The 'object' has more characteristics than we can include in the explicit or implicit definition of the label for the 'object' Besides, the definition (implicit or explicit) of the 'object' *is not* the object itself, which always holds many surprises for us The latter has the '*individuality of the object*', as we may call it Every one who uses a car, or a gun, or a typewriter, or who has had a number of wives, or husbands, or children, knows that well In spite of the fact that these objects are, to a large extent, standardized, every individual object has individual peculiarities With modern methods of physical, chemical, and astronomical investigation, scientists find that even their special materials and equipments have also peculiar individualities which must be taken into account in the more refined researches

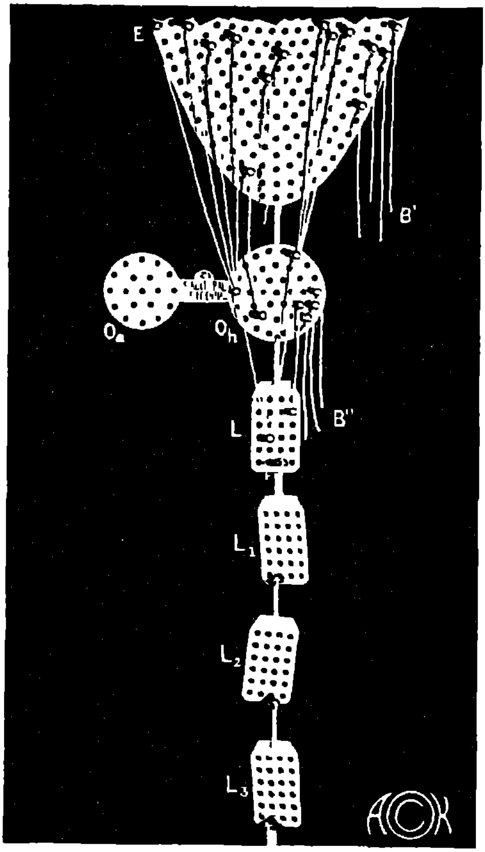


FIG 1
THE STRUCTURAL DIFFERENTIAL

If we take any ordinary object and expect to find such and such characteristics, ascribed to the objects by *definition,* we may be disappointed As a rule, we find or can find, if our analysis is subtle enough, these peculiar individualities The reader can easily convince himself by looking over a box of matches, and by noticing the peculiar individuality of each match. But since, *by definition*, we expect that when we strike a match it should ignite, we may disregard all other characteristics as irrelevant for our purpose. A similar process is at work in other phases of life. We often live, feel happy or unhappy, *by what actually amounts to a definition,* and not by the empirical, individual facts less coloured by semantic factors. When Smith$_1$ marries Smith$_2$, they mostly do so *by a kind of definition*. They have certain notions as to what 'man', 'woman', and 'marriage' 'are' *by definition*. They actually go through the performance and find that the Smith$_1$ and his wife, Smith$_2$, have unexpected likes, dislikes, and particularities—in general, characteristic and semantic reactions *not included* in their definition of the terms 'man', 'woman', 'husband', 'wife', or 'marriage'. Characteristics 'left out' in the definitions make their appearance. 'Disappointments' accumulate, and a more or less unhappy life begins.

The above analysis applies to all phases of human life, and appears entirely general because of the structure of 'human knowledge'. Characteristics are discovered when it is *too late*. The *not knowing* or the *forgetting* of the relations explained above does the semantic havoc. On verbal, 'definitional', or doctrinal semantic grounds, we expect something else than what the experiences of life give us. The non-fulfillment of expectation produces a serious affective and semantic shock. If such shocks are repeated again and again, they disorganize the normal working of the nervous system, and often lead to pathological states. An indefinitely large number of experimental facts fully supports the above conclusions. Many of them have been supplied during the World War. Curiously enough, when the soldier *did* expect horrors, and later experienced them, he seldom became deranged 'mentally'. If he did not fully expect them, and yet had to experience them, he often broke down nervously.

The attack of hay fever at the sight of *paper roses*, referred to already, gives a similar semantic example. The attack followed from the semantic *'definition'* of 'roses', of 'hay fever', and from the situation as-a-whole, and was not due to *inspection* of the objective 'roses', or to the physico-chemical action of the 'roses'. If the patient had been blindfolded when the paper 'roses' were brought into his presence, no attack would have occurred.

We are now ready to define 'consciousness of abstracting' in *simpler terms,* namely, in terms of 'memory'. The term 'memory' is structurally a physico-chemical term. It implies that the events are interconnected, that everything in this world influences everything else, and that happenings leave some traces somewhere

A similar analysis can be carried on in connection with the object and the event Briefly, the object represents structurally an abstraction of some order, does not, and cannot, include all the characteristics of the event, and so, again, we have some characteristics *left out* as indicated by the lines (B′)

Here we have the possibility of making a series of most general, and yet entirely true, *negative* statements of great semantic importance, that the label *is not* the object, and that the object *is not* the event, For the number of *m o* characteristics which we ascribe to the label by *definition* does not cover all the characteristics we recognize in the object, and the number of characteristics which we perceive in the object is also not equal to the infinite numbers of characteristics the event has The differences are still more profound Not only do the numbers of *m o* characteristics differ, but also the *character* of these abstractions differs from level to level of the successive abstractions

We can now define 'consciousness of abstracting' as '*awareness* that in our process of abstracting we have *left out* characteristics' Or, consciousness of abstracting can be defined as '*remembering* the "*is not*", and that some characteristics have been *left out*' It should be noticed that in this formulation, with the aid of the Structural Differential, we have succeeded in translating a *negative* process of forgetting into a *positive* process of *remembering* the denial of identity and that characteristics are left out Such a positive formulation makes the whole system workable and available for the semantic training and education

The use of the Structural Differential becomes a necessity for any one who wants to receive full semantic benefit from the present work A book is, by necessity, *verbal* Whatever any author can say is verbal, and nothing whatsoever can be *said* which is *not verbal* It seems entirely obvious that in life we deal with an enormous number of things and situations, 'feelings' , which are *not verbal* These belong to the 'objective level'. The crucial difficulty is found in the fact that whatever can be said *is not* and *cannot* be on the objective level, but belongs *only* to the verbal levels This difference, being *inexpressible* by words, cannot be expressed by words We must have *other means* to indicate this difference We must show with our hand, by pointing our finger to the object, and by being silent outwardly as well as inwardly, which silence we may indicate by closing our lips with the other hand The verbal denial of the 'is' of identity covers this point also when shown on the Differential If we burst into speech based on the 'is' of identity, as we usually do, we find ourselves obviously on the verbal levels indicated by the labels L, L_1, L_2, . . . L_n, but never on the objective level (O_h). On this last level, we can look, handle . . ., but *must be silent*. The reason that we nearly all identify the two levels is that it is impossible to train an individual in this semantic difference by *verbal means alone*, as all verbal means belong to the levels of labels and never to the objective un-speakable levels. With a visual and tactile *actual object* and labels on the Structural Differential, to point our finger at, handle . . ., we now have simple means to convey the tremendously important semantic difference and train in *non-identity*.

We should notice that the consciousness of abstracting, or the remembering that we abstract in different orders with omission of characteristics, depends on the denial of the 'is' of identity and is connected with limitations or 'non-allness', so characteristic of the new non-systems.

The consciousness of abstracting eliminates *automatically* identification or 'confusion of the orders of abstractions', both applying to the semantic confusion on all levels. If we are *not* conscious of abstracting, we are bound to identify or confuse the object with its finite number of characteristics, with the event, with its infinite numbers of *different* characteristics. Confusion of these levels may misguide us into semantic situations ending in unpleasant shocks. If we acquire the consciousness of abstracting, and remember that the object *is not* the event and that we have abstracted characteristics fewer than, and different from, those the event has, we should expect many unforeseen happenings to occur. Consequently, when the unexpected happens, we are saved from painful and harmful semantic shocks.

If, through lack of consciousness of abstracting, we identify or confuse words with objects and feelings, or memories and 'ideas' with experiences which belong to the un-speakable objective level, we identify higher order abstractions with lower. Since this special type of semantic identification or confusion is extremely general, it deserves a special name. I call it *objectification*, because it is generally the confusion of words or verbal issues (memories, 'ideas' . . .) with objective, un-speakable levels, such as objects, or experiences, or feelings . . . If we objectify, we *forget*, or we *do not remember* that words *are not* the objects or feelings themselves, that the verbal levels are always different from the objective levels. When we identify them, we disregard the inherent differences, and so proper evaluation and full adjustment become impossible.

Similar semantic difficulties arise from the confusion of higher order abstractions, for instance, the identification of inferences with descriptions This may be made clearer by examples In studying these examples, it should be remembered that the organism acts as-a-whole, and that 'emotional' factors are, therefore, always present and should not be disregarded In this study, the reader should try to put himself '*emotionally*' in the place of the Smith we speak about, then he cannot fail to understand the serious semantic disturbances these identifications create in everybody's life

Let us begin with a Smith who knows nothing of what has been said here, and who is *not* conscious of abstracting For him, as well as for Fido, there is, in principle, no realization of the 'characteristics left out' He is 'emotionally' convinced that his words entirely cover the 'object' which '*is* so and so' He identifies his lower abstractions with characteristics left out, with higher abstractions which have all characteristics included He ascribes to words an entirely false value and certitude which they cannot have He does not realize that his words may have different meanings for the other fellow He ascribes to words 'emotional' *objectivity* and value, and the verbal, *A* 'permanence', 'definiteness', 'one-value' , to objects. When he hears something that he does not like, he does not ask 'what do you mean?', but, under the semantic pressure of identification, he ascribes his own meanings to the other fellow's words For him, words '*are*' 'emotionally' overloaded, objectified semantic fetishes, even as to the primitive man who believed in the 'magic of words' Upon hearing anything strange, his *s r* is undelayed and may appear as, 'I disagree with you', or 'I don't believe you', There is no reason to be dramatic about any unwelcome statement One needs definitions and interpretations of such statements, which probably are correct from the speaker's point of view, if we grant him his informations, his *undefined terms,* the structure of his language and premises which build up his *s r* But our Smith, innocent of the 'structure of human knowledge', has mostly a semantic belief in the one-value, absoluteness , of things, and thinghood of words, and does not know, or does not *remember,* that words *are not* the events themselves Words represent higher order abstractions manufactured by higher nerve centres, and objects represent lower order abstractions manufactured by lower nerve centres Under such *identity-delusions,* he becomes an absolutist, a dogmatist, a finalist, He seeks to establish 'ultimate truths', 'eternal verities' , and is willing to fight for them, never knowing or remembering, otherwise forgetting, the 'characteristics left out', never recognizing that the noises he makes *are not* the objective actualities we deal with If somebody contradicts him, he is much disturbed Forgetting characteristics left out, he is always 'right' For him his statement is not only *the* only statement possible, but he actually attributes some cosmic objective evaluation to it

The above *description* is unsatisfactory, but cannot be much improved upon, since the situation involves *un-speakable* affective components which *are not* words We must simply try to put ourselves in his place, and to live through his experiences when he identifies and believes without question that his words *'are'* the things they only stand for To give the full consequences of such identification resulting in wrong evaluation, I might add most tedious *descriptions* of the interplay of situations, evaluations , in quarrels, unhappinesses, disagreements , leading to dramas and tragedies, as well as to many forms of 'mental' illness effectively described only in the *belles-lettres* Thus, $Smith_1$, who is *not* conscious of abstracting, makes the statement, 'A circle is not square' Let us suppose that $Brown_1$ contradicts him $Smith_1$ is angered, for his *s r*, his statement 'is' the 'plain truth', and $Brown_1$ must be a fool. He objectifies it, ascribes to it undue value For him, it 'is' 'experience', a 'fact' , and he bursts into speech, denouncing $Brown_1$ and showing how wrong he 'is' From this semantic attitude, many difficulties and tragedies arise

But if $Smith_2$ (conscious of abstracting) makes the statement, 'A circle is not square', and $Brown_2$ contradicts him, what would $Smith_2$ do? He would smile, would not burst into speech to defend *his* statement, but would ask $Brown_2$, 'What do you mean? I do not quite understand you' After receiving some answer, $Smith_2$ would explain to $Brown_2$ that his statement is not anything to quarrel about, as it is verbal and is true only by *definition* He would also grant the right of $Brown_2$ *not* to accept *his definition,* but to use another one to satisfy himself The problem would then, naturally, arise as to what definition both could accept, or which would be generally acceptable And the problem would then be solved by purely pragmatic considerations Words appear as creatures of definitions, and optional, but this attitude involves important and new *s r*

This fact seems of tremendous semantic importance, as it provides the working foundation for a theory of 'universal agreement' In the first part of the above example, $Smith_1$, according to the accepted standards, was 'right' ('a circle is not square') Is he 'more right' than $Brown_1$, for whom the 'circle is square'? Not at all Both statements belong to the verbal level, and represent only forms of representation for *s r inside their skin* Either may be 'right' by some explicit or implicit 'definitions' Are the two statements equally valid? This *we do not know a priori,* we must investigate to find out if the noises uttered have meanings outside of pathology, or which statement structurally covers the situation better, carries us structurally further in describing and analysing this world,. Only scientific structural analysis can give the preference to one form over another Smith and Brown can only produce their 'definitions' according to their *s r*, but they are *not* judges as to which 'definitions' will *ultimately* stand the test of structure

The moment we eliminate identification we become conscious of abstracting, and permanently and instinctively remember that the object is *not* the event that the label is *not* the object, and that a statement about a statement is *not* the first statement, thus, we reach a semantic state, where we recognize that everybody 'is right' by his own 'definitions' But any individual or unenlightened public opinion is not the sole judge as to what 'definitions' and what language should prevail. Only structural investigation (science) can decide which appears as the structurally more similar form of representation on the verbal levels for what is going on on the un-speakable, objective levels

When it comes to 'description of facts', the situation is not fundamentally altered Mistakes seem always possible and often occur Besides, the semantic impressions which 'facts' make on us are also individual, and often in conflict, as comparison of the testimonies of eye-witnesses shows But there is no need for permanent disagreement, more structural investigation of the objective and verbal levels will provide a solution Once such an investigation is carried far enough, we can always reach a semantic basis where all may agree, provided we do not identify, do not objectify, and do not confuse description and inference, descriptive and inferential words,

As our analysis is carried out from the structural and *non-el* point of view, we should not miss the fact that semantic components associated with words and statements are, outside of very pathological cases, never entirely absent, and become of paramount importance In the older days, we had no simple and effective means by which we could affect painful, misplaced, or disproportionate evaluations, meanings , through a semantic re-education, which are supplied by the present analysis and the use of the Structural Differential The means to eliminate identification consist of first, an *objective* relief diagram to which we can *point our finger*, and second, a convincing explanation (pointing the finger to the labels) that the verbal levels, with their distressing and disastrous older *s r*, *are not*, and differ entirely from, the levels of objects and events Whatever we may say or feel, the objects and events remain on the un-speakable levels and cannot be reached by words Under such natural structural conditions, we can only reach the objective level by seeing, handling, actually feeling , and, therefore, by pointing our finger to the object on the Structural Differential and being silent—all of which cannot be conveyed by words *alone*

In experiments with the 'mentally' ill in whom the semantic disturbances were very strong, it took several months to train the patients in non-identity and in silence on the objective levels But, as soon as this was achieved, either complete or partial relief followed

The main disturbances in daily life, as well as in 'mental' illness, are found in the affective field We find an internal pressure of identifications, expressed by bursting into speech, and unjustified semantic over-evaluation of words, the ascribing of objectivity to words,. In such cases, suppression or repression of words does not accomplish much, but often does *considerable harm* and must be avoided by all means Under such conditions, the use of the relief diagram becomes a necessity in pointing to the difference between different orders of abstractions and inducing the semantically beneficial silence on the 'objective level' without repression or suppression

With the use of the Structural Differential, we can eliminate identification, and so attain the benefits, avoiding the dangers If any one identifies, and his *s r* drive him into an outburst of speech, we do not repress or suppress him, we say, instead 'At your pleasure [since it makes him feel better], but remember that your words occur on the verbal levels [showing with a gesture of the hands the hanging labels], and that they *are not* the objective level, which remains untouched and unchanged' Such a procedure, when repeated again and again, gives him the proper semantic *evaluation of orders of abstractions, frees him from identification, yet without repression or suppression* It teaches him, also, to enquire into alleged 'facts', and then to try to find structurally better forms of representation If such results are not forthcoming, we may use the older forms, but by proper evaluation we do not semantically put 'belief' in these forms of representation Such beliefs always appear as the result of identification somewhere

The technique of training is simple We live on the 'objective' or lower order of abstraction levels, where we must see, feel, touch, *perform* , but *never* speak In training, we must use our hands , It is very useful, after the Structural Differential has been repeatedly explained, stressing, in particular, the rejection of the 'is' of identity, not to interrupt the other fellow Let him speak, but wave the hand, indicating the verbal levels , then point the finger to the objective level, and, with the other hand, close your own lips, to show that on the objective level one can only be silent When performed repeatedly, this pantomime has a most beneficial, semantic, pacifying effect upon the 'over-emotionalized' identification-conditions The neurological mechanism of this action is not fully known, but some aspects are quite clear

The more elaborate a nervous system becomes, the further some parts of the brain are removed from immediate experience Nerve currents, having finite velocity, eventually have longer and more numerous paths to travel, different possibilities and complications arise, resulting in 'delayed action' It is known that the thalamus (roughly) appears connected with affective and 'emotional' life, and that the cortex, farther removed and isolated from the external world, has the effect of inducing this delay in action' In unbalanced and 'emotional' 'thinking', which is so prevalent, the thalamus seems overworked, the cortex seems not worked enough The results take on the form of a low kind of animalistic, primitive, or infantile behaviour, often of a pathological character in a supposedly civilized adult It appears that the 'silence on objective levels' introduces this 'delayed action', unloading the thalamic material on the cortex This psychophysiological method is very simple, scientific, and entirely general The standard 'mental' therapy of today applies also a *method of re-education* of *s r*, as if relieving the thalamus, and putting more of the nerve currents through the cortex, or eventually furnishing the cortex with different material, so that the thalamic material returning from the cortex could be properly influenced

If we succeed in such a semantic re-education, the difficulties vanish The older experimental data show that in many instances we have succeeded, and that in many we have failed The successful cases show that we actually know the essential semantic points involved, the failures show that we do not know enough, and that our older theories are not sufficiently general At present, only the more pronounced and morbid semantic disturbances come to the attention of physicians, and very little is done by way of *preventive* measures Besides the pronounced disturbances in daily life, we see an enormous number of semantic disturbances which we disregard, and call 'peculiarities' In the majority of cases, these 'peculiarities' are undesirable, and, under unfavorable conditions, may lead to more serious consequences of a morbid character They usually involve a great deal of unhappiness for all concerned, and unhappiness appears as a sign of some semantic maladjustment somewhere, and so may be destructive to 'mental' and nervous health

In advanced 'mental' illnesses, such as usually come to the attention of psychiatrists, there are certain psycho-logical symptoms which are generally present The symptoms of interest to us in this work are called 'delusions', 'illusions', and 'hallucinations' All of them involve the semantic identification or *confusions of the orders of abstractions,* the evaluation of lower orders of abstraction as higher, or higher as lower It was explained already that some components of identification are invariably present there, and so identification may be considered as an elementary type of semantic disturbances from which all the other states differ only in intensity.

The main point is to find psychophysiological preventive means whereby this identification can be forestalled or eliminated To date, experience and analysis show that all forms of identification may be successfully eliminated by training in *visualization,* if this semantic state can be produced For this purpose the Structural Differential is uniquely useful and necessary With its help we train all centres The lower centres are involved, as we see, feel, hear , the higher centres are equally involved, as we 'remember', 'understand' , *with the result that all centres work together without conflict* The 'consciousness of abstracting' is inculcated, replacing vicious *s r* of confusion of orders of abstractions and identification

This harmonious working of all centres on their proper levels has extremely far-reaching, practical consequences in 'mental' and physical hygiene We become co-ordinated, adjusted, and difficulties which might otherwise occur in the future are eliminated in a preventive way It must be remembered that, at present, it is impossible to foresee to how great an extent the elimination of identification on all levels will have a beneficial effect At this stage we know even experimentally that the benefits are very large, but we may expect that they will become still more numerous when more experimenting has been done Delusions, illusions, and hallucinations represent manifestations which occur in practically all 'mental' difficulties, and they only represent a semantic identification of orders of abstractions of different degrees of intensity When this confusion is eliminated, we may expect general changes in the symptoms But as the correspondence is probably not *one-to-one,* it is impossible to foretell theoretically what improvements may be expected in pronounced illness In the slighter disturbances, which affect us in daily life, the results are much easier to foresee, and are *always* beneficial

To how great an extent the consciousness of abstracting benefits semantically *the whole organism,* I may illustrate by one of my own experiences Once I was travelling on a ship A gentleman visited my cabin, and, seeing the Structural Differential, asked questions about it After a short explanation, he asked about practical applications

My guest was sitting on my berth, I was sitting on a small folding chair I got up, went to the door, then pretended that I was coming in, and, at my suggestion, he said, 'Please have a seat' I remained standing while explaining how, if I were not 'conscious of abstracting', to me his word 'seat' would be identified with the chair (objectification) and my *s r* would be such that I would sit down with great confidence If the chair were to collapse I would have, besides the bump, an affective shock, 'fright', which might do harm to my nervous system But if I were conscious of abstracting, my *s r* would be different. I would remember that the *word*, the *label* 'seat' is *not* the thing on which I am supposed to sit I would remember that I am to sit on this individual, unique, un-speakable object, which might be strong or weak,. Accordingly I would sit carefully In case the chair should collapse, and I should hurt myself physically, I would still have been saved an affective nervous shock

During all these explanations I was handling the little chair and shaking it I did not notice that the legs were falling out, and that the chair was becoming unfit for use Then, when I actually sat on the relic, it gave way under me However I did *not fall* on the floor. I caught myself in the air, so to say, and saved myself from a painful experience It is important to notice that such physical readiness requires a very elaborate, nervous, unconscious co-ordination, which was accomplished by the semantic state of *non-identification* or *consciousness of abstracting*. When such a consciousness of abstracting is acquired it works instinctively and automatically and does *not* require continual effort Its operation involves a fraction of a second's *delay in action*, but this small delay is not harmful in practice, on the contrary it has very important psycho-logical and neurological 'delayed action' effects

It seems that 'silence' on the objective levels involves this psycho-physiological delay No matter how small, it serves to unload the thalamic material on the cortex In a number of clinical cases, Dr Philip S. Graven has demonstrated that the moment such a delay can actually be produced in the patient, he either improves or is entirely relieved The precise neurological mechanism of this process is not known, but there is no doubt that this 'delayed action' has many very beneficial effects upon the whole working of the nervous system It somehow balances harmful *s r*, and also somehow stimulates the higher nervous centres to more *physiological* control over the lower centres

A very vital point in this connection should be noticed That this 'delayed action' is beneficial is acknowledged by the majority of normally developed adults in the form of delay in action and finds its expression in such statements as 'think twice', 'keep your head', 'hold your horses', 'keep cool', 'steady', 'wait a minute'., and such functional recipes as 'when angry, count ten', In daily life, such wisdom is acquired either by painful experience, or is taught to children in an *A* language, which, as practice shows, is rarely effective because of its inadequacy It is seldom realized that the mechanism of these functional observations and familiar advices have very powerful and workable underlying *neurological processes,* which can be *reached and directly affected by psychophysiological,* ordinal, *non-el* methods in connection with *the structure of the language we use* Thus, under an infantile, *A*, and prevailing system we use and teach our children a language involving the 'is' of identity, and so we must confuse orders of abstractions, preparing for ourselves and the children the harmful semantic predispositions for 'bursting into speech', instead of 'wait a minute', which, neurologically, means abusing our thalamus and keeping our cortex 'unemployed'. In a $\bar{A}$ ∞-valued system we reject the 'is' of identity, we cannot confuse orders of abstractions, we cannot identify words with the un-speakable objective levels or inferences with descriptions, and we cannot identify the different abstractions of different individuals, This semantic state of proper evaluation results in discrimination between the different orders of abstractions, an automatic delay is introduced—the cortex is switched completely into the nervous circuit The semantic foundation is laid for 'higher mentality' and 'emotional balance'

We have already had occasion to mention the mechanism of projection in connection with identification, as a semantic state of affective ascribing of lower centre characteristics to higher order abstractions and vice versa, and in connection with the introverted or extroverted attitudes Likewise, we have already reached the conclusion that a well-adjusted and, therefore, well-balanced individual should be neither of the extremes, but a balanced extroverted introvert By training with the Differential this important semantic result may be brought about By training with the 'object' on its level, we become extroverted, and we learn to observe, this results in semantic freedom from 'preconceived ideas', such as we have when we start with the evaluation, label first and object next, instead of the natural order, object first and label next By passing to higher order abstractions and evaluating the successive ranks of labels, we train in introversion The result, as a whole, is that we may achieve the desirable and balanced semantic state of the extroverted introvert

That in the training with the Differential we use all available nerve centres is beneficial, because the lower centres are in closer connection with the vegetative nervous system than are the others.

CHAPTER XXVII

HIGHER ORDER ABSTRACTIONS

> The characters which science discerns in nature are subtle characters, not obvious at first sight They are relations of relations and characters of characters (573) A N. WHITEHEAD

> In this connection one should particularly remember that the human language permits the construction of sentences which do not involve any consequences and which therefore have no content at all—in spite of the fact that these sentences produce some kind of picture in our imagination, e g, the statement that besides our world there exists another world, with which any connection is impossible in principle, does not lead to any experimental consequence, but does produce a kind of picture in the mind. Obviously such a statement can neither be proved nor disproved. One should be especially careful in using the words "reality," "actually," etc, since these words very often lead to statements of the type just mentioned (215) W HEISENBERG

Section A General

In the previous chapters I demonstrated that there is a short cut which enables us to grasp, acquire, and apply what has been advanced in the present work This semantic short cut is 'consciousness of abstracting' It is a psycho-logical attitude toward all our abstracting on all levels, and so involves the co-ordinated working of the organism-as-a-whole.

The use of the Structural Differential is necessary, because some levels are un-speakable We can see them, handle them, feel them , but under *no* circumstances can we reach those levels by speech alone We must, therefore, have a diagram, by preference in relief form, which represents the empirical structural conditions, and which indicates the un-speakable level by some other means than speech We must, in the simplest case, either point our finger to the object, insisting upon silence, or must perform bodily some activity and similarly insist upon silence, as the performing and feelings are also *not* words

In such semantic training it is enough to insist upon the non-identity or the difference between the objective, *un-speakable* levels of lower order abstractions, (O_h), and the verbal or higher order abstractions, (L_n) When this habit and feeling are acquired, no one should have difficulties in extending the non-identity method to daily-life occurrences To achieve these semantic aims, we must first emphasize the common-sense fact that an object *is not* the event To do this, we start with the 1933 scientific structural 'metaphysics' about the event and stress the fact that the object, being a nervous abstraction of lower order, has fewer and different *m o* characteristics than the event has This is best accomplished by stressing the fact that in abstracting from the event to the object we left out some characteristics. We did not *abstract 'all'* characteristics, this would be a self-contradiction in terms, an impossibility

We do not even need to stress a full understanding of the event Common-sense examples, showing that what we recognize as a 'pencil' is not 'all', often suffice No one will have difficulties, provided he trains himself in this direction, in *remembering* continually and instinctively the free hanging strings (B'), (B"), which indicate the non-abstracted or left-out characteristics and which help to train in *non-identity* With the relief diagram, the *s r* of the student are trained *through all nervous centres* He sees, he handles , the hanging strings, and he also hears about them This gives the *maximum probability* that the organism-as-a-whole will be affected In this way an 'intellectual' theory engages the 'senses', feelings and reflex mechanisms To affect the organism-as-a-whole, organism-as-a-whole methods must be employed

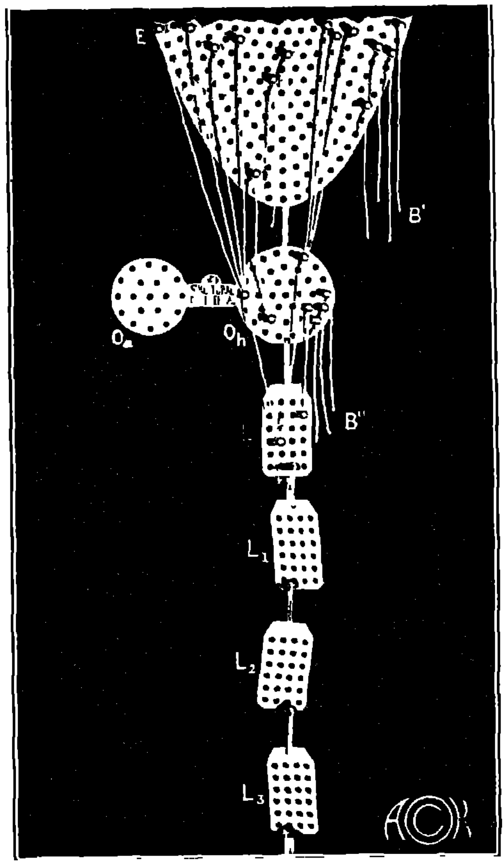


FIG 1
THE STRUCTURAL DIFFERENTIAL

A similar structural situation is found when we deal with higher order abstractions A word, or a name, or a statement is conveyed in spoken form or by writing, and affects first the lower centres and then is abstracted, and again transformed, by the higher centres The order is generally not changed when the verbal issues are neither seen nor heard but originate in ourselves Most 'impulses', 'interests', 'meanings', 'evaluations' , originate in lower centres and follow the usual course, from lower centres to higher When 'experience' (reaction of lower centres) is transformed into 'memories' (higher centres) , the order is similar. Difficulties begin when the order is pathologically *reversed* and 'ideas' are evaluated as experience, words as objects, In the building of language a similar process can be observed We observe the absolute individuals with which we actually deal, we label them with individual names, say, $A_1, A_2, \ldots, A_{11}, A_{12}, \ldots, A_{21}, A_{22}, \ldots, A_{31}, A_{32}, \ldots$ By a process of abstracting and *disregarding,* for instance, the characteristic subscripts '1', we would have only the ones which have the characteristic subscripts 2, 3, ... , 9, 22, 23 ... , 29, *Disregarding* the characteristic subscripts '2', we would have the ones with the characteristics 3, 4, ... , 9, 33, 34, ... , 39, Finally, if we should eliminate all *individual* characteristic subscripts, we would have a 'general' name A for the whole group without singling out individual characteristics.

All words of the type of 'man', 'animal', 'house', 'chair', 'pencil' , have been built by a similar process of abstraction, or *disregard* for individual differences. In each case of disregard of individual characteristics a *new* neurological process was involved

Similarly, with 'statements about a statement'. When we hear a statement, or see it in a written form, such a statement becomes a stimulus entering through the lower centres, and a statement about it represents, in general, a new process of abstraction, or an abstraction of higher order

It becomes obvious that the introduction of a language of 'different order of abstractions', although it is not familiar, yet structurally it represents very closely, in terms of *order,* most fundamental neurological processes going on in us As we already know, a natural order has been established by evolution, namely, lower order abstractions first, higher next, the identifications of orders or the reversal of orders appears pathological for man and appears as a confusion of orders of abstractions, resulting in *false evaluation* identification, illusions, delusions, and hallucinations

Historically, the first to pay serious attention to the above problems in a consistent, yet very limited, way were mathematicians In the investigation of the problems of the foundation of mathematics, mathematical 'logic', and the theory of aggregates, we came across self-contradictions which would make mathematics impossible To avoid such a disaster, Russell invented what is called the 'theory of mathematical types'. The status of this theory is a very interesting and instructive one The theory solves the mathematical difficulties, thus saving mathematics, but has *no* application to life Practically all mathematicians, if I am not mistaken, the author of the theory included, somehow 'dislike' the theory and make efforts to solve the problems in a different way and possibly to abandon the theory altogether

We have already shown that the introduction of a language of 'different orders of abstractions' is structurally entirely justified and physiologically natural, as it describes, in terms of order, the activities of the nervous system Such facts are important, but if, in addition the introduction of a language of a new $\bar{A}$ structure would give us further demonstrable advantages, then the introduction of such a language would become increasingly desirable

Although the majority of mathematicians 'dislike' the theory of types, yet, at present, this theory is unconditionally necessary for non-self-contradictory mathematics The author was pleasantly surprised to find that after his $\bar{A}$-system was formulated, this simple and natural, actional, functional, operational, *non-el* theory covers the theory of mathematical types and generalizes it, making the theory applicable not only to the solution of mathematical paradoxes but to the solution of the majority of purely human and scientific difficulties One general rule of 'non-confusion of orders of abstractions', and the acquiring of the simple and workable 'consciousness of abstracting' based on the denial of the 'is' of identity, offers a *full* structural and semantic solution The disregard of the issues involved leads fatalistically to the manufacture of endless and unnecessary human sufferings and unhappiness, the elimination of which is one of the main points in a theory of sanity There is no mystery in 1933 that continuous small painful shocks may lead to serious semantic and physical disturbances Psycho-logicians and psychiatrists will find it increasingly difficult to work at their problems if they disregard these semantic issues Parents and teachers will find simple yet effective structural means for training the reactions of children *in sanity,* with all the ensuing semantic benefits to the individuals and to society

When Whitehead and Russell were working at the foundations of mathematics, they came across endless paradoxes and self-contradictions, which, of course, would make mathematics impossible After many efforts they found that all these paradoxes had one general source, in the rough, in the expressions which involve the word 'all', and the solution was found by introducing 'non-allness', a semantic forerunner of non-identity Consider, for example, 'a proposition about *all* propositions' They found that such totalities, or such 'all' statements, were not legitimate, as they involved a self-contradiction to start with A proposition cannot be made legitimately about 'all' propositions without some restriction, since it would have to include the new proposition which is being made If we consider a *m o* term like 'propositions', which we can manufacture without known limits, and remember that any statement *about* propositions takes the form of a proposition, then obviously we cannot make statements about *all* propositions In such a case the statement must be limited, such a set has *no total,* and a statement about 'all its members' cannot be made legitimately Similarly, we cannot speak about *all* numbers

Statements such as 'a proposition about *all* propositions' have be called by Russell 'illegitimate totalities' In such cases, it is neces to break up the set into smaller sets, each of which is capable of ha a totality This represents, in the main, what the theory of types a to accomplish In the language of the *Principia Mathematica,* the pri ciple which enables us to avoid the illegitimate totalities may be express as follows 'Whatever involves *all* of a collection must not be one of t collection', or, 'if, provided a certain collection had a total, it would ha members only definable in terms of that total, then the said collectio has no total' [1] The above principle is called the 'vicious-circle principle because it allows us to evade the vicious circles which the introductio of illegitimate totalities involve Russell calls the arguments which involve the vicious-circle principle, 'vicious-circle fallacies'

As an example, Russell gives the two-valued law of 'excluded third', formulated in the form that 'all propositions are true or false' We involve a vicious-circle fallacy if we argue that the law of excluded third takes the form of a proposition, and, therefore, may be evaluated as true or false Before we can make any statement about 'all propositions' legitimate, we must limit it in some way so that a statement about this totality must fall outside this totality

Another example of a vicious-circle fallacy may be given as that of the imaginary sceptic who asserts that he knows nothing, but is refuted by the question—does he *know* that he *knows* nothing? Before the statement of the sceptic becomes significant, he must limit, somehow, the number of facts concerning which he asserts his ignorance, which represent an illegitimate totality When such a limitation is imposed, and he asserts that he is ignorant of an extensional series of propositions, of which the proposition about his ignorance is not a member, then such scepticism cannot be refuted in the above way

We do not need to enter into further details concerning the elaborate and difficult theory of types In my $\bar{A}$ psychophysiological formulation, the theory becomes structurally extremely simple and natural, and applies to mathematics as well as to a very large number of daily experiences, eliminating an unbelievably large number of misunderstandings, vicious circles, and other semantic sources of human disagreements and unhappiness

It should be noticed that, in the given examples, we always made a statement *about* another statement, and that the vicious circle arose from identifying or from the confusion of the orders of statements The way out is found in the consciousness of abstracting, which leads to the semantic discrimination between *orders of abstractions* If we have certain propositions, p_1, p_2, p . . p_n, and make a new proposition about these propositions, say P, then, according to the present theory, the statement P about the statements p_1, p_2 . must be considered as an abstraction of higher order and so different, and must not be identified as to order with the propositions p_1, p_2, . p_n

The above psychophysiological formulation is entirely general, yet simple and natural in a $\bar{A}$-system To make this clearer, I shall take several statements concerning the theory of types from the *Principia Mathematica,* shall designate them by (Pr), shall reformulate them in my language of *orders of abstractions,* and shall designate them as general semantics (G S)

Thus, 'The vicious circles in question arise from supposing that a collection of objects may contain members which can only be defined by means of the collection as a whole' (Pr) Objects as individuals and 'collections of objects' obviously belong to different orders of abstractions and should not be confused (G S) A 'Proposition about *all* propositions' (Pr) This involves a confusion of orders of abstractions, for if we posit propositions p_1 [illegible] then a proposition P *about* these propositions represents a higher o[illegible]straction and should not be identified with them (G S) 'More generally, given any set of [illegible] such that, if we suppose the set to have a total it will contain members which presuppose this total, then such a set cannot have a total By saying that a set has "no total", we mean, primarily that no significant statement can be made about "all its members" ' (Pr) A set of statements or objects or elements, or the like, and a statement *about* them belong to different orders of abstractions and should not be confused (G S) In the language of Wittgenstein 'No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the "whole theory of types").'[2]

In the language of the present general semantics a statement about a statement is not the 'same' statement, but represents, by structural and neurological necessity, a higher order of abstraction, and should not be confused with the original statement.

Similar reformulations apply to all cases given in the *Principia Mathematica,* and so it becomes evident that the present theory covers a similar ground as the theory of types, and also covers an endless list of daily-life applications which are of crucial semantic importance in a theory of sanity. We must stress here a simple, natural, and *single semantic law of non-identity which covers all confusions of orders of abstraction.* This one rule and training teach us not to confuse the higher orders with the lower, not to identify words with objects (not to objectify), as well as not to confuse higher abstractions of different orders. This generality and structural simplicity constitute an argument in favor of the present $\bar{A}$-system. It is easier to teach a single, simple, and natural rule which covers a vast field of semantic sources of human difficulties. For when the rule is explained, and the learner is trained with the Structural Differential, the semantic problem resolves itself simply into the showing with one's finger different orders of abstractions, and insisting that 'this *is not* this'.

If we consider the natural, structural, and *empirical* fact that our lives are lived in a world of non-identical abstractions of different orders, the discrimination between different orders becomes of paramount semantic importance for evaluation. Under such conditions we should become thoroughly acquainted with the mechanism of these different orders of abstractions. We should notice, first, that the language of the *Principia Mathematica* is A, and involves the 'is' of identity. Such a language leads to identifications and to confusions, and makes simple issues difficult and perplexing. The term 'class' is very confusing. What do we mean by this term? In life we have, and deal with, *individuals* on *objective, un-speakable* levels. If we take a number of individuals, we have a number of them, yet they all remain individual. If we produce an abstraction of higher order, so that the individuality of each member is lost, then we have an abstraction of a higher order ('idea' in the old language), but no more the absolute individuals of our collection. The term 'class' in this respect is seriously confusing, as it tends to conceal a simple experimental fact, and leads to confusion of the orders of abstractions if the *multiordinality* of the term 'class' is not formulated.

Many critics and reviewers of the *Principia Mathematica* somehow feel this to be so, but their criticisms are not bold enough, and do not go to the roots of the A semantic difficulty. They do not pay attention to the A, 'logical', 'philosophical', and 'psychological' *elementalistic* method and language involving the 'is' of identity, in which the Introduction of the *Principia* is written. Doctor Alonzo Church is the first, as far as my knowledge goes, to suggest that, following Peano, numbers should be defined in the language of abstractions. He does not carry his analysis further, however, and does not state that it involves a language of entirely different $\bar{A}$ structure.[3] If we abandon the term 'class' and accept the language of 'abstractions of *different* orders', then we are led to the rejection of the 'is' of identity and to the present system, of which the theory of mathematical types becomes a necessary part. The problems of 'class' cease to be an 'assumption', as the different orders of abstractions are descriptions of experimental facts, and so the 'axiom of reducibility' becomes unnecessary. In my language, this axiom is also an *aristotelian description* of the experimental fact that we can abstract in different orders.

Section B. Multiordinal terms.

In the examples given in Section A, we used words such as 'proposition', which were applied to all higher order abstractions. We have already seen that such terms may have different uses or meanings if applied to different orders of abstractions. Thus originates what I call the *multiordinality* of terms. The words 'yes', 'no', 'true', 'false', 'function', 'property', 'relation', 'number', 'difference', 'name', 'definition', 'abstraction', 'proposition', 'fact', 'reality', 'structure', 'characteristic', 'problem', 'to know', 'to think', 'to speak', 'to hate', 'to love', 'to doubt', 'cause', 'effect', 'meaning', 'evaluation', and an endless array of the most important terms we have, must be considered as *multiordinal terms*. There is a most important semantic characteristic of these *m.o.* terms, namely, that they are ambiguous, or ∞-valued, in general, and that each has a definite meaning, or one value, only and exclusively in a given context, when the order of abstraction can be definitely indicated.

These issues appear extremely simple and general, a part and parcel of the structure of 'human knowledge' and of our language. We cannot avoid these semantic issues, and, therefore, the only way left is to face them explicitly. The test for the multiordinality of a term is simple. Let us make any statement and see if a given term applies to it ('true', 'false', 'yes', 'no', 'fact', 'reality', 'to think', 'to love', ...). If it does, let us deliberately make another statement *about* the former statement and test if the given term may be used again. If so, it is a safe assertion that this term should be considered as *m.o.* Any one can test such a *m.o.*

term by himself without any difficulty The main point about all such *m o* terms is that, *in general,* they are *ambiguous,* and that all arguments about them, 'in general', lead only to *identification of orders of abstractions and semantic disturbances, and nowhere else* Multiordinal terms have only definite meanings on a given level and in a given context Before we can argue about them, we must fix their orders, whereupon the issues become simple and lead to agreement As to 'orders of abstraction', we have no possibility of ascertaining the 'absolute' order of an abstraction, besides, we *never* need it In human semantic difficulties, in science, as well as in private life, usually no more than three, perhaps even two, neighbouring levels require consideration When it comes to a serious discussion of some problem, errors, ambiguity, confusion, and disagreement follow from confusing or identifying the neighbouring levels In practice, it becomes *extremely simple* to settle these three (or two) levels and to keep them separated, *provided we are conscious of abstracting, but not otherwise*

For a theory of sanity, these issues seem important and structurally essential In identifications, delusions, illusions, and hallucinations, we have found a *confusion* between the orders of abstractions or a false evaluation expressed as a reversal of the natural order

One of the symptoms of this confusion manifests itself as 'false beliefs', which again imply comparison of statements about 'facts' and 'reality', and involve such terms as 'yes', 'no', 'true', 'false',. As all these terms are multiordinal, and, therefore, ambiguous, 'general' 'philosophical' rigmaroles should be avoided With the consciousness of abstracting, and, therefore, with a *feel* for this peculiar stratification of 'human knowledge', all semantic problems involved can be settled simply

The avoidance of *m o* terms is impossible and undesirable Systematic ambiguity of the most important terms follows systematic analogy They appear as a direct result and condition of our powers of abstracting in different orders, and allow us to apply one chain of ∞-valued reasoning to an endless array of different one-valued facts, all of which are different and become manageable only through our abstracting powers

For further details about the theory of types, the reader is referred to the literature on the subject and Supplement II[4], here I shall give only a few examples of the complexities and difficulties inherent in language, and show how simply they become solved by the aid of $\bar{A}$ general semantics and the resulting 'consciousness of abstracting'

As an example, I quote Russell's analysis of the 'simple' statement 'I am lying', as given in the *Principia.* 'The oldest contradiction of the kind in question is the *Epimenides* Epimenides the Cretan said that all Cretans were liars, and all other statements made by Cretans were certainly lies Was this a lie? The simplest form of this contradiction is afforded by the man who says "I am lying", if he is lying, he is speaking the truth, and vice versa

'When a man says "I am lying", we may interpret his statement as. "There is a proposition which I am affirming and which is false" That is to say, he is asserting the truth of some value of the function "I assert p, and p is false" But we saw that the word "false" is ambiguous, and that, in order to make it unambiguous, we must specify the order of falsehood, or, what comes to the same thing, the order of the proposition to which falsehood is ascribed We saw also that, if p is a proposition of the nth order, a proposition in which p occurs as an apparent variable is not of the nth order, but of a higher order Hence the kind of truth or falsehood which can belong to the statement "there is a proposition p which I am affirming and which has falsehood of the nth order" is truth or falsehood of a higher order than the nth Hence the statement of Epimenides does not fall within its own scope, and therefore no contradiction emerges

'If we regard the statement "I am lying" as a compact way of simultaneously making all the following statements "I am asserting a false proposition of the first order," "I am asserting a false proposition of the second order," and so on, we find the following curious state of things As no proposition of the first order is being asserted, the statement "I am asserting a false proposition of the first order" is false This statement is of the second order, hence the statement "I am making a false statement of the second order" is true This is a statement of the third order, and is the only statement of the third order which is being made Hence the statement "I am making a false statement of the third order" is false Thus we see that the statement "I am making a false statement of order $2n+1$" is false, while the statement "I am making a false statement of order $2n$" is true. But in this state of things there is no contradiction'[5]

Clearly, if we should apply the language of orders of abstractions to the above case, a similar outcome is reached more generally and more simply. If we should confuse the orders of abstractions, we might naturally have an endless argument at hand This example shows how a confusion of orders of abstractions might lead to insoluble verbal problems, and how semantically important it is that we should not identify, and that we should be conscious of abstracting, with the resulting instinctive feeling for this peculiar structural stratification of 'human knowledge' We should notice that with the confusion of orders of abstractions, and by the use of *m o* terms, *without realizing their ∞-valued* character, we may always construct an endless array of such verbal arguments to befog the issues, but that as soon as we assign a definite order to the *m o* terms, and so settle a specific single meaning in a given context for the many meanings any *m o* term may have, the difficulties vanish

As the above analysis applies to all *m o* terms, and these terms happen to be most important in our lives, there is no use in trying to avoid these terms and the consequences of using them Quite the contrary, often it is structurally necessary to build a *m o* term—for instance, 'abstracting'—we must take for granted that it has many meanings, and indicate these meanings by assigning to the term the definite order of abstraction Thus, such a term as 'abstracting' or 'characteristic' , might be confusing and troublesome, but 'abstracting in different orders' , is not, as in a given context we may always assign the definite order and single meaning to the term

It has been repeatedly said that a *m o* term has, by structural necessity, many meanings No matter how we define it, its definition is again based on other *m o* terms If we try to give a *general* 'meaning' to a *m o* term, which it cannot have, further and deeper analysis would disclose the multiordinality of the terms by which it is defined, restoring once more its multiordinality As there is no possibility of avoiding the above structural issue, it is more correct and also more expedient to recognize at once the fundamental multiordinality of a term If we do so, we shall not get confused as to the meaning of such a term in a given context, because, in principle, in a context its meaning is single and fixed by that context

The semantic benefits of such a recognition of multiordinality are, in the main, sevenfold (1) we gain an enormous economy of 'time' and effort, as we stop 'the hunting of the snark', usually called 'philosophy', or for a one-valued general definition of a *m o* term, which would not be formulated in other *m o* terms, (2) we acquire great versatility in expression, as our most important vocabulary consists of *m o* terms, which can be extended indefinitely by assigning many different orders and, therefore, meanings, (3) we recognize that a definition of a *m o* term must, by necessity, represent not a proposition but a propositional function involving variables, (4) we do not need to bother much about formal definitions of a *m o* term outside of mathematics, but may use the term freely, realizing that its unique, in principle, meaning in a given context is structurally indicated by the context, (5) under such structural conditions, the freedom of the writer or speaker becomes very much accentuated, his vocabulary consists potentially of infinite numbers of words, and psycho-logical, semantic blockages are eliminated, (6) he knows that a reader who understands that ∞-valued mechanism will never be confused as to the meaning intended, and (7) the whole linguistic process becomes extremely flexible, yet it preserves its essential extensional one-valued character, in a given case.

In a certain sense, such a use of *m.o.* terms is to be found in poetry, and it is well known that many scientists, particularly the creative ones, like poetry. Moreover, poetry often conveys in a few sentences more of lasting values than a whole volume of scientific analysis. The free use of *m.o.* terms without the bother of a structurally impossible formalism outside of mathematics accomplishes this, *provided we are conscious of abstracting, otherwise only confusion results*.

It should be understood that I have no intention of condemning formalism. Formalism of the most rigorous character is an extremely important and valuable discipline (mathematics at present), but formalism, as such, in experimental science and life appears often as a handicap and not as a benefit, because, in empirical science and life, we are engaged in exploring and discovering the unknown structure of the world as a means for structural adjustment. The formal elaboration of some language is only the consistent elaboration of its structure, which must be accomplished independently if we are to have means to compare verbal with empirical structures. From a $\bar{A}$ point of view, both issues are equally important in the search for structure.

Under such structural empirical conditions the *m.o.* terms acquire great semantic importance, and perhaps, without them, language, mathematics, and science would be impossible. As soon as we understand this, we are forced to realize the profound structural and semantic difference between the A and $\bar{A}$ systems. What in the old days were considered propositions, become propositional functions, and most of our doctrines become the doctrinal functions of Keyser, or system-functions, allowing multiple interpretations.

Terms belong to verbal levels and their meanings *must* be given by definitions, these definitions depending on undefined terms, which consist always, as far as my knowledge goes, of *m.o.* terms. Perhaps it is necessary for them to have this character, to be useful at all. When these structural empirical conditions are taken into account, we must conclude that the postulational method which gives the structure of a given doctrine lies at the foundation of all human linguistic performances, in daily life as well as in mathematics and science. The study of these problems throws a most important light on all mysteries of language, and on the proper use of this most important human neurological and semantic function, without which sanity is impossible

From a structural point of view, postulates or definitions or assumptions must be considered as those relational or multi-dimensional order structural assumptions which establish, conjointly with the undefined terms, the structure of a given language Obviously, to find the structure of a language we must work out the given language to a system of postulates and find the minimum of its (never unique) undefined terms This done, we should have the structure of such a system fully disclosed, and, with the structure of the language thoroughly known, we should have a most valuable tool for investigating empirical structure by predicting verbally, and then verifying empirically

To pacify the non-specialist, let me say at once that this work is very tedious and difficult, although a crying need, nevertheless, it may be accomplished by a single individual Because of the character of the problem, however, when this work is done, the semantic results have always proved thus far—and probably will continue so—quite simple and comprehensible to the common sense, even of a child

One very important point should be noted Since language was first used by the human race, the structural and related semantic conditions disclosed by the present analysis *have not been changed,* as they are inherent in the structure of 'human knowledge' and language Historically, we were always most interested in the immediacy of our daily lives We began with grunts symbolizing this immediacy, and we never realize, even now, that these historically first grunts were the most complex and difficult of them all Besides these grunts, we have also developed others, which we call mathematics, dealing with, and elaborating, a language of numbers, or (as I define it semantically) a language of *two symmetrical and infinitely many asymmetrical unique, specific relations* for exploring the structure of the world, which is, at present, the most effective and the simplest language yet formed Only in 1933, after many hundreds of thousands of years, have the last mentioned grunts become sufficiently elaborate to give us a sidelight on structure We must revise the whole linguistic procedure and structure, and gain the means by which to disclose the structure of 'human knowledge' Such semantic means will provide for the proper handling of our neurological structure, which, in turn, is the foundation for the structurally proper use of the human nervous system, and will lead to human nervous adjustment, appropriate *s r*, and, therefore, to sanity

Human beings are quite accustomed to the fact that words have different meanings, and by making use of this fact have produced some rather detrimental speculations, but, to the best of my knowledge, the structural discovery of the multiordinality of terms and of the psychophysiological importance of the treatment of orders of abstractions resulting from the rejection of the 'is' of identity—as formulated in the present system—is novel In this mechanism of multiordinality, we shall find an unusually important structural problem of human psychologics, responsible for a great many fundamental, desirable, undesirable, and even morbid, human characteristics The full mastery of this mechanism is only possible when it is formulated, and leads automatically to a possibility of a complete psychophysiological adjustment This adjustment often reverses the psycho-logical process prevailing at a given date, and this is the foundation, among others, of what we call 'culture' and 'sublimation' in psychiatry

Let me recall that one of the most fundamental functional differences between animal and man consists in the fact that no matter in how many orders the animal may abstract, its abstractions stop on some level beyond which the animal cannot proceed Not so with man Structurally and potentially, man can abstract in indefinitely many orders, and no one can say legitimately that he has reached the 'final' order of abstractions beyond which no one can go In the older days, when this semantic mechanism was not made structurally obvious, the majority of us copied animals, and stopped abstracting on some level, as if this were the 'final' level In our semantic training in language and the 'is' of identity given to us by our parents or teachers or in school, the multiordinality of terms was never suspected, and, although the human physiological mechanism was operating all the while, we used it on the conscious level in the animalistic way, which means ceasing to abstract at some level Instead of being told of the mechanism, and of being trained consciously in the fluid and dynamic s r of *passing to higher and higher abstractions as normal,* for Smith, we preserved a sub-normal, animalistic semantic blockage, and 'emotionally' stopped abstracting on some level

Thus, for instance, if, as a result of life, we come to a psychological state of hate or doubt, and stop at that level, then, as we know from experience, the lives of the given individual and of those close to him are not so happy But a hate or doubt of a higher order reverses or annuls the first order semantic effect Thus, hate of hate, or doubt of doubt—a second order effect—has reversed or annulled the first order effect, which was detrimental to all concerned because it remained a *structurally-stopped or an animalistic* first order effect

The whole subject of our human capacity for higher abstracting without discernible limits appears extremely broad, novel, and unanalysed It will take many years and volumes to work it out, so, of necessity, the examples given below will be only suggestive and will serve to illustrate roughly the enormous power of the $\bar{A}$ methods and structure, aiming to make them workable as an educational, powerful, semantic device.

Let us take some terms which may be considered as of a positive character and represent the structure of 'culture', science, and what is known in psychiatry as 'sublimation', such as curiosity, attention, analysis, reasoning, choice, consideration, knowing, evaluation, The first order effects are well known, and we do not need to analyse them But if we transform them into second order effects, we then have curiosity of curiosity, attention of attention, analysis of analysis, reasoning about reasoning (which represents science, psycho-logics, epistemology ,), choice of choice (which represents freedom, lack of psycho-logical blockages, and shows, also, the semantic mechanism of eliminating those blocks), consideration of consideration gives an important cultural achievement, knowing of knowing involves abstracting and structure, becomes 'consciousness', at least in its limited aspect, taken as consciousness of abstracting, evaluation of evaluation becomes a theory of sanity,.

Another group represents morbid semantic reactions Thus the first order worry, nervousness, fear, pity , may be quite legitimate and comparatively harmless But when these are of a higher order and identified with the first order as in worry about worry, fear of fear , they become morbid Pity of pity is dangerously near to self-pity Second order effects, such as belief in belief, makes fanaticism To know that we know, to have conviction of conviction, ignorance of ignorance , shows the mechanism of dogmatism, while such effects as free will of free will, or cause of cause , often become delusions and illusions

A third group is represented by such first order effects as inhibition, hate, doubt, contempt, disgust, anger, and similar semantic states, the *second order reverses and annuls* the first order effects Thus an inhibition of an inhibition becomes a positive excitation or release (see Part VI), hate of hate is close to 'love', doubt of doubt becomes scientific criticism and imparts the scientific tendency, the others obviously reverse or annul the first order undesirable *s r*

In this connection the pernicious effect of identification becomes quite obvious In the first and third cases beneficial effects were *prevented*, because identification of orders of abstractions, as a semantic state, produced a semantic blockage which did not allow us to pass to higher order abstractions, in the second case, it actually produced morbid manifestations

The consciousness of abstracting, which involves, among others, the full instinctive semantic realization of non-identity and the stratification of human knowledge, and so the multiordinality of the most important terms we use solves these weighty and complex problems because it gives us structural methods for semantic evaluation, for orientation, and for handling them By passing to higher orders these states which involve inhibition or negative excitation become reversed Some of them on higher levels become culturally important; and some of them become morbid Now consciousness of abstracting in all cases gives us the semantic *freedom* of all levels and so helps *evaluation* and selection, thus removing the possibility of remaining animalistically fixed or blocked on any one level Here we find the mechanism of the 'change of human nature' and an assistance for persons in morbid states to revise by themselves their own afflictions by the simple realization that the symptoms are due to identifying levels which are essentially different, an unconscious jumping of a level or of otherwise confusing the orders of abstractions Even at present all psychotherapy is unconsciously using this mechanism, although, as far as I know, it has never before been structurally formulated in a general way.

It should be added that the moment we eliminate identification and acquire the consciousness of abstracting, as explained in the present system, we have already acquired the permanent semantic feeling of this peculiar *structural stratification* of human knowledge which is found in the psycho-logics of the differential and integral calculus and mathematics, similar in structure to the world around us, without any difficult mathematical technique Psycho-logically, both mathematics and the present system appear structurally similar, not only to themselves, but also to the world and our nervous system, and at this point it departs very widely from the older systems

Let me give another example of how the recognition of order of abstractions clears up semantic difficulties

I recall vividly an argument I had with a young and very gifted mathematician Our conversation was about the geometries of Euclid and Lobatchevski, and we were discussing the *dropping* and *introduction* of assumptions I maintained that Lobatchevski *introduced* an assumption, he maintained that Lobatchevski *dropped* an assumption On the surface, it might have appeared that this is a problem of 'fact' and not of *preference* The famous fifth postulate of Euclid reads, 'If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than two right angles' We should note, in passing, that a straight line is *assumed* to be of 'infinite' length, which involves a definite type of structural metaphysics of 'space', common to the *A* and older systems This postulate of Euclid can be expressed in one of its equivalent forms, as, for instance, 'Through a point outside a straight line one, and only one, parallel to it can be drawn'. Lobatchevski and others decided to build up a geometry *without* this postulate, and in this they were successful Let us consider what Lobatchevski did. For this, we go to a deeper level—otherwise, to a higher order abstraction—where we discover that what on *his level* had been the *dropping* of an assumption becomes on our deeper level or higher order abstraction the *introduction* of an assumption, namely, the assumption that through a point outside a straight line there passes *more* than one parallel line

Now such a process is *structurally inherent in all human knowledge* More than this, it is a unique characteristic of the structure of human knowledge We can always do this If we pass to higher orders of abstractions, situations seemingly 'insoluble', 'matters of fact', quite often become problems of *preference* This problem is of extreme semantic importance, and of indefinitely extended consequences for all science, psychiatry, and education in particular

The examples I have given show a most astonishing semantic situation, namely, that one question can sometimes be answered 'yes' or 'no', 'true' or 'false', depending on the order of abstractions the answerer is considering The above facts alter considerably the former supposedly sharply defined fields of 'yes' and 'no', true' and 'false', and, in general, of all multiordinal terms Many problems of 'fact' on one level of abstraction become problems of 'preference' on another, thereby helping to diminish the semantic field of disagreement

It is interesting to throw some light on the problem of 'preference' Which statement or attitude is preferable? The one claiming that Lobatchevski *dropped* a postulate, or the one claiming that Lobatchevski *introduced* a new postulate? Both are 'facts', but on different levels, or of different orders The *dropping* appears as an historical fact, the *introducing* as a psycho-logical fact *inherent* in the structure of human knowledge The preference is fairly indicated, the psycho-logical fact is of the utmost generality (as all psycho-logical facts are) and, therefore, more useful, since it applies to all human endeavours and not merely to what a certain mathematician did under certain circumstances

Section C Confusion of higher orders of abstractions

We have already seen that Fido's power of abstracting stops somewhere If we are finalists of any kind, we also assume that *our* power of abstracting stops somewhere In some such way the finalistic, dogmatic and absolutistic semantic attitudes are built

If, however, by the aid of the Structural Differential we train the *s r* of our children in $\bar{A}$ non-identity and the inherent stratification of human knowledge and power of abstracting, we *facilitate* the passing to higher order abstractions and establish *flexible s r* of *full conditionality* which are unique for Smith and of great preventive and therapeutic value We thus build up 'human mind' for efficiency and sanity by eliminating the factors of semantic blockages, while, by engaging the activity of the higher nerve centres, we diminish the vicious overflow of nervous energy upon the lower nerve centres, which, if allowed, must, of necessity, make itself manifest in arrested or regressive symptoms

The above issues are of serious semantic importance in our daily lives and in sanity All semantic disturbances involve evaluation, doctrines, creeds, speculations , and vice versa Under circumstances such as described above, which appear inherent with us, it is dangerous not to have means to see one's way clear in the maze of verbal difficulties with all their dangerous and ever-present semantic components

By disregarding the orders of abstractions, we can manufacture any kind of verbal difficulties, and, without the consciousness of abstracting, we all become nearly helpless and hopeless semantic victims of a primitive-made language and its underlying structural metaphysics Yet the way out is simple, non-identity leads to 'consciousness of abstracting' and gives us a new working sense for *values*, new *s r*, to guide us in the verbal labyrinth

Outside of 'objectification', which is defined as the evaluation of higher order abstractions as lower, namely, words, memories , as objects, experiences, feelings , the most usual identification of different *higher order* abstractions appears as the confusion of inferences and inferential terms with descriptions and descriptive terms

Obviously, if we consider a description as of the nth order, then an inference from such a description (or others) should be considered as an abstraction of a higher order $(n+1)$ Before we make a decision, we usually make a more or less hasty survey of happenings, this survey establishing a foundation for our judgements, which become the basis of our action This statement is fairly general, as the components of it can be found by analysis practically everywhere. Our problem is to analyse the general case Let us follow up roughly the process.

We assume, for instance, an hypothetical case of an ideal observer who observes correctly and gives an impersonal, unbiased account of what he has observed Let us assume that the happenings he has observed appeared as: ●, ◆, ▮, ▶,   , and then a new happening ⧗ occurred. At this level of *observation,* no speaking can be done, and, therefore, I use various fanciful symbols, and not words. The observer then gives a *description* of the above happenings, let us say $a, b, c, d, \ldots x$ then he makes an inference from these descriptions and reaches a conclusion or forms a judgement A about these facts We assume that facts unknown to him, which always exist, are not important in this case. Let us assume, also, that his conclusion seems correct and that the action A'' which this conclusion motivates is appropriate Obviously, we deal with at least three different levels of abstractions, the seen, experienced lower order abstractions (un-speakable), then the descriptive level, and, finally, the inferential levels

Let us assume now another individual, Smith$_1$, ignorant of structure or the orders of abstractions, of consciousness of abstracting, of s r , a politician or a preacher, let us say, a person who habitually identifies, confuses his orders, uses inferential language for descriptions and rather makes a business out of it Let us assume that Smith$_1$ observes the 'same happenings' He would witness the happenings ●, ◆, ▮, ▶, . . ., and the happening ⧗ would appear new to him. The happenings ●, ◆, ▮, ▶,   , he would describe in the form $a, b, c, d, \ldots$, from which fewer descriptions he *would form a judgement, reach a conclusion, B*, which means that he would pass to another order of abstractions When the new happening ⧗ occurs, he handles it with an already formed opinion B, and so his description of the happening ⧗ is *coloured* by his older s r and no longer the x of the ideal observer but $B(x) = y$ His description of 'facts' would *not* appear as the $a, b, c, d, \ldots x$, of the ideal observer but $a, b, c, d, \ldots, B(x) = y$. Next he would abstract on a higher level, form a new judgement, about 'facts' $a, b, c, d, \ldots, B(x) = y$, let us say, C We see how the semantic error was produced The happenings appeared the 'same', yet the unconscious identification of levels brought finally an entirely different conclusion to motivate a quite different action, C''.

A diagram will make this structurally clearer, as it is very difficult to explain this by words alone On the Structural Differential it is shown without difficulty.

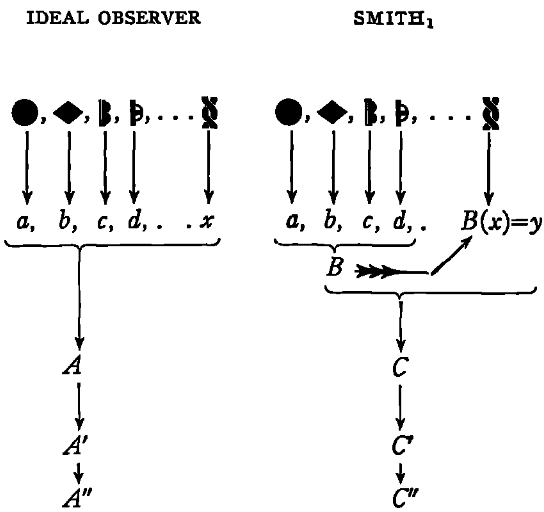
Seen happenings (unspeakable)
(First order abstractions) ....

Description
(Second order abstractions) ....

Inferences, conclusions, and what not.
(Third order abstractions).............

Creeds and other semantic reactions ...

Action ....



Let us illustrate the foregoing with two clinical examples. In one case, a young boy persistently did not get up in the morning. In another case, a boy persistently took money from his mother's pocketbook. In both cases, the actions were undesirable. In both cases, the parents unconsciously identified the levels, x was identified with $B(x)$, and confused their orders of abstractions. In the first case, they *concluded* that the boy was *lazy*, in the second, that the boy was a *thief*. The parents, through semantic identification, read these inferences into every new 'description' of forthcoming facts, so that the parents' new 'facts' became more and more semantically distorted and coloured in evaluation, and their actions more and more detrimental to all concerned. The general conditions in both families became continually worse, until the reading of inferences into descriptions by the ignorant parents produced a semantic background in the boys of driving them to murderous intents.

A psychiatrist dealt with the problem as shown in the diagram of the ideal observer. The net result was that the one boy was not 'lazy', nor the other a 'thief', but that both were ill. After medical attention, of which the first step was to clarify the symbolic semantic situation, though not in such a general way as given here, all went smoothly. Two families were saved from crime and wreck.

I may give another example out of a long list which it is unnecessary for our purpose to analyse, because as soon as the 'consciousness of abstracting' is acquired, the avoidance of these inherent semantic difficulties becomes automatic. In a common fallacy of '*Petitio Principii*', or 'Begging the Question' fallacy, we, by self-deceptive semantic evaluation, *assume the conclusion* to be proved In other words, we confuse the orders of abstractions Beside the wilful use of this fallacy by lawyers in courts to influence juries of low intelligence , a similar fallacy is widely committed in the reasonings of daily life and leads to many unnecessary semantic difficulties Particularly vicious is the use of the so-called 'question-begging epithets' We postulate the fact which we wish to prove, label it by another name, and then use the new higher order name in our premise It represents clearly a confusion of orders of abstractions

All such terms as 'un-patriotic', 'un-christian', 'un-american', 'pro-german' (during the World War), 'wet', 'dry' , fall into this group It is probably no secret that a large part of the population of this world was swayed by such methods during the war. In times of peace, large countries are continually swayed by such use of terms which play upon the pathological *s r* of the population, thereby facilitating the 'putting over' of different propagandas Similar procedures lead to many semantic difficulties in daily life It is easy to see that the difficulty is general, namely, 'the confusion of orders of abstractions' The antidote is equally general, and is found in the elimination of the 'is' of identity, resulting in the 'consciousness of abstracting' It should be noticed that these pathological reactions have long been known, and that they are extremely general We are told about them in schools under the name of 'logical fallacies', disregarding their semantic character, and so it is practically impossible to eliminate them or to apply the wisdom we are taught It is not difficult to see why this should occur In the older days, all the 'wisdom' was taught to us by purely 'intellectual', 'verbal', classical *A* and *el* methods We had no simple psycho*physiological* method of *complete* generality, which could be taught in a *non-el* way affecting *all* nerve centres It is known how difficult it is to 'change human nature', which simply means that the older verbal educational methods could not properly affect the lower centres It seems that the first step in developing a method to accomplish these ends is to use the Structural Differential, without which it is practically impossible to teach 'silence on the objective level' and 'delayed action' and to train through *all* centres in non-identity, 'stratification', natural order, and so in appropriate *s r* It appears that now, to begin with, we have acquired a workable and simple psychophysiological method for changing identification into visualization, and, in general, for the prevention or elimination of identification or confusion of orders of abstractions We have now discovered a mechanism which involves and deals directly with the reactions of the lower centres, 'senses', affects, 'emotions', . The older, difficult 'change in human nature' becomes an easily accomplished fact in a structural, $\bar{A}$ semantic education 'Human nature' can best be described, perhaps, as a complex of *s r*, which *can* be educated and 'changed' to a large extent

It seems unnecessary to enlarge further upon this subject Every attentive reader can supply endless examples of this kind of semantic disturbances from his own observation or experience Naturally, the generality, simplicity, and *physiological* character of the method proposed in this work become powerful assets, and instruction in the $\bar{A}$ methods can easily be given to, or acquired by, everybody It can be taught in homes and schools It gives a preventive psychophysiological method of training the *s r* in the millions and millions of cases in which human life becomes wrecked through the lack of a *working structural educational theory* concerned with these reactions But it is not enough to preach these 'platitudes', they must be practised as well If the parents and the boys mentioned above had been trained as children with the Structural Differential, it would have been an impossibility for the situation to have become so acute

Let us follow our *daily experiences* by the aid of the Structural Differential. We find ourselves on at least five levels The first represents the un-speakable event, or the scientific object, or the unseen physico-chemical processes on the sub-microscopic levels which constitute stimuli registered by our nervous system as objects The second consists of the external, objective, also un-speakable, levels on which we see with our eyes, On this level, we could make a moving picture, including actions , (writing a book is also behaviour) The third level represents the equally un-speakable psycho-logical 'pictures' and *s r* On the fourth level of abstractions we describe verbally our facts, that humans (*a*) eat, sleep , (*b*) cheat, murder , (*c*) moralize, philosophize, legislate ; (*d*) scientize, mathematize, Finally, in the present context, our inferences belong to the fifth level

Unfortunately, we usually abstract facts (*a*), identify the levels, and form a conclusion 'man is an animal', From this *conclusion* we confuse the levels again and colour the description of the facts (*b*), (*c*), (*d*) , jump again to higher levels and build conclusions from descriptions (*a*) and from *distorted,* coloured descriptions (*b*), (*c*), (*d*), and so obtain the prevailing doctrines in all fields These again lead us, in the field of action, to the mess we all find ourselves in In this dervish dance between the levels we entirely *disregarded uncoloured facts* (*d*)

The ideal observer would observe *all* forms of human behaviour at a given date, *not leaving out facts* (*d*) , then, without confusing his levels, and also without confusing descriptions with inferences, he would reach his higher order of abstractions properly, with very different resultant doctrines, which would produce entirely different semantic evaluation, and motivate equally different action

We may understand now why we must constantly revise our doctrines, for the above analysis throws a considerable light on the fact that scientists need training with the Differential as much as other mortals (the author included) History shows that they have not officially checked themselves up sufficiently to become aware of this fatal habit of confusion of orders of abstractions through identification

It might appear, at first glance, that all that has been said here is simple and easy On the contrary, it is *not* for the grown-ups, it is easy only for children and the young In all my studies and experimenting I have found that, for the reasons already given, the use of the Differential appears essential, and that it requires a long while and training to accomplish new semantic results As a rule, unless they are very unhappy, people try to trust their 'understanding', and dislike to train repeatedly with the Differential For some reason or other, they usually forget that they cannot acquire structural familiarity with, or reflex-reactions in, spelling, or typewriting, or driving a car , *by verbal means alone* Similar considerations apply in this case Without the actual training with the Differential, certainly the best results cannot be expected

To gain the full benefit involves the uprooting of old habits, taboos, 'philosophies', and private doctrines, the worst being the structure of our primitive A language with the 'is' of identity, all of which are deeply rooted and work unconsciously Only the semantic training with the Differential in *non-identity* can affect the 'habitual' and the 'unconscious' Rationalization, lip-service to the 'understanding' of it, will be of no use whatsoever Persistent training seems the only way to acquire this *special structural sense for proper evaluation,* and the habit of *feeling* when identification, or the confusion of orders of abstractions becomes particularly dangerous This feeling, as it involves most important factors of evaluation, is difficult to acquire, as difficult, perhaps, as reflex-learning to spell or to typewrite But, when acquired, it makes us aware of the continuous, necessary utilization of many levels of abstractions, which becomes dangerous only when we identify them or are *not conscious* of this fact We can then utilize the different orders of abstractions *consciously,* without identification, and thus keep out of danger. Most of the important *terms* appear as multiordinal, and, although they *belong* to verbal levels, they *apply* often to all levels, an important structural fact impossible to avoid, and one which makes this special semantic sense uniquely necessary to acquire

It seems unnecessary to repeat that everything that has been said above applies in the fullest extent to our ethical, social, political, economic, and international relations Before any sanity can be brought into the analysis of these relations, before they can be rationally analysed, the investigators would have to be trained to observe correctly and to avoid verbal structural pitfalls For the lack of such semantic training and re-education, the 'time-honoured' 'Fido' debates involving the 'is' of identity, continue on all sides, and lead to naught else but a waste of 'time' and effort

I say waste of 'time', simply because there seems no end to the paradoxes which, with a little ingenuity, we can build up when we begin to gamble with confusion of orders of abstractions and disregard multiordinality Any doctrine, no matter how structurally true or beneficial, can be defeated, confused, or delayed, by the use of such methods These problems appear of crucial semantic importance, because our lives are lived in a *permanent* structural interplay between different orders of abstractions All our achievements depend upon this interplay, yet the most acute and painful dangers also have their sources in the non-realization of this dervish dance between different orders of abstractions

Since we cannot evade the passing from level to level, or the use of multiordinal terms, our wisdom should consist only in not abusing these semantic conditions of human life As we must do that, let us do it, but let us not identify the orders, and thus let us evade the dangers Consciousness of abstracting gives us the complete *psychophysiological* solution of this complex situation, as it allows us to have the psychological benefits and to avoid the dangers by the use of *physiological* means

In conclusion, I must stress once more the importance of the structure of the language in which we analyse any given problem In the $\bar{A}$-system I am proposing, the term *order* is accepted as one of its very foundations In 1933, we know that as words *are not* the things spoken about , structure, and structure alone, becomes the only possible content of knowledge, and the search for structure, the only possible aim of science If we try to define structure, we can do so in terms of relations and multi-dimensional order The recent advances of science show, beyond doubt, that the day will come when all science will be formulated in terms of structure and, therefore, of physics, and physics formulated as a form of multi-dimensional geometry, based on multi-dimensional order, giving us, ultimately, multiordinal structure

The application of the term *order,* which involves physiological, as well as semantic, mechanisms of evaluation, to the analysis of human behaviour, has led me to the present $\bar{A}$-system and the investigation of the structure of language The discovery that some of the most important terms we use appear multiordinal, a character *concealed* by the 'is' of identity, has disclosed to us a most vital and inherent psychological mechanism, responsible in humans for many most desirable, many undesirable, and many morbid human characteristics It disclosed, also, the psycho-logical *structure* of these characteristics, and so we have obtained *physiological* means by which to enhance the development of desirable characteristics, and to prevent or transform the others

Further analysis has disclosed a natural survival order in evaluation the event first, the object next, the object first, the label next, description first, inferences next , in inherent importance We have also found that the majority of human difficulties, 'mental' ills included, involve semantic disturbances and exhibit, *not* the natural survival order, but the identification of different orders, resulting in a reversed (pathological) order

It is impossible in this book to review the data of psychiatry from the $\bar{A}$ point of view, as this would require a separate large volume which, I hope, will be written some day, but any one can verify the statements made above by himself from clinical literature, and also by analysing his own or other persons' life-difficulties, quarrels, disagreements , which generally involve quite unnecessary sufferings Psychotherapeutic literature shows abundantly that the success of the physicians depends mostly on reversing the pathological reversed order in a given field, and so restoring the natural order in the *s r* It is easily verified that, in most cases, when 'mental' illness originated through different life experiences, these would have affected very little, if at all, a child or an adult who was conscious of abstracting, and whose nervous processes and corresponding semantic states followed the natural order

With the aid of the Structural Differential and a $\bar{A}$ language of new structure, it is easy to train the *s r* of an infant, a child, or a young person, and possible, although much more difficult, to train a grown-up in the natural order Such a training becomes a potent preventive structural *physiological* method, as it eliminates the psycho-logical states of identification or reversed order, both of which represent the raw semantic material out of which future nervous disorders are produced

The *non-el* term 'order' is equally applicable in life and science, gives us, in 1933, the simplest structural common base, and allows us to attempt the formulation of a science of man, which ultimately becomes a theory of sanity, a consequence of a *non-aristotelian system*

It should be recalled that order is accepted in the present system as undefined and fundamental; yet its use is easily explained by the aid of the term 'between', and can be shown and applied in reference to empirical structures

If we can formulate a method which, through the application of a psychophysiological term such as *order,* and a simple device such as the training of *s r* in the natural survival order, or reversing the pathological reversed order, includes the mechanism of non-identity and one of the most important human nervous functionings, such a method, because of its structural simplicity and physiological character, may be expected to prove very workable I desire to stress most emphatically the very important *general, impersonal, preventive, semantic, reflex-character* of such a method

In actual life, we deal, for the most part, with persons who are 'mentally' or nervously disturbed in different degrees We could, eventually, divide them for our purpose into two groups (1) those who do not want to improve or get well, but who somehow like their fictitious worlds and the maladjustments connected with them, (2) those who genuinely want to get over their difficulties

In general, it is extremely difficult or impossible to achieve anything at all with the first group The second group is greatly helped if we give them means to work by themselves at their problems Very often it is most effective to explain to them this simple 'natural order', 'identification', and 'reversed order' mechanism, the multiordinality of terms , and so give them a *definite psychophysiological symptom* to struggle against These symptoms of identification or reversed order, in their generality and structural neurological fundamentality, underlie the process of formation of practically all known semantic difficulties of evaluation

The reader should not assume that it is always possible to eliminate identification and so achieve this coveted natural order, or the reversal of the reversed pathological order, but, whenever this *is* possible, the person is relieved in a great many psycho-logical fields The simplicity and generality, the physiological and structural character of this method seems its main recommendation, particularly as a preventive measure or semantic training for sanity The training is a laborious process, requiring great persistence, but, to my knowledge, very few trainings are easy, and, perhaps, none leads to more important results than does this one.

CHAPTER XXVIII

ON THE MECHANISM OF IDENTIFICATION AND VISUALIZATION

"Did you say 'pig,' or 'fig'?" said the Cat.
"I said pig," replied Alice, "and I wish you wouldn't keep appearing and vanishing so suddenly: you make one quite giddy!"
"All right," said the Cat, and this time it vanished quite slowly, beginning with the end of the tail, and ending with the grin, which remained some time after the rest of it had gone.
"Well! I've often seen a cat without a grin," thought Alice, "but a grin without a cat! It's the most curious thing I ever saw in all my life!"*

LEWIS CARROLL

The significance of the paradoxical phase is not limited to pathological states such as those previously observed, and it is highly probable that it plays an important part in normal men too, who often are apt to be much more influenced by words than by the actual facts of the surrounding reality. (394)

I. P. PAVLOV

In the case of an imbecile, repetition without comprehension, psittacism, may prevail; the rôle of visual impressions is null or nearly so among the illiterate; the deaf from birth who have learned to speak have no auditory impressions to intervene. But, normally, it is feelings and ideas that appear in action, in the form of language. (411)

HENRI PIÉRON

The specific neurones necessary for sensation are also necessary for the associative reawakening of that sensation, which is called the image—a dynamic process and not a photographic negative resting miraculously in the nervous substance, where some subtle spirit might go to consult it. (411)

HENRI PIÉRON

It is none the less true that certain cultivated persons can use visual images, and can even use these images in preference to others. (411)

HENRI PIÉRON

Objectification and visualization are usually not differentiated. The first represents a very undesirable semantic process, whereas the second, visualization, represents one of the most beneficial and efficient forms of human 'thought'. From a $\bar{A}$ point of view, such a lack of differentiation between the two reactions appears as a very serious problem, requiring an analysis of the respective mechanisms.

To visualize, we must have such forms of representation as lend themselves to visualization; otherwise, we must fail. The A-system, which could not adequately handle asymmetrical relations, and could not be built explicitly on structure, necessarily involves identification. In the A period, we were able to visualize objects and a few objective situations but all the higher abstractions were, in principle, inaccessible to visualization, making scientific theories needlessly difficult. A $\bar{A}$-system, free from identification, must be based explicitly on *structure* on all levels (structure defined in terms of relations and ultimately multi-dimensional

*Alice in Wonderland.

order), which can be easily visualized It should be recalled that structure, relations, and multi-dimensional order supply us with a language which completely bridges daily-life experiences with all science, leading toward a *general theory of values* Mathematics and mathematical physics then become the representatives and the foundation of all science, and in the human field a general theory of values will lead to adjustment or sanity and will some day include ethics, economics,

For these reasons, the Structural Differential is uniquely useful, as, at a glance, it conveys to the eye structural differences between the world of the animal, the primitive man, and the infant, which, no matter how complex, is extremely simple in comparison with the world of the 'civilized' adult The first involves a *one-valued orientation* which, if applied to the ∞-valued facts of life, gives extremely inadequate, wasteful, and ultimately painful adjustment, where only the few strongest survive The second involves ∞-valued orientation, similar in structure to the actual, empirical, ∞-valued facts of life, allowing a one-to-one adjustment in evaluation with the facts in each individual case, and producing a semantic flexibility , necessary for adjustment This flexibility is known to be the foundation for balanced semantic states, 'higher intelligence',

Visualization requires a definite elimination, through differentiation, of harmful identification, which, as usual, is based on incorrect evaluation of structural issues Thus, we have had endless, bitter, and futile arguments as to whether or not the 'mechanistic' point of view of the world and ourselves is legitimate, adequate, The average person, as well as the majority of 'philosophers', identifies '*me*chanistic' with '*ma*chinistic' Roughly, mechanics is a name for a science which deals with dynamic manifestations on all levels, thus, we have macroscopic classical mechanics, colloidal mechanics now being formulated, and the sub-microscopic quantum mechanics already being well-developed disciplines In the rough, 'machine' is a label applied to a man-made apparatus for the application or transformation of power But even machines differ greatly, thus, a dynamo is entirely different in principle, in theory, and in applications from a lathe or an automobile

If we ask 'Is the *ma*chinistic point of view of the world justified?', the answer is simple and undeniable, namely, that this point of view is grossly inadequate and should be entirely abandoned But it is not so with the *me*chanistic point of view, understood in its modern sense and including the quantum mechanics point of view, which is entirely *structural* In 1933, we know positively that even the gross macroscopic physico-chemical characteristics of everything we are dealing with depend on the sub-microscopic *structure* (see Part X) The details are not yet fully known, but the principles are firmly established. With a $\bar{A}$ understanding and evaluation of the unique importance of structure as the only possible content of 'knowledge', these 'firmly established' principles become *'irreversibly established'*. We may go further and say that the quantum mechanics point of view becomes the first structurally correct point of view and, as such, should be accepted fully in any sane orientation. If we stop identification, then we will differentiate between some simple facts. For instance, we will understand that any semantic state, reaction, or process has its corresponding sub-microscopic, structural, colloidal, and ultimately quantum mechanical processes going on in the nervous system; however, the *s.r.*, or feelings of pain or pleasure , *are not* the sub-microscopic processes. These belong to different levels, but with ∞-valued semantics we can establish in principle a one-to-one correspondence between them. Thus, when we differentiate adequately, the older *machinistic* objections disappear entirely; and, in its proper field, for structural reasons, we must preserve the *mechanistic,* and entirely abandon the too crude *machinistic* attitudes. The *mechanistic* (1933) attitude is based on *structure* and so is indispensable for visualization, and *training in visualization automatically abolishes objectification,* which represents an important special case of all identification. From the point of view of a $\bar{A}$-system, adjustment and sanity in humans depend, to a large extent, on their 'understanding', which is entirely structural in character, therefore, we must accept a *mechanistic* (1933) *attitude,* which, in the meantime, can be visualized.

The finding of structural means of representation facilitates *visualization,* imagining, picturing, In the adjustment trend we start with lower nerve-impressions, 'senses', 'feelings' , lower abstractions, and these are abstracted again by the higher centres. The higher centres produce the 'very abstract' theories, which cannot be visualized for a while. The lower centres, which are involved in visualization, can deal only with structures which can be 'concretely pictured'. So we always try to invent mechanistic or geometrical theories, such as can be handled by the lower centres.

Individual 'experiences', supplied by the lower centres of different individuals, do not blend directly. They are blended in the higher centres. In them manifold experiences, whether individual or accumulated by the race (time-binding), are abstracted further, integrated, and summarized. Once this has been accomplished, structural means are sought *and discovered* to translate these higher abstractions into lower, the only ones with which the lower centres can deal. Then we can 'visualize' our theories, and the higher centres not only influence the lower centres, but the lower centres have appropriate means by which to co-operate with the higher centres in their new *non-el* quests

The lack of explicitly structural forms of representations is responsible, also, for the difficulties which arise when the higher order abstractions are translated into the reflex-reactions of the lower centres, which can deal with 'intuitions', 'orientations', 'visualization', The so-called 'geniuses' have a very subtle nervous system in which the translation of higher order abstractions into lower and vice versa is easily accomplished From the point of view of forms or representations, we can have two issues. (1) we may have *el* forms of representations which are not based on structure, visualization , and cannot efficiently affect the activities of the lower centres, (2) we may have a *non-el* system based on structure, visualization , which can be translated simply, easily, and efficiently into the terms of the lower centres These problems are of educational importance and should be worked out more fully

In my experience with grown-ups who have had only a *short* contact with my work, I find, in many cases, that, although they may have even given their complete verbal approval of the main point of the system, yet, invariably, in practice, the full application is lacking Obviously, the semantic importance of the present findings is not in the verbal approval alone, when that approval is not applied, but in the consistent and permanent instinctive acquisition of the new semantic attitude which involves a complete elimination of identification, allness, elementalism,

We can teach any one to repeat verbally, by heart, instructions for operating an automobile, a piano, or a typewriter, but no one could operate them satisfactorily by reflex-action after such verbal training alone To operate effectively and skilfully any structural complex, we must become intimately familiar with its structural working through actual reflex-training, and only then can we expect the best results In my experience, this is true with language, and, without the *visual* Structural Differential on which we can point our finger to the objective level and urge silence , such basic semantic *reflex-training* cannot properly be given

If we ask a man 'Do you know how to drive a car?', and he answers 'Yes', we assume that he has acquired the proper *reflexes* If he answers 'No, but I know *about* it', he means that he has *not* acquired the proper reflexes, but that his 'knowledge' is on purely verbal levels, non-effective in application on *non-verbal* reflex-levels This applies fully to *s r*, we may 'know' *about* them, but we may never apply successfully what we supposedly 'know' To 'know' represents a multiordinal process which involves equally the activities of the lower nerve centres and of the higher. In our *el* systems we had no such distinction, and so we confused them The older 'knowledge', when presented in *el* language, could not have been absorbed easily by the *non-el* organisms-as-a-whole As the main task, at present, is to unlearn the older *s r*, the new reactions need a persistent training, particularly by the grown-ups The *non-el*, $\bar{A}$ language and method prove to have psychophysiological importance

Although the neurological mechanism underlying identification, objectification, visualization.., is not well known (1933), neurology gives us evidence that in these states, as well as in delusions and hallucinations, the actual lower nerve centres are somehow engaged We may assume that different 'resistances', 'blockages' , in some parts of the nervous system make the passage of nervous impulses more difficult, and it seems reasonable to suppose that, in such cases, the paths travelled by the nervous currents are different

In Fig 1, an hypothetical and over-simplified scheme of the different types of distribution of nervous currents, as is known functionally, is suggested The ordering is not anatomical but functional in terms of degrees of intensity In this scheme, we may consider that the nervous impulse (A) reaches the lower nerve centres, the brain-stem and the thalamus, passes through the sub-cortical layers and the cortex, continuously being transformed Finally, in returning, it may take either the beneficial and adaptive semantic form of visualization (V), free from identification and semantic disturbances, or may involve identification, with semantic disturbances, such as objectifications of different orders (O), delusions (D), illusions (I), or, finally hallucinations (H)

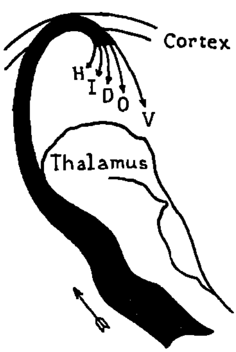


FIG 1

Identification, or confusion of orders of abstractions, consists of erroneous evaluation that which is going on inside of our skin has objective existence outside of our skins, the ascribing of external objectivity to words, the identification in value of 'memories of experiences' with experience, the identification of our *s r* and states with words, the identification of inferences with descriptions, Identification is greatly facilitated, if not actually induced, by the *A* structure of language in which we have *one name* for at least *four* entirely different entities Thus, the *A* 'apple' (without subscripts and date) is used as a label for the physico-chemical process, for an object, say, '$\text{apple}_{1\ \text{Feb 23 1933}}$', for a 'mental' picture on the un-speakable semantic level, and for the verbal definition Under such linguistic conditions, it is practically impossible, *without special training*, not to identify the *four* entirely different abstractions into *one* , with all the following sinister consequences

Delusions represent incorrect notions and inappropriate *s r* formed, not by insufficient knowledge or 'logic', but by affective pressure in a definite evaluational direction; as, for instance, delusions of grandeur; delusions of persecution, delusions of 'sin', delusions of reference,

Illusions appear more like real perceptions, but pathologically changed For instance, anything may be semantically coloured or interpreted, or evaluated as an offense, or a threat, or a promise,.

Hallucinations consist of 'perceptions', with all their vividness, but *without* any external stimuli Patients hear voices, see visions, feel pricks or burnings , when there is nothing to hear, or see, or to be pricked by

In *visualization,* identification does not occur, orders of abstractions are not confused, semantic disturbances do not appear, the *evaluation is correct,* a 'picture' is evaluated as a picture and not as the events,. In other words, because of the consciousness of abstracting, the natural order of evaluation is preserved But once, through identification, this natural order is reversed, it marks a pathological condition more or less morbid, and often of a non-adaptive character

Identification represents, in affective tension, the mildest semantic disturbance, consisting of an error in meanings and evaluation Objects are evaluated as events, 'ideas', or *higher* order abstractions, are evaluated as objects, as experience, as the un-speakable semantic states or reactions, otherwise, as *lower* order abstractions The confusion in the field of higher order abstractions follows a similar rule Inferences obviously represent higher order abstractions than descriptions, so, when they are not differentiated, higher order abstractions are again identified with the lower We all know from daily-life experience the fantastic amount of suffering we can, and do, actually produce for ourselves and others with such identifications.

In delusions, a similar but more intense identification occurs, resulting in erroneous semantic evaluation, wishes, feelings, and other semantic states inside of our skins are projected into the external world, giving delusionally strong objective evaluation

In illusions, we also ascribe to, or identify our complex semantic states with, different perceptions and evaluate our higher order abstractions as lower

In hallucinations, this process of reversing the natural order comes to a culminating point higher order abstractions are translated into, and have the full vividness and 'reality' of, lower order abstractions.

We see that the pathological processes of 'mental' illnesses involve identification as a generalized symptom, which means the reversal, in different degrees, of the natural order of evaluation based on the intensified confusion of orders of abstractions The more intense this process of reversal becomes, the more non-adaptive and morbid the manifestations It should be noticed that this analysis becomes a necessity once we decide to accept a *non-el* language This analysis is far from exhaustive, but an analysis in new *non-el*, structurally correct terms, throws a new light on old problems

Hallucinations which result from 'physical' illness do not represent a permanent danger, but when a patient seems 'physically' well, and his confusions of orders of abstractions, delusions, illusions, and hallucinations become completely 'rationalized', then these are unmistakable signs of serious 'mental' illness, suggesting sub-microscopic colloidal lesions Now this 'rationalization' represents nothing else but a nervous disturbance and involves *identification* somewhere In 'physical' ills the nervous system may be disturbed, but the illness does not usually originate in nervous disturbances, and so, as such, is not dangerous

The distinction between visualization and objectification based on a $\bar{A}$-system seems new, the difference is subtle, but when it is formulated we can discover a simple means whereby to control the situation If we were to take a 'bone' made of papier-maché and smear it with fat or meat, Fido would, perhaps, *objectify* (identify) such a 'bone' from the smell and the form of the papier-mache with an edible one, and would fight for it We do a similar sort of thing when we objectify Religious wars, the 'holy inquisition', the persecution of science, which we are witnessing even at the present day in some countries and communities, are excellent examples

We should notice that Fido was able to *trust* his natural, even 'objectified', instinct, for nature does not play such tricks on him, such as producing 'bones' of papier-maché If nature did, dogs that objectify and persist in their liking for such 'food' would soon be eliminated. These particular objectifications would be dangerous and painful to those particular kinds of dogs with that particular nervous system, and would ultimately prove of no survival value Thus identification, which represents an inappropriate evaluation, is harmful to all life, but is little noticed at present, because the main periods of the animal racial adjustment have been accomplished long ago Experiments on flies show that the number of mutants which may be produced in a laboratory is large, but very few would survive outside of a laboratory In unaided nature, these mutants probably occur, but seldom leave observable traces[1]

However, even today, as Pavlov has shown in his laboratories, we can impose, by an interplay of a four-dimensional order of stimuli, such conditions upon animals for which their nervous survival structure was not naturally adapted, and so induce nervous pathological states. Wrong evaluation is, indeed, harmful to all life and accounts for such rigid survival laws in nature, which science teaches humans how to make more flexible. Practically word for word, this applies to ourselves. We are constantly producing more and more complex conditions of life, man-made, man-invented, and deceptive for the non-prepared. These new conditions are usually due to the application of the work of some genius, and the nervous system and *s.r.* of most of us are not prepared for such eventualities. In spite of inventions and discoveries of science, which are *human* achievements, we still preserve *animalistic* systems and doctrines which shape our *s.r.* Hence, life becomes more strained and increasingly more unhappy, thereby multiplying the number of nervous break-downs.

It is known that not all people are able to visualize equally well. In the older days this fact was taken for granted, and did not suggest further analysis. Under present conditions in many human beings and also in animals, as shown in the experiments of Pavlov, the visual stimuli are physiologically weaker than the auditory ones; in man, however, the visual stimuli should be physiologically stronger than the auditory. This difference does not affect the *general* mechanism of the cyclic nerve currents and orders of abstractions. In the auditory type the main returning currents are deviated into different paths. The division between 'visual' and 'auditory' types is not sharp. In life we deal mainly with individuals who have no more than a special inclination for one or the other types of reaction.

In the case of 'mental' processes, human adjustment has to be managed on higher, more numerous, and more complex levels. Obviously, then, the auditory types are more enmeshed by words, further removed from life than the visual ones, and so cannot be equally well adjusted. This fact should not be neglected, and on the human levels we should have educational methods to train in visualization, which automatically eliminates identification.

The auditory channels which connect us with the external world are much less subtle and effective than the visual ones. The eye is not merely a 'sense-organ'. Embryology shows that the eye is a part of the brain itself, and what is called the 'optic nerve' must be considered not a nerve but as a genuine nervous tract. This fact, of course, would assign to the eye a special semantic importance, not shared with other 'sense-organs' or receptors We ought not to be surprised to find that the visual types are better adjusted to this world than the auditory types In pathological states, such as identifications, delusions, illusions, and hallucinations, there seems to be involved a translation of *auditory* semantic stimuli into visual images In these pathological cases the order of evaluation appears as label first and object next, while the adaptive order seems to require object first and label next, There seems little doubt that visualization is very useful, and that identification is especially harmful The *most effective means to transform the s r of identification is found in visualization, which indicates its special semantic importance*

The semantic *disturbance* of identification may have many sources, auditory included, but the only adaptive trend is in visualization, which involves in some way the optical neural structure Some structural light is thrown on this subject when we realize that, physiologically, the eye is more closely related to the vegetative nervous system, which regulates our vital organs, than the ear is In man the optic thalamus is greatly enlarged, so that the whole thalamus is often called the 'optic thalamus' Actually, the thalamus has many functions, other than visual, and is connected with affective manifestations

As most of our observations are accomplished by the aid of the eye, we should expect auditory types to be *poor observers,* and so racially, in the long run, not so well adjusted semantically Observation shows that the auditory types often have infantile reactions—a serious handicap From an adaptive point of view the 'normal', non-infantile, best-adjusted individual ought to be a visual type Auditory types must also be further detached from actualities than the visual types, as auditory stimuli involve more inferences than descriptions, which is the opposite of the functioning of the visual types If inferences, rather than descriptions, are involved, we naturally deal with higher abstractions first, and with the lower next, and so there is always a danger of the semantic confusion of orders of abstractions, which necessarily involves inappropriate evaluation, of which objectification is only a particular case

Even to common sense it seems clear that there is a significant difference between 'knowing' this world by hearing and 'knowing' it by seeing There is, likewise, a difference between the translation of higher abstractions into lower terms by the visual path, and the corresponding translation by the auditory path In daily life we never say 'I hear' when we wish to convey that we understand, but we say 'I see' When we say 'I hear', we usually wish to convey that we have heard something which we did not fully grasp or approve. The above relation is rather important, but has not been sufficiently analysed. The problems of introversion and extroversion are connected with it.

The relation between the problems of identification and the *number* of values *found* in the empirical world in connection with the *number* of values *ascribed*, or *assumed*, by our semantic processes, is most important.

The following analysis is, by necessity, one-sided, over-simplified, as a fuller analysis would require a separate volume. I consider many problems 'in principle' only, this allows me a briefer treatment necessary for my purpose, but it must be realized that our language and general semantics, which, *in practice*, we use unconsciously, are *extremely complex* and involve one-, two-, three-, and ∞-valued components, never, as yet, sharply differentiated nor formulated. Investigation shows that the ∞-valued semantics is the most general and includes the one-, two-, and few-valued semantics as particular cases. The one-valued semantics of literal identifications are found only among animals, primitive people, infants, and the 'mentally' ill, although more or less serious traces of some identification are found in practically all of us, because these are embodied in the structure of our language and prevent the acquisition of the ∞-valued systems necessary for sanity. For my purpose, it is enough to formulate the problems for the complete elimination of primitive identification, and then modern, ∞-valued, $\bar{A}$ semantics follow automatically. Under such conditions, I must concentrate on the vital problem of one-valued identification and treat the two-, and few-valued systems sketchily, 'in principle', although we must realize that the last systems have been made more flexible by the use of many ingenious verbal devices which I do not even mention in the present work.

Let me repeat that the attitudes, flexibility, or fixity, of our *s.r.* depend to a large extent on the structure of language used, which involves also its appropriate general semantics. The 'logic' of our schooldays represents a composite affair, in the main *A*, and we call it by this last name. This 'logic' can be considered as a two-valued 'logic' because of the fundamental 'law of the excluded third', expressed as 'A is B or not B', by which a third possibility is excluded. But even the traditional 'logic' had to admit in its scheme what was called 'modality', namely, some degrees of certainty or uncertainty with which a given statement is made. Lately, Łukasiewicz has shown that a three-valued 'logic' can be so formulated as to include modality. Later, he and Tarski generalized it to an *n*-valued 'logic'. When *n* tends toward infinity, this 'logic' becomes the 'logic' of probability. If these disciplines are made *non-el*, we have what I call one-, two-, three- , and ∞-valued *general semantics* Theoretically, and in practice, we are interested mostly in the one-, two-, three-, few-valued, and ∞-valued general semantics For my purpose, and for simplicity, I shall deal only with identification; that is, the primitive one-valued semantics, the influence of which is found in both the two- and three-valued semantics, *and may only be completely eliminated in an ∞-valued semantics*

We live in a four-dimensional space-time manifold which, on all levels, consists of absolutely individual events, objects, situations, abstractions , and we must conclude that structurally we live in an *indefinitely many-valued* or ∞-valued world, the possibilities of which follow in principle the laws of combinations of higher orders The above statement represents a description of a structural observation about the empirical world, independent of our pleasure, and can be contradicted only by exhibiting empirically, actual 'identity' or 'absolute sameness' , of different events, objects, or situations , which exhibiting becomes an impossibility if we decide to investigate facts more fully

Under such empirical conditions, for adjustment and so for sanity, we must have on semantic levels such theories, systems, methods which would allow us in a given case, under given conditions, at a given date , to evaluate the individual happenings *uniquely*, or which would allow us to establish a one-to-one correspondence between the essentially ∞-valued facts of experience and our semantic states It becomes obvious that this can be accomplished only if we have ∞-valued and *non-el* general semantics We see that the two-, or three-valued, *el A* 'logic', 'psychology' , and, in general, the *A*-system, being structurally different from the empirical world, will prevent, in principle, such an adjustment and, therefore, sanity

Identification may be considered as the remains of pre-human, or primitive, or infantile, one-valued semantics, which establishes, or results from, semantic states, by which the essentially ∞-valued facts of experience are not differentiated or evaluated properly, and so the indefinitely many values of these facts are identified into a single value Such identification is always structurally unjustified and dangerous, and may be the result of a great many factors, such as low development, ignorance, insufficient observation, 'wishful thinking', fears, pathological states of our nervous system, different semantic disturbances, 'mental' ills, infantilism in the grown-ups,. But among humans we cannot avoid training, through the mechanism of language and its structure, in some, most often unconscious, general semantics, and so a great deal depends on what kind of semantics or methods of evaluation we impose on our children

We should notice an important fact which is usually disregarded, namely, that a language, and often one word, involves a definite type of semantics. Thus, in primitive 'polysynthetic' languages, it is not a question of associations or superstitions, the mystic characteristics and the thing simply are not differentiated, but literally identified into one whole Thus, we have one-valued semantics where the 'good' and 'evil spirits' actually participate in everything considered as a synthetic whole [2]

A language of 'true' and 'false' involves two-valued semantics, the introduction of adverbs or their equivalents introduces modality and so three-valued semantics The introduction of indefinitely many degrees between the 'true' and 'false' leads finally to ∞-valued semantics

A diagram may help to make this clearer

A, B, C , ∞-valued and different facts of experience, which, in a given case, have, by necessity, *indefinitely many, single, individual* values

a, b, c., ∞-valued non-aristotelian orientation structurally *similar* to the empirical world which allows us, in a given case, to assign indefinitely many single, one-to-one corresponding values to the individual facts

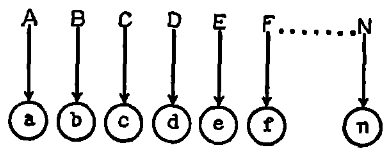


FIG 2

A, B, C , ∞-valued and different facts of experience, which, in a given case, have, by necessity, *indefinitely many, single, individual* values

Σ_1, Σ_2., two-, three- , and few-valued aristotelian orientation structurally non-similar to the empirical world, which compels us to ascribe two , or few values to the essentially indefinitely many-valued and different facts, resulting in identification of the many values into a few, which improper evaluation is projected on the facts.

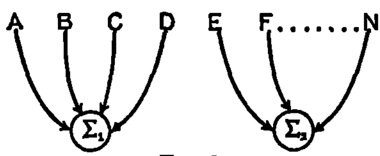


FIG. 3

A, B, C , ∞-valued and different facts of experience, which, in a given case, have, by necessity, *indefinitely many, single, individual* values

Ω, one-valued, animal, primitive , orientation, structurally non-similar to the empirical world, which compels us to ascribe one value to the essentially indefinitely many-valued and different facts, resulting in *identification* of the many values into one, which improper evaluation is projected on the facts

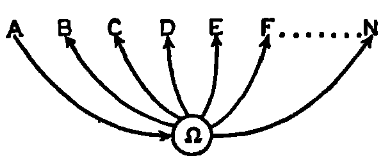


FIG. 4

In Fig 2, the arrows Aa, Bb , indicate the $\bar{A}$ one-to-one correspondence between the ∞-valued individual facts of life, A,B,C , and the corresponding *s r*, or a,b,c orientations , which ascribe single values to the different facts, establishing a foundation for structurally correct *proper evaluation* which helps adjustment and so sanity

In Fig 3, the A two-., few-valued orientation and type of correspondence is shown

In Fig 4, Ω indicates a single, say, proper evaluation of the *one* fact A The arrows ΩB, ΩC, ΩD ΩE, ΩF, , ΩN indicate the *projection* of the one-valued semantic state, or orientation on the essentially unchanged ∞-valued facts A, B, C , *distorting them* In other words, the ∞-valued facts, through the identification of many values into one, and by pathological projection, have been given wrong evaluation, thereby preventing, *in principle,* adjustment and sanity, particularly for a civilized human 1933

If we train our children in one-, two-, three-, and more generally few-valued *el, A* reactions based on corresponding languages, 'logics , the result must be that they will have great difficulty in adjusting themselves to a world of *non-el* ∞-valued facts, and that, even if they succeed, this would ultimately happen only after a great waste of efforts and unnecessary sufferings If we approach the ∞-valued facts of life with one-, two-, or even few-valued semantic attitudes, we must identify some of the indefinitely many values into one or a few values, and so approach the ∞-valued world with an orientation which *projects* ignorantly or pathologically our *restricted,* few-valued semantic evaluations on the ∞-valued individual facts of experience.

The above explanations apply in the fullest extent to the structure of language The daily language, as well as our attitudes toward it, still reflects primitive structural *s r* of the period before it was known that on the objective levels we deal *exclusively* with *∞-valued, four-dimensional processes* The language in the A-system represents, *in principle,* what may be called a three-dimensional and one-, two- , more generally few-valued linguistic system structurally non-similar to the ∞-valued, four-dimensional event-process conditions Let us analyse, for instance, the A term 'apple' This term represents, *in principle,* a name for a verbal, one-valued, and constant intensional definition, in which space-time relations do not enter What are the structural facts of experience? The object which we call 'apple represents a process which changes continually ; besides, every single apple that ever existed, or will exist was an absolute individual, and different from any other objective 'apple' In applying such a three-dimensional and one-valued language to essentially ∞-valued processes, we only make proper evaluation, and so adjustment and sanity, very difficult

Yet the structural adjustment is simple in a $\bar{A}$-system The A 'apple' was a name for a verbal *intensional definition;* in a $\bar{A}$-system, we manufacture indefinitely many names for the indefinitely many objective and different 'apples' by subscripts, 'apple_1', 'apple_2', 'apple_3' , supplementing the subscript with the date, thus, in '$\text{apple}_{1,\text{Feb 23 1933}}$', we gain the possibility of considering '$\text{apple}_{n,t}$' as ∞-valued, and so, in a given case , we are enabled to have a single name which we could relate to single values of the objective, absolute individuals, and absolute individual stages of the process Similarly with multiordinal terms Before the multiordinality of terms was discovered and formulated by me in 1925, these terms were silently assumed *in principle,* to be one-valued, and we were either prevented from using them in connection with ∞-valued orders of abstractions, or, if used by semantic necessity, we identified the indefinitely many values into one Both results were undesirable, the first established semantic blockages to creative scientific work, the other promulgated semantic disturbances But once the multiordinality of terms is established, we have ∞-valued terms to which, in a given context (by differentiating the different orders of abstractions which a context indicates), we can ascribe single values

Such a pioneering analysis may appear difficult at first, but this is only due to the lack of familiarity and established pre-A and A one-, two-, three-, or few-valued *s r*, all of which involve ultimately *identification* somewhere Once identification is abolished, however, and this is childishly simple, although not easy and rather laborious for grown-ups, ∞-valued semantics become natural and *automatic*, evading very serious theoretical difficulties In the present volume, I had to elaborate in detail upon different issues, simply because my readers will be mostly grown-ups with established *pre-A*, and A reactions, who must first be made to recognize the benefits of a $\bar{A}$ evaluation before they will be willing to undergo a laborious re-education of their older *s r* The procedure in the training of infants and children is extremely simple and entirely on their levels

There is, however, one point that I wish to make entirely clear From the older point of view, one might say that a $\bar{A}$-system may lead to 'over-rationalization' and, consequently, take 'all the joy out of life' Such objections are entirely unjustified First of all, the A-system leads to shallow, but often clever verbal interplay of definitions, mostly non-similar in structure to the world and ourselves, representing a species of apologetics, and usually called 'rationalization' The $\bar{A}$-system leads to structural adjustment of language and *s.r.*, and a *structural* enquiry, resulting in *understanding*. It makes shallow infantile 'rationalization', 'wishful thinking', and apologetics of different brands impossible, but leads to a higher order of adult intelligence, based on *proper evaluation*. In mere 'rationalization', we often have clever, but shallow, infantile evaluation, based on the ignorance or disregard of structural facts, which alone make up the content of all 'knowledge'. In a $\bar{A}$-system, by eliminating the sources of infantile evaluation and reactions, we supply the nervous system of the infant with uniquely appropriate material, so that it may develop into a 'normal' adult. In the older system, instead of helping, we hindered the development of adult standards of evaluation, with well-known results. There is nothing wrong with 'human nature' or the majority of nervous systems as such, but there is something definitely wrong with our educational methods inside and outside our schools.

There is another point which is still more convincing and, perhaps, even more decisive. The above-mentioned older objections are due to *s.r.* based on the play upon *elementalistic* terms and are a *neurological impossibility*. The organism works as-a-whole, and in the cyclic nerve currents it is impossible, by any known educational methods, to abolish 'emotions'. But what *can be* accomplished is this. By training in silence on the un-speakable objective levels and in differentiation between different orders of abstractions, we automatically abolish the infantile identifications and evaluations, we introduce a 'delay in action', which is the physiological means for getting our 'emotions' under control and for engaging the fuller co-operation of the cortex. Infantile 'over-emotionalism' is abolished in the adult. Infants would behave as infants, but this infantile behaviour would not be carried into the period when adulthood should begin. The 'emotions' are not abolished but 'sublimated'.

It is true that many standards would be changed. For instance, we might roughly say that an infantile type is often bored by a symphony and that jazz satisfies his infantile make-up. If we were to take such an infantile grown-up and compel him to listen only to symphonies, this would not be kind, nor would it transform his infantile *s.r.* into adult reactions. But, if unhampered by inappropriate semantic and so neurological training, he would be free to develop normally into an adult, and his own preference would be toward a symphony rather than toward primitive throbbings, his enjoyment, then, would not be diminished, but, perhaps, made fuller.

Similar analysis could be made of all human interests, with the result that the forcing of adult standards on infantile types would remain unkind, but the sad part of it is that, in spite of repressions, impositions., these imposed standards remain largely ineffective and are abandoned as soon as compulsion ends Not so, if, by proper semantic education, we allow the infant to develop normally into adulthood The *new* standards are not imposed, but become his own We do not then need compulsion from without, because the new standards act from within and become pleasurable and lasting

A similar process is very obvious in the practice of psychotherapy The standards of evaluation of the patients are usually inappropriate to the conditions of modern life and often clash sharply with the accepted standards Moralizing without changing by *other means* his standards of evaluation never accomplishes any satisfactory therapeutic results, quite the contrary, it often does a great deal of harm A physician would be very unwise to censure or condemn a symptom, as this would preclude any beneficial results What physicians usually do is to treat any symptom, no matter how repulsive, with great sympathy and understanding They *do not attempt* to change the symptom directly, but, by the understanding of its main mechanism, they try to *change the patient's standards of evaluation,* of which the symptom is only a consequence. If at all successful and the physician succeeds in changing the inappropriate standards of evaluation, the symptom then automatically disappears In daily life, we usually attack only the symptoms, disregarding mostly the underlying structural foundations, this method accounts for the doubtful results

Under infantile standards we apply similar methods to society Many may want to abolish wars, revolutions, 'depressions' , but they do not investigate structurally deep enough They attack the symptoms, instead of analysing the structural issues which produce these symptoms

In conclusion, let us notice that the analysis of a semantic mechanism on a printed page requires new terms and the co-ordination of many details , which, at first, do not always appear so simple, although, once the theoretical side is mastered, the educational application is genuinely simple Thus, the analysis of the one-, two-, three- , and ∞-valued semantics may appear difficult, yet, in practice, it only amounts to imparting through our educational systems a semantic flexibility, instead of fixity, to acquiring the inclination of starting with observations, followed by descriptions, from which we pass to inferences, in connection with awareness of these ordered processes. In training, it is enough to abolish identification, and this is easily achieved once we have produced the proper method, based on a language of new $\bar{A}$ structure. This last actually consists of but a few, new, simple, and common-sense terms, the analysis of which helps us to discover a few simple and invariant psychophysiological relations. Thus, identification is eliminated by starting with an *ordinal* language and method Once we get the feel of the horizontal and vertical stratification, and learn to differentiate between orders of abstractions, identification disappears. Silence on the objective levels produces a 'delay', involves and trains the cortex, our reactions become more and more intelligent in the human sense . . , and the most important results are reached by the simplest means

The training in visualization and the abolishing of objectification are the first and most important steps for a complete elimination of identification. When this first step is achieved, the rest is comparatively a very simple task

But the reader may ask why we should have to use such unfamiliar and, therefore, seemingly difficult methods to achieve such obvious results. Do we really need a $\bar{A}$-system to achieve the results which, even in an A-system, are known to be desirable? The answer is weighty with consequences and should be taken very earnestly In the A-system, these desirable results could not be attained generally, because the structure of our old languages and the method hampered rather than helped us New theories, new systems . are built precisely with the aim to facilitate adjustment Those questions which in the older days were supposed to be 'philosophical', 'metaphysical' , the application of which required a high grade of intelligence, knowledge , to start with, become, in the new way, simply *a problem of the structure of the language we use* All the issues appear closely interrelated. We do not require 'high intelligence' nor 'higher education' to begin with, in order to obtain these desirable results, as the results follow *automatically* from the structure of the language we accept and teach our children Thus, the older impossibilities are achieved simply and automatically, with the greatest possible efficiency and the most lasting results.

CHAPTER XXIX

ON NON-ARISTOTELIAN TRAINING

If the preliminary experiments described above should be fully upheld, an important fact in the physiology of the cortex will be disclosed—namely, that new connections can be established in the cortex, not only in the areas of optimal excitability, but also in those areas which are in one or another phase of inhibition (394) I P PAVLOV

That wretched monosyllable "all" has caused mathematicians more trouble than all the rest of the dictionary (23) E T BELL

these observations . . point to the view . that the mechanism of development of a conditioned reflex and the mechanism of external inhibition are somehow similar, and that the process of external inhibition bears some relation to the development of new connections between different cortical elements (394) I P. PAVLOV

In particular the factor of duration of time was shown to act as a real physiological stimulus, and experiments were described in which definite time intervals appeared as effective stimuli (394) I. P PAVLOV

The procedure for training in the present system by the aid of the Differential follows directly from the theoretical considerations which have been explained in the foregoing chapters The contentions of the system have been verified experimentally in all cases where it has been consistently applied

The main aim is to acquire the coveted 'consciousness of abstracting', on which non-delusional evaluation is based, and which becomes the foundation for non-pathological *s r* and sanity As we deal with different aspects of an organic process which inherently works as-a-whole, all these aspects appear strictly interrelated We have found by analysis two main aspects which underlie the others It appears that the *A* structure leads to semantic states which can be formulated as the feeling of 'allness', and that, through the 'is' of identity, it leads to the confusion of orders of abstractions Thus, for training, the program is readily sketched we must first eliminate the 'allness', then we must impart this peculiar stratification of 'human knowledge' which follows from the rejection of the 'is' of identity, in other words, eliminate identification It becomes also obvious that a theory of sanity cannot be separated from a $\bar{A}$-system

Since the organism works as-a-whole, all nerve centres should be trained so as to impart a permanent, lasting, and ingrained feeling of abstracting Once this has been achieved, the recognition of the vertical and the horizontal stratification of human knowledge becomes, also, a permanent semantic state This gives us a kind of semantic co-ordinate system, in which we can represent any life situation or scientific situation, or any difficulty, with great clarity, and so evaluate them properly In verbal theoretical explanations this procedure appears complex, in practice, it is not so It is extremely simple, provided we persistently follow the instructions, which are based on theory and practice Above all, we must not expect results too quickly.

For reasons already explained, students should not only hear and see the explanations, but should also *perform for themselves,* should handle the labels and indicate with their hands the different orders of abstractions After preliminary explanations, the children should be called to the Differential, and, using their hands, they should explain it This applies, also, to grown-ups and to patients The Differential is not only a permanent structural and semantic reminder which affects many nerve centres, it is more, for, in training, it conveys the *natural order* through all centres Any reader who refuses to use his hands in this connection handicaps himself seriously, because *ordering* abolishes identification

Fundamentally, there is no structural difference between the use of language and the use of any other mechanical device, they all involve reflex-action It is well known that any pianist, telegraph operator, typist, or chauffeur would not be a successful performer if he had to meditate about every move he makes As a rule, *verbal* explanations of the working of the respective machines are necessary at first, yet the structural reflex-skill required is actually acquired by prolonged practice, where again all nervous centres are involved We all know what amazing *unconscious* reflex-adjustments a good driver of a car can make in case of unexpected danger

A similar semantic reflex-skill is required in handling our linguistic apparatus, and, in case of danger, of sudden turns and twists, our orientation should also work unconsciously That is why the structural *feeling* for the working of the apparatus is required All nerve centres should be trained to employ the most effective means to affect the organism and its working as-a-whole

The semantic training of grown-ups and that of children do not differ in essentials Children have fewer established habits, have more fluid *s r* than adults, and, therefore, the results with children are achieved more quickly and last better

I shall now explain how to train children A similar method applies to adults, also; but an adult should not trust himself too much that he has completely acquired 'consciousness of abstracting' He must train very thoroughly I speak from personal experience Although I have the Differential before my eyes practically always and am the author of the present system, yet every once in a while I catch myself with one of the old vicious semantic habits Habits, and particularly linguistic habits, may be very pernicious and difficult to change.

We do not need to start with profound theoretical considerations, we may start with any familiar daily-life objects and a microscope or magnifying glass We bring the Differential into the classroom, with labels (except one) detached, but do *not* proceed to explain it We start with a little semantic *experiment* upon the subject of 'allness' We take any actual object, an apple, a pencil, or anything else which is familiar to the children The principles involved are entirely general and apply to all objective levels in a very similar way We tell them that we will have some fun Then we ask them to tell us 'everything' or 'all' about the object in question, in this case, the apple When the children begin to tell us 'all' about it, we write the characteristics down on the black-board *This last is vital* We must have a visual and extensional record of the ascribed characteristics When the children have exhausted their ingenuity in telling 'all' about the apple, we should *not* be satisfied We should make them doubt, urging them that, perhaps, they did not tell 'all' about it, using the word 'all' continually The term 'all' should be stressed and repeated to the point of the children's being thoroughly

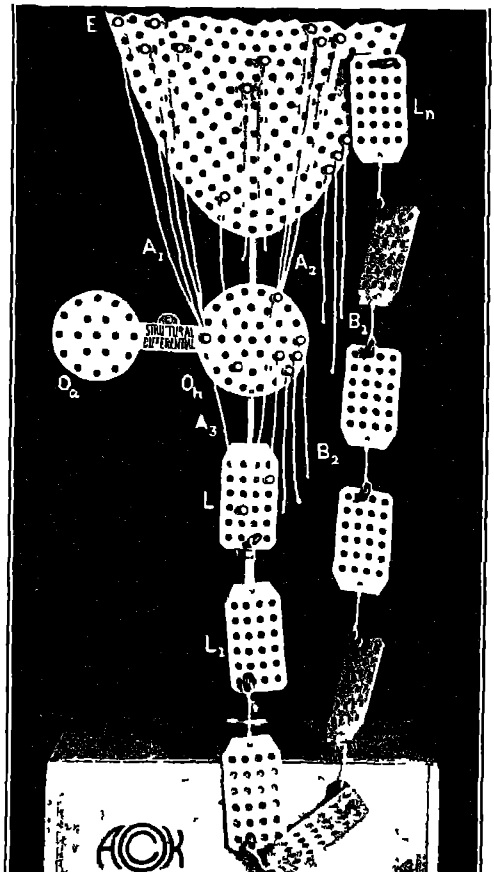


FIG 1
THE STRUCTURAL DIFFERENTIAL

annoyed with the term The more they learn to dislike this term, the better. We are already training a most important *s.r.*

We should *not* be satisfied with the best answers made by the most intelligent children In a large class there may even be a child who tells us bluntly that it is impossible to tell 'all' about the apple We should concentrate on the *less* intelligent children and deal particularly with them There are many and important reasons for this For one thing, the children become more eager and more interested in their own achievement. Then, too, they easily learn by example what a difference in intelligence means This understanding of the shortcomings of others has an important semantic, broadening effect In life, numerous serious 'hurts' occur precisely because we do not appreciate some natural shortcomings, and expect *too much* Expecting too much leads to very harmful semantic shocks, disappointments, suspicions, fears, hopelessness, helplessness, pessimism,

The less bright children benefit also The experiment is conducted on their level, so that they also have the maximum chance to benefit Soon the children begin to argue about the new method and to explain it to each other by themselves, for we have touched very vital and complex semantic processes of 'curiosity', 'achievement', 'ambitions' , characteristics strongly represented in the child's life We evade, also, the danger of taking clever, yet shallow, replies as standard The last error would be fatal, as the issues are fundamental, and we should not rest content with mere verbal brilliancy

When the subject seems exhausted, and the list of characteristics of the apple 'complete' (we repeatedly make certain that the children assume they have told us 'all' about it), we cut the apple into pieces and show the children experimentally, using eventually a microscope or a magnifying glass, that they did *not* tell us 'all' about the apple

It may appear to some educators that such training might involve some undesirable psycho-logical results But later, when consciousness of abstracting is acquired as a lasting semantic state, this fear appears entirely unjustified, as explained further on The first step in dealing with 'reality' seems to demand that we abandon entirely the older delusional methods

When the children have become thoroughly convinced of the non-allness and the *impossibility* of 'allness', we are ready to explain to them what the word abstracting means, using again the terms 'all' and 'not all' We show them a small rotating fan and explain to them about the separate blades which when rotating we see as a disk In such demonstrations we can go as far as desired All science supplies data (e g , the dynamic structure of seemingly solid materials) We must select the data according to the age of the children or the knowledge of the grown-ups Everything said should be demonstrated empirically from a structural point of view

The next step is to demonstrate practically that an object taken from different points of view has different aspects for different observers We may use different objects or wooden geometrical figures painted with different colours on different sides We may place the object in different positions and ask the children their descriptions, which should be written down The descriptions will, of course, be different, and the children should be made thoroughly aware of this In all these preliminary exercises the ingenuity of the teacher has a vast field for exercise, and we do not need to enter into details.

When all these results have been accomplished on the level of the *least developed* child, we then proceed to explain the Differential as a structural diagrammatic summary of the above results It is a *positive condition* that the new language be used, and that an object be described as an *abstraction* of some order. If this vital structural point is disregarded, most of the psycho-logical semantic benefits of 'non-allness' are either lost or greatly lessened We should make this term clear to the child, and should train him in its use, as it appears uniquely in accordance with the structure and the functioning of his nervous system The child should be warned that the old languages are not structurally suitable for their future understanding and semantic adjustment This warning should be repeated seriously and persistently

Having eliminated 'allness', we begin to eliminate the 'is' of identity, which, at the primitive and infantile stages of racial human development, happens to be extremely ingrained in our *s r*, embodied, as it is, in the structure of our daily language As was explained before, identification is a natural reaction of the animal, the primitive man, and the infant, reflected and systematized in the A and older linguistic systems, which, through the ignorance or neglect of parents and teachers, is not counteracted and so is continued into the lives of children and grown-ups, until, finally, it becomes embodied in the structure of what we call 'civilization' (1933) In a theory of adjustment or sanity we must counteract this animalistic, primitive, or infantile *s r* by building a $\bar{A}$-system, which entirely rejects the 'is' of identity

In the A-system, through the use of this 'is', different orders of abstractions were unconsciously identified in values, in obvious contradiction to empirical facts In other words, being identified in values, they were treated as of one order or on one level and so did not necessitate indefinitely many expanded orders of horizontal and vertical differences Similarly with the objectively meaningless 'infinite velocity' of a process, it does *not* allow *order* But once we have a finite velocity of a process, *order* makes its appearance as an indispensable aspect of a process *The finite and known velocity of nerve currents on the physico-mathematical levels results in ordered series on physiological levels, in non-identity and proper evaluation on semantic levels, and in orders of abstractions and a non-aristotelian system and general semantics on verbal levels*

Once we abolish in our language the always false to fact 'is' of identity, we automatically stop identifying different orders of abstractions We do not assume that they represent one level, which becomes expanded into a natural ordered series of indefinitely many different orders of abstractions, with different values Adjustment, therefore, sanity and adulthood of humanity, depend on proper evaluation, impossible under conditions of delusional identification of fundamentally different orders of abstractions We must then train the *s r* in the natural *physiological order* of the process of abstracting which, on the psychological levels, become non-pathological semantic evaluation

In the case of training in the 'non-allness', it was necessary to start with the analysis of an ordinary object, to give the child a simplified theoretical explanation, and then to demonstrate it empirically The child will be easily 'convinced', but this conviction is not enough, because it will not affect permanently his *s r* We explain this difficulty very simply, telling him that, although he 'agreed' with our presentation, he will very soon 'forget' it, and so we need a permanent visual reminder which is supplied by the strings, freely hanging from the event and from the object, and indicating those 'characteristics left out', or not abstracted

In the elimination of the 'is' of identity, we have also structurally interconnected aspects The rejection of this 'is' becomes an equivalent to the stressing of the *stratification* in the structure of 'human knowledge' To facilitate training, we should *stress both aspects* by all available means, and should involve as many nerve centres as we can Thus, through the ear we stress verbally the formula of the rejection of the 'is' of identity by indicating with our finger the different orders of abstractions, in the meantime, affecting the eye while we repeat 'this *is not* this' We utilize the kinesthetic centres, not only by pointing the finger to different levels, but also by making broad motions with our hands, indicating the stratifications We should train in both horizontal and vertical stratifications, always using the hands The horizontal stratification indicates the difference, or ordering of different order abstractions, the vertical stratification indicates the difference between 'man' and 'animal' and the differences between the different absolute individuals. In both cases, the semantic effect of the 'is' of identity is counteracted

The above procedure in training has an important neurological foundation Besides what has been explained already, we find that a word has four principal characteristics with corresponding cortical representations A word can be heard, seen, spoken, and written Language, thus, involves many nervous functions, e g, auditory, visual, and diversified motor nerve centres, interconnected in a most complex network of 'horizontal' and 'vertical' fibres The use of the Differential involves all available nervous channels, we see, we hear, we speak, we move our hands, indicating stratification, 'non-allness', engaging large cortical areas, and so have the maximum probability of affecting, through *non-el* methods, the organism-as-a-whole. The Differential gives us a special, simplified, yet advanced interracial *structural symbolism* (1933), which affects wide nervous areas of the illiterate, or nearly illiterate person, or of the infant, which otherwise could not be affected It is known that extensive reading and writing, as well as speaking a number of languages, has a very marked cultural effect and helps visualization and consciousness of abstracting The reason can be found, perhaps, in the fact that a learned polyglot, or a scholar, utilizes many nerve centres in co-ordination In the older days, unless one became a scholar of some sort, it was extremely difficult to train these nerve centres in co-ordination With the Differential we can train simply, and comparatively quickly, all necessary nerve centres, and so impart to children and to practically illiterate persons the cultural results of prolonged and difficult university training without any complicated technique This last should always be regarded as a means and not as an end

In my experience with children, and with men from the lowest 'mentality' to the highest, the non-identity of different orders of abstractions is usually taken lightly It all seems so simple and self-evident that no one assumes that there could be serious, unconscious, structural, semantic, linguistic, and neurological delusional mechanisms involved, which cannot be reached without specially devised non-identity training The delusional feelings of 'allness' and 'identity' are peculiar in that, like other pathological states, they tend to appear as all-pervading It is the most difficult in daily, as well as in medical, experience to make a breach in this all-pervading tendency, but once this delusional state is even partially replaced by glimpses of *m o* reality, the further elaboration and training in adjustment to 'reality' becomes comparativel simple Thus, in practice, if we *start* with ordinary objects, feelings and words, and train in the non-allness and non-identity, any child, or any grown-up, even an imbecile, can follow this easily Once this feeling has been acquired, and in most cases it is only a matter of method and persistence to acquire it, the main *semantic* blockage has been eliminated, and the rest is comparatively easy I have had no opportunity yet to verify it, but I am convinced that even a superior imbecile could be trained to differentiate between descriptions and inferences, after he has learned to differentiate between the objective levels and words In such a training with imbeciles we can go in simplicity as far as desired, thus, if the given individual is hungry and says he wants 'bread', we hand him a label which is attached to the objective bread, and he would be quickly made to realize that the symbol *is not* the thing symbolized

It should be realized that in the training we should impart the obvious fact that words or labels represent conveniences, and *are no* the objects or feelings themselves We should carry the labels in or pockets, so to say, as we carry our money, or checks for hats or trunks and not identify them 'emotionally' with what they eventually stand for because monetary standards change, and hats and trunks get exchange lost or burnt To accomplish this, we must have *objective labels*, whic we may handle and carry in our pockets, and also an objective somethin to which we can attach the labels In the present $\bar{A}$-system the rejectio of the 'is' of identity is complete and applies to all levels Thus, th event *is not* the object, the object *is not* the label, description *is n* inference, a proper name *is not* a class name , the characterist ascribed to events, objects, or labels *are not* identical, an object, a situ tion, or a feeling *is not* identical with another object, situation or fee ing , , all of which establishes a *structure of horizontal and vertic stratification* At an early stage of the training, we must begin wit what appears the simplest and most obvious to the child, namely, th absence of identity between the word and the object, or that the wor *is not* the object We accomplish this by stressing that one cannot si on the *word* 'chair', that one cannot write with the *word* 'pencil', o drink the *word* 'milk', These simple facts should always be trans lated into the *generalized form*, indicating with the hand the two level on the Differential, conjointly with the fundamental formula 'this *is no* this'. We should always tell the child that the formula is entirely gen eral, but for the present we should not go into any further details

At this stage we can advance one step further, still using *onl* ordinary objects as examples, and explain the un-speakable characte of the [illegible]ely, that whatever we can see, taste, smell, handle., is an [illegible]dividual (demonstrated empirically) and *un-speakable*. We [illegible] the apple, bite it (actually performing), and explain that, alt[illegible] object is *not words*, yet we are very much interested, and t[illegible] so, in this un-speakable level. Then we explain repeatedly [illegible]ngth, emphasizing the important principle of evaluation, that [illegible] we must deal with the objective level, yet this level cannot be [illegible]d by *words alone*. As a rule, it takes a few weeks, or even months, [illegible]re this simple s.r. is established, the old identification being psycho-[illegible]siologically very much ingrained. Once this is established, we stress [illegible] fact that we must handle, look, and listen, never speak, but remain [illegible]nt, outwardly as well as inwardly, in order to find ourselves on the [illegible]ective level. Here we come to one of the most difficult steps in the [illegible]ole training. This 'silence on the objective level' involves checking [illegible]n neutral grounds of a great many 'emotions', 'preconceived ideas', [illegible] step, in the meantime, appears as the first, the simplest, most obvi-[illegible] and most effective psychophysiological 'reality-factor' in eliminating [illegible]delusional identifications.

[illegible]ce the child is thoroughly aware of the absence of identity be-[illegible]words and objects, we may attempt the expanding of the notion [illegible] to the 'objective levels'. Such training requires persistence, even [illegible]t seems fundamentally simple. We demonstrate and explain that tion[illegible] actual bodily performance, and all objective happenings, *are not* [illegible]. At a later stage we explain that a toothache, or demonstrate that [illegible]al pain of a prick, *are not* words, and belong to the *objective* [illegible]*kable levels*. Still later, we enlarge this notion to cover all ordi-[illegible]bjects, all actions, functions, performances, processes going on [illegible] our skin, and also all immediate feelings, 'emotions', 'moods'., [illegible]on inside our skins which also *are not* words. We enlarge the [illegible]' to all happenings on the objective levels and the animalistic, [illegible] nature' begins to be 'changed' into quite a different *human* [illegible].

When this is accomplished the rest is much simplified, although much more subtle. We explain, as simply as we can, the problems of evaluation and s.r., stressing and making obvious the fact that our actual lives are lived *entirely* on objective, un-speakable levels. We illustrate this all the time by simple examples, such as our sleeping, or eating, any activities, or pain, or pleasure, or immediate feelings, 'emotions', which *are not* words. If words are not translated into the first order un-speakable effects, with the result that we do not do something, or do not feel something, or do not learn or remember something [illegible]rds take *no effect* and become useless noises

One fact should be stressed, namely, that the prob[illegible] one of 'inadequacy of words' We can always invent 'adequate[illegible] but even the most ideal and structurally adequate language will [illegible] things or feelings themselves On this point there is *no poss[illegible] promise* Many people still utter quite happily, pessimistic exp[illegible] about the present *language*, based on silent assumptions connecte[illegible] unconscious delusional identification, and believe that in an 'adequ[illegible] language the word by some good primitive magic would be identical wi[illegible] the thing The more the denial of the 'is' of identity is driven home, a[illegible] the sooner it becomes a part of one's *s r*, the sooner the 'consciousness [illegible] abstracting' is acquired

We are now ready to go further into the theory of *natural eval[illegible] tion* based on natural *order* As a preliminary step, we must show [illegible] peatedly the difference between descriptions and inferences, using sim[illegible] examples We must stress the fact that words, as such, must be divid[illegible] into two categories a first, of descriptive, in the main, functio[illegible] words, and a second, of inferential words, which involve assumptions [illegible] inferences Thus, 'A does not get up in the morning' may be consider[illegible] as descriptive If A explicitly refuses to get up, the statement 'A *refus[illegible]* to get up in the morning' may also be considered as descriptive If A d[illegible] not explicitly refuse, this statement becomes inferential, because A m[illegible] be dead or paralysed If we would say simply, 'A is lazy', such a stat[illegible] ment represents an illegitimate inference of high orders based on igno[illegible] ance, because in 1933 it is known that 'laziness' represents a symptom o[illegible] physico-chemical, colloidal, or semantic disturbances It should b[illegible] stressed that this discrimination between descriptive and inferentia[illegible] words, although extremely important, is not based on any 'absolute' differences, but, to a large extent, depends on the context I shall not analyse this problem further, because any parent or teacher who has acquired the consciousness of abstracting himself will find more examples at hand than are needed

We should notice here a very vital, yet generally diregarded, structural fact—that *human* life is lived under conditions which establish a *natural order* of importance between different orders of abstractions This natural order should be made the basis of natural adaptive evaluation and so survival *s r* As our lives are lived *entirely* on the *un-speakable level,* which includes not only scientific objects and ordinary objects, but, also, actions, functions, processes, performances, feelings, 'emotions' , this level is obviously first in importance, and the verbal level, which is only auxiliary, comes next in importance The analysis of the relative evaluation between description and inferences appears extremely complex and would require a separate volume, beyond the scope of the present work Here we may assume the generally accepted opinion that the reliability of inferences depends on the reliability of the descriptive premises, and that description is more reliable than inference In importance and in temporal and neurological natural *order,* description comes first, inferences, next If we consider different orders of inferences, or inferential words, inferences or inferential words of lower order are more reliable and so more important than inferences of higher orders (inferences from inferences of lower orders)

As science is a *racial* product and so represents structural descriptions and inferences of an enormous amount of constantly revised observations and formulations of past generations, this racial product, 'science', is more reliable and important *in principle,* particularly in its negative results, than the individual abstractions of individuals If some individuals happen to be 'geniuses', who upset racial scientific abstractions, they are under the scrutiny of other scientists who, no matter how biased or slow, remain judges of their products In 1933 the opinion of scientists is the most dependable opinion we have We must accept, at a given date, the racial, particularly negative, abstractions as more reliable, establishing in evaluation the event (scientific object) first, and the ordinary object next It should be stressed that the 'object' of daily experience, in human life, is by far not so reliable as that in the life of animals entirely without human interference Thus, a high tension wire, or a third rail, or high explosives are not found in unaided nature and do not forewarn us as ordinary objects do These 'objects' possess characteristics concealed or not obvious on the objective level of our ordinary inspection of, let us say, sight, hearing, or smell, yet these characteristics appear just as 'real' and dangerous as ever It appears, then, that the 'scientific object', or the event, in contradistinction to the ordinary object, is more important than the daily object, no matter how important the latter might be In fact, the only macroscopic importance of objects, outside of aesthetic and symbolic values, may be found in those not obvious physico-chemical, microscopic, and sub-microscopic characteristics Thus the importance of food, or air, or a chair is found precisely in these physico-chemical effects which result from eating, from breathing, and from resting on a chair, and so again these hidden characteristics, revealed *only* by science, appear much more important than the gross characteristics manufactured by our nervous systems which we recognize as an object

We come thus to a *natural scale* of a definite *natural order*, which also establishes the *natural order* of *genetic importance* and represents the *natural* base for *survival semantic evaluation*. For our purpose the relative order may be represented as the scientific object or the event first, ordinary object next, the ordinary object first, the label next, description first, inferences next, extended to descriptive and inferential words.

If we use the 'is' of identity and identify in value or importance the different, ultimately non-identical, levels, we nullify in principle the natural order of evaluation, which, by psychophysiological necessity, appears as a *reversal* of natural order in various degrees. We find many reasons for this curious fact, but, for our purpose, it will be enough to suggest that (1) words are simpler and take less effort to handle than objects, (2) inferences being higher order abstractions than descriptions, are psycho-logically closer to our feelings and easier for any individual to manage than impersonal descriptions which require developed linguistic training, power of observation, self-mastery, and, in general, consciousness of abstracting. The reversal of the natural order must lead to non-adjustment and results in pathological symptoms in different degrees. The natural order consists of asymmetrical relations expressed by an ordered series, not only as to space-time, but as to values. All our experiences and all we know indicate definitely that ordinary materials ('objects') are extremely rare and very complex special cases of the beknottedness of the plenum, that the organic world and 'life' represent extremely rare and still more complex special cases of the material world, and, finally, so-called 'intelligent life' represents increasingly complex and still rarer special cases of 'life'. When we identify the members of these series, we disregard the asymmetrical character of this series and transform it into a fictitious, or delusional, or false to facts symmetrical relation of identity. It becomes also obvious why in the A-system, which did not allow asymmetrical relations, proper evaluation, adjustment, and sanity in general were, in principle, impossible.

Although the language used in this connection is not familiar, it is not entirely arbitrary. It appears experimentally that four-dimensional order has physiological importance on the one hand, on the other, that on the psycho-logical levels it involves the semantic factors of evaluation. In training in the *physiological natural order*, we train the *evaluation* or appropriate human and adult *s.r.* on the *psycho-logical levels*.

In the difference between the un-speakable 'scientific object' and the ordinary object, the objective level and the verbal level, we find the precise spot at which we differ most radically from the animals. If we disregard these differences and retain the 'is' of identity, we must somehow copy animals in our nervous processes. Through wrong evaluation we are using the lower centres too much and cannot 'think' properly. We are 'over-emotional'; we get easily confused, worried, terrorized, or discouraged, or else we become absolutists, dogmatists,. The results of such copying of animals are usually tragic, as might be expected. Owing to wrong evaluation we add self-made semantic difficulties to the difficulties which we actually find in nature When we live in a *delusional* world, we multiply our worries, fears, and discouragements, and our higher nerve centres, instead of protecting us from over-stimulation, actually multiply the semantic harmful stimuli indefinitely. Under such circumstances 'sanity' is impossible

It seems that here in the elimination of the 'is' of identity we have put our hands on an extremely powerful reflex-mechanism for the education, or re-education, of our 'emotional' life As has already been said, suppressing or repressing our feelings is dangerous, and should be avoided The old animalistic educational systems were built on repression and suppression, with sad results But since we had no other means of education, we had to use the older means or else abandon this special education altogether Not so in the new $\bar{A}$ way with the Structural Differential. We *do not* repress or suppress We teach silence on the objective level *in general,* which is a most impressive *'emotional' education,* on perfectly *neutral* grounds, one of the consequences of the elimination of the 'is' of identity Any bursting into speech is not repressed, a gesture of the hand to the labels reminds us that words are *not* objects, or action, or happenings, or feelings Such a procedure has a most potent semantic effect It gives a semantic jar, yet this jar is *not repression,* but the realization of a most fundamental, natural, structural fact of evaluation in which we should all be well trained Disturbing *s r* subside, and no one is 'hurt' It takes long and persistent training, but the results are most beneficial.

We must note an important difference between a statement involving the 'is' of identity, that 'we are animals'—which has nothing to do with the actual facts, all of us (the animals, too) *are not* words, but represent absolute individuals and all different—and the statement that we 'copy animals' in our nervous reactions In the first case, nothing can be done In the second case, although the results are *equally sad,* we can stop 'copying animals' the moment the mechanism is discovered and we begin to realize that we are doing so Thus, the old hopeless becomes hopeful

I have already mentioned that some educators may assume the eventual harmfulness of training in the consciousness of abstracting on the ground that children should be kept 'close to reality'. The answer to such an argument can be found in the recognition that what in the older days seemed 'reality' must now, in the light of new knowledge, be considered delusions, and the older training as preparatory for acquired un-sanity. The modern conditions of human life appear much more complex than those of animal life, or of the primitive man. Every year, perhaps even every month, new human 'realities' make their appearance, complexities arise and our educational systems do not equip the children semantically to meet these new conditions. After investigation one may find by himself that the older 'allness' and identifications represent delusional factors found nowhere in the empirical world, and thus have to conclude that if we train children in such delusions, adjustment to the actual world is made extremely difficult, if not impossible It is true that some beneficial results do not appear at once, but only after the full consciousness of abstracting is acquired Thus, at an early stage of the training, when the student begins to realize the delusional character introduced by the 'is' of identity, the general and well-known tendency may struggle hard to *retain* the *delusions* His *first* reaction may be that of disappointment, with its many concomitants, depending on his temperament, metaphysics , but when he acquires the freedom of the full consciousness of abstracting, all levels will be evaluated properly and he will be able to adjust himself to conditions of *m o* reality described in the present work, which cannot be avoided by any one 'Knowledge or 'intelligence' is only possible with abstracting, and, therefore, it fundamentally involves 'non-allness' 'Omniscience' would involve a 'knowledge' of every point-event These are fundamentally different, and such a world would be one of chaos, where knowledge would be impossible Life, *m o* abstracting, and *m o* intelligence start together and are *conditioned by the m o process of abstracting*

Among the many semantically beneficial results of such training, besides the training in sanity and, therefore, in adjustment, a few other benefits should be mentioned. Our life, our *m o* mentation, the structure of our language, with its syllogisms, fallacies , consist for the most part of the constant utilization of the different levels of abstractions This appears as an inherent characteristic of 'human knowledge', and, therefore, we cannot abolish it without abolishing *m o* intelligence altogether Intelligence requires the passing from level to level in both directions All the benefits we possess follow from this, but also many semantic dangers are hidden in it Similar remarks could be made about an automobile, A great many beneficial results follow from the use of automobiles , but there are also great dangers involved For instance, at present we have regulations for the driving of an automobile A driver has to pass his examination, demonstrate his practical reflex-ability in driving , before he is allowed to drive in public Similarly with our language, we find the greatest benefits in it, and we should utilize them Proper training in the use of language should teach us how to avoid dangers Obviously, 'consciousness of abstracting' teaches us how to avoid these dangers, likewise, once we become trained in the passing to higher and higher order of abstractions, we become capable of the performance of what we call 'high intelligence'. The difference between 'high intelligence' and 'low intelligence' consists in the fact that a 'high intelligence' has a larger outlook backwards as well as forward, a 'low intelligence', as suggested in Fig 2, sees only a little backwards (ignorance) and foresees only a little A 'high intelligence' has a larger span or field, it knows more about the past and looks further into the future

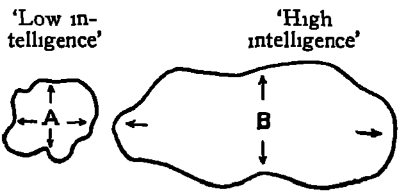


FIG 2

It is no mystery that when we want to look further into the past and the future we need higher and higher order abstractions By training in this passing to higher and higher abstractions we train the 'mind' to be more efficient, this 'mental' expanding should be the structural and semantic aim of every education

Once we eliminate identification, we must accept *structure* as the only possible content of 'knowledge' and also realize that no 'knowledge' is ever free from some structural assumptions. Sometimes it is pathetic to watch the metaphysical performances of some otherwise very eminent scientists, who seem entirely innocent of these facts They often attempt to divorce their metaphysics from science, and miss the point that primitive metaphysics represents 'science' or the structural assumptions of that period, whereas modern science represents structural assumptions or metaphysics of modern 'times', which cannot be reconciled with the older 'science' The difference appears in dates, not in kind The real problem before mankind presents itself in the selection of a structural metaphysics. If we select the primitive structural assumptions and have to live under present conditions, we must become a split personality which cannot adjust itself If we accept modern structural assumptions called science, we may adjust ourselves In no case can we free ourselves entirely from some structural assumptions. The problem becomes one *of dates,* and of un-sanity versus sanity. These problems appear of unusual importance, because the difficult scientific technique does not enter into this field at all, and the few structural data (1933) can be given in the simplest form to children and even to the feeble-minded. In the older days this problem was entirely misunderstood. We tried to 'popularize' science in the sense that we translated structurally correct language into the daily language of *primitive structure,* this resulted only in bewilderment, we did not analyse the structure of language and its role in our lives and *begin* with a structural linguistic revision. Once this revision is accomplished, and we build a $\bar{A}$ language, the semantic background is prepared for a natural acceptance of modern *structural* metaphysics (science) of each date and the older 'popularization' becomes unnecessary. Such procedure would help to integrate the individual, while the older methods only help to split him.

Let us recall that the animal stops somewhere in his abstracting. When we come to a *stop,* and consider it 'final' or that we 'know all about it', we copy animals in our nervous reactions. Training in passing from order to order of abstractions as such, trains the particularly *human,* fluid, non-blocked *s.r.,* counteracts, and ultimately abolishes, the animalistic blockage. In a language of a given structure we can express ourselves in a definite way; and, if that way is incomplete, we must leave the field open, for in a structurally different language the issues may look entirely different.

It is fundamental to stress that the old 'unknowable' becomes entirely abolished. This 'unknowable' originated in the primitive identification and elementalism. Our ancestors could not miss indefinitely that identification was false to facts, yet somehow the emphasis, which the ecclesiastical authorities (for their purpose) laid on the importance of the *A-system,* prevented them from completely rejecting the 'is' of identity. The un-speakable was called the 'unknowable', a very gloomy term, indeed. The use of this term prevented them from discovering, long ago, that the only content of 'knowledge' appears as structural, with all the following *non-el* consequences. 'Knowledge' was expected to represent somehow more than 'knowledge'—a silent self-contradiction. On this foundation whole systems of delusions were built. With the newer realizations, we understand that the only possible content of 'knowledge' appears as *structural,* so that we can know all which belongs to the structural legitimate field of 'knowledge'. What does not belong to the field of 'knowledge' must be considered meaningless, and making noises about it, one way or another, will not help us at all, quite the contrary, it involves us in delusional states Students of the history of 'philosophy' may realize, in this case, particularly, the drama and the dangers which the playing on such *m o* terms as 'knowledge' may needlessly produce

Through the semantic mechanism which it involves, 'consciousness of abstracting' abolishes many fears, despairs, and other disturbances which follow from the confusion of orders of abstractions We become introverted extroverts, in other words, we become affectively well-balanced, and ready to deal with empirical first order effects on their levels, and with verbal problems on their different levels We learn, also, to *observe*, as soon as we have learned 'silence' on the 'objective levels' Realizing that we abstract in different orders, we slowly acquire the most creative structural feeling that human knowledge is inexhaustible, we become more and more interested in knowledge, our curiosity becomes aroused, our sporting spirit stimulated and our level of *m o* intelligence raised

It is well known that the higher intelligence is characterized by a critical attitude By training with the Structural Differential until the memory of the characteristics left out and the non-identity become a permanent semantic acquisition with us, this critical attitude is also developed No one who feels habitually these 'characteristics left out'—'this is *not* this'—will ever *take a word or a statement for granted* He will enquire, investigate, will always ask 'what do you mean', a question which automatically leads to further investigation, and finally strikes the bottom of the undefined terms which divulge our silent structural creeds and metaphysics

We should avoid the mistake of assuming that the average man or a moron does not 'think' His nervous system works continually, as does that of a genius The difference consists in that its working is not productive or efficient Proper training and understanding of the semantic mechanism must add to efficiency and productiveness By the elimination of semantic blockings, as in identification, we release the creative capacities of any individual We release him from the primitive semantic bondage in the daily and constant use of a powerful instrument called language—full of benefits, but also full of dangers—the structure of which he totally misunderstands Such misunderstanding must lead to inefficiency in the use, and so to the abuse, of this function Instead of being a semantic slave of the structure of language, he becomes its master

When we become more civilized and enlightened, no public speaker or writer will be allowed to operate publicly without demonstrating first that he knows the structure and semantic functioning of the linguistic capacities. Even at present no professor, teacher, lawyer, physician, or chemist , is allowed to operate publicly without passing examination to show that he knows his subject. The above statement does not mean control or censorship. Far from it. Our language involves a much more intricate, beneficial, or dangerous semantic mechanism than any automobile ever had or will have. We do not control the drivers in their destinations. They come and go as they please, but for *public safety* we demand that they should have acquired the necessary reflex-skill for driving, and so we eliminate unnecessary tragedies. Similarly with language, of which the ignorant or pathological use becomes a public danger of a very serious semantic character. At present public writers or speakers can hide behind ignorance (1933) of the verbal, semantic, and neurological mechanism. They may 'mean well', yet, by playing upon the pathological reactions of their own and those of the mob, they may 'put over' some very vicious propaganda and bring about very serious sufferings to all concerned. But once they would have to pass an examination to get their licence as public speakers or writers, they could not hide any longer behind ignorance. If found to have *misused* the linguistic mechanism, such an abuse on their part would be clearly a *wilful act*, and 'well meaning' would cease to be an alibi.

We must accept the obvious facts which make the older theoretical 'democracy' or the older theoretical 'socialism' a scientific impossibility. If, in 1933, 99% of the population of the globe appear as infantile or 'mentally' deficient, how can any one expect that the majority or the mass could ever have proper evaluation or non-pathological *s.r.*? All history shows at present, and this evidence should not be taken lightly by *scientifically enlightened society*, that the majority appears 'always wrong', and that all that we call 'progress', 'civilization', 'science' , has been achieved by a very small minority. Such an understanding should guide our future conduct if we desire better results than we have at present. Under $\bar{A}$ conditions, not the state, nor different private societies, but professional scientific bodies would have to set the standards and perfect the technique of the linguistic structural examinations. They would also select members who would serve on the examination boards. It might seem that such a $\bar{A}$ innovation would not be important or far-reaching. This would be a mistake. It seems that most of those public writers and speakers may be considered privately as 'honest' men, who do not realize that under A conditions they often impose on defenceless masses delusional states which too often become of a pronounced morbid character. Once such an examination would force them to look into structural, semantic, and linguistic problems, we may take for granted that a great many of them would become able to evaluate properly their own activities and comprehend the harm they do As a result, quite probably, a great amount of useless, befogging issues, delusional writings and speeches would *not* be produced, with great benefit to all concerned No one would censure them Consciousness of abstracting would accomplish that They would become their own censors, aided, also, by the newly developed consciousness of abstracting on the part of some members of the audiences or readers

It would be desirable to experiment and introduce parallel classes in schools for a while, one group continuing in the old A-system, the other being trained in the $\bar{A}$-system We may expect that at the end of a year the results would be fairly tangible The ones which acquire the 'consciousness of abstracting' should show a marked improvement in character, should behave better, and should also show better results in scholarship, not to mention, in addition, the *preventive* semantic benefits in their future life and adjustment Experimenting under various conditions is very desirable, as we deal with such a tremendously broad and fundamental structural problem that it is impossible at present to foresee more than the main results and bearings

In a school one three-dimensional Differential should be enough, but in each classroom a large printed diagram, which is published also, should be permanently exhibited on the walls and applied in all studies This is necessary, not only because such a reminder makes the children thoroughly familiar with the 'characteristics left out', 'natural order' , but also because the children will discuss it and settle their educational and personal difficulties by its aid and so train themselves in $\bar{A}$ reactions In my practice, I have found that one of the main difficulties of the learner, or in 'thinking', in general, consists in the fact that in any verbal discussion we must utilize different orders of abstractions and *m o* terms If we do not realize this, the problem often seems very involved, once we are conscious of it, however, the problem becomes simple In fact, it may be said that this special flexibility which is entirely absent in animals and little developed in the primitive man, represents the working mechanism of 'high intelligence', and that this special flexibility can be acquired through proper $\bar{A}$ training

The dealing with reflex-reactions and with experimental theories in general has one very encouraging characteristic, namely, that no matter how difficult the theoretical side may be, the practical is invariably extremely simple Thus, a theoretical treatise on the Einstein theory, or the new quantum mechanics, or on an automobile, radio, or piano, or on music, or the conditional reflexes of Pavlov , may present, and, in fact, generally does present, difficulties, because it is formulated on purely verbal and analytical levels But these levels are most important, for we find that on these levels the *full evaluation*, and so the full realization, of existing or possible *relations* and *meanings* is accomplished In these verbal levels we find also economical and effective means to analyse further developments on which the possible range of applications ultimately depends Such a treatise can be produced by a single man and thereby becomes available for the rest of us

A description of the application is, however, very simple; we label the related parts of some structure, describe, mostly in terms of order, their interrelations, and then give instructions how to act, push, pull, or turn a given part to get such and such results These descriptions, although verbal, refer exclusively to some physical structure, so that men of very low 'mentality' can soon become acquainted with the practical problems concerned When the reflex-handling of the physical structure is acquired, the experimental and behaviouristic aspects become childishly simple A child can see the experimental results of any theory, or notice the ease and simplicity of the reflex-adjustments a good driver can make

But what an infant, a savage, or an ignorant man can*not* do, is *appreciate* the *meanings* of given occurrences and *evaluate* them, in other words, they cannot *relate* the given occurrences to other occurrences which alone give the *significance* of the happenings Thus, not only physicists, but even the average man, knew of the equality of gravitational and inertial mass, it took the genius of an Einstein, however, to *evaluate* properly, to have the proper *semantic reaction* toward this 'commonplace fact'. The present work shows clearly that all semantic disturbances exhibit a lack of proper evaluation, or in getting hold of the *meanings*, or *relations*, or *order* of different order abstractions. Only a full *theoretical* understanding can supply us with those meanings and produce in us the proper *s r* of evaluation—a necessary psychophysiological step for further advance, and for full application of the conquests already made

As the present work is experimental throughout, and deals with verifiable subjects, such as the structure of languages on record, the natural order of development, the pathological reversal of order, which, if again reversed, restores the natural order , and, when applied, brings about most beneficial experimental results, all that has been said about experimental theories applies fully in our case

Just as in other disciplines, the instructions are simple 'push these', 'pull this', or 'turn that', so, in our case, this simple descriptive rule which refers to the objective Differential is given as 'this *is not* this' Once the reflex-activities have been acquired, we can, for instance, enjoy the pleasure of an automobile trip, the music of a radio, or a semantic trip toward sanity in harmony with ourselves and others, very simply, in spite of the underlying theoretical complexities which are always means and not ends in themselves

But here we must face an important difference It is easy to demonstrate empirically to the majority of us the usefulness or pleasurableness of automobiles and radios, but it is very difficult to demonstrate the benefits of consciousness of abstracting to those who have not acquired it Before the experimental data begin to accumulate and become common knowledge, the main evaluation will have to be made on theoretical grounds Besides, before children can be trained by the simple and easy methods outlined above, adults must first re-train and re-shape their own *s r*, which re-training and re-shaping are not easy, and require even more difficult theoretical considerations Because of this, the present work had to be written in the form of a text-book for parents, teachers, physicians, and workers in 'mental' hygiene, and for future students and research workers in psychophysiology and semantic hygiene

At the beginning, in the application of the method, a number of difficulties will be discovered which will have to be overcome As a rule, the training in non-pathological *s r* proves to be easiest and simplest with very young children Most of it, or at least the laying down of the semantic foundations for such reactions, should be accomplished at home by specially trained teachers, if the parents are unable to do that themselves In countries or communities where the national or local governments show an interest in the health of the population by providing, for instance, specialists in preventive vaccination, specialists to train in preventive measures against semantic disturbances will probably also be provided

In elementary schools the teachers at first will have to train themselves as best they can with the help of specialists, but in high schools, colleges, and universities, special instructors will have to be employed

The first concern, then, is to start the education and training of teachers With this end in view, the present work has been written so as to form a fairly complete outline of the whole problem, reference literature has been indicated, so that any one who wants to specialize in the subject can find some suitable text-book as a primer As to qualifications for the professional $\bar{A}$ instructors,—it is, at present, very difficult to foresee details, but, as the full consciousness of abstracting leads to *s r* which, also, follow unconsciously, or which should so follow, from the study and acquiring the *feel* of the calculus,—students of mathematics would, perhaps, be the most desirable Any specialist in a new line of work has to learn a great deal, and this cannot be avoided, but it makes a difference what kind of training one has had as a young person Thus, it is simpler for a student of mathematics to learn about psychiatry, or psycho-logics, than for a psychiatrist or a psycho-logician to learn mathematics However, for a person with university training, this is less important than a genuine willingness to master the subject Once the consciousness of abstracting has been acquired by such a student, his semantic blockages will be eliminated. He will then have no difficulties whatsoever with details, or even in doing creative work along this line

With very young children, in the beginning an hour a day for several months should be devoted to the subject. When they have acquired the consciousness of abstracting, the results should not be entirely trusted as to permanence, but, at least, once a week the problems should be recalled to them How many hours a week should be devoted to it in high schools, colleges, and universities I shall not venture to suggest, because the working hours in these institutions are already very crowded The training in consciousness of abstracting automatically eliminates an enormous amount of semantic blockages, and would facilitate the acquisition of learning in all branches of knowledge, and so save 'time' and effort—the more so, if the respective teachers themselves were to become conscious of abstracting

The beneficial results which are to be expected may be found in better scholarship, more interest in studies, improved character, higher *m o* intelligence, and a better general adjustment All of this seems quite apart from the *preventive* character of the training as a protection against many semantic disturbances in the future But when teachers of all subjects acquire consciousness of abstracting themselves, they will probably discover new means and methods of conveying more simply and more effectively what they wish to convey to their pupils. I am convinced that the hours spent on semantic training would actually turn out to be an important *economy of efforts* Moreover, it would effectively give the children and students the highest grade of *cultural training*, which at present we acquire only occasionally and with difficulty, without the conscious co-operation of our teachers.

CHAPTER XXX

IDENTIFICATION, INFANTILISM, AND UN-SANITY VERSUS SANITY

Common sense, do what it will, cannot avoid being surprised occasionally The object of science is to spare it this emotion and create mental habits which shall be in such close accord with the habits of the world as to secure that nothing shall be unexpected (457) BERTRAND RUSSELL

Medicine is to-day an Art or Calling, to whose exercise certain Sciences are no doubt ancillary, but she has forfeited pretension to be deemed a Science, *because* her Professors and Doctors decline to define fundamentals or to state first principles, and refuse to consider, in express terms, the relations between Things, Thoughts and Words involved in their communications to others (122) F G CROOKSHANK

Unless physiology like any other of the sciences basic to medicine will teach less fact and more method, it might as well be deleted from the catalogue

Can anything be done to help the situation? Not, I think, without large vision The student of medicine needs not more external but more internal discipline * MARTIN H FISCHER

I wonder how soon we shall be far enough along to have the physician ask How much and what, if anything, is *structural?* how much *functional, somatic* or *metabolic?* how much *constitutional, psychogenic* and *social?*** ADOLF MEYER

Section A General

The name of Freud is usually associated with the term 'the unconscious' This term appears as a general *descriptive* term standing for a great many psycho-logical semantic processes In 1933 the work of Freud is generally accepted as important and very suggestive, although further experiments by many research workers and practicians have shown that the freudian formulations have not the exclusiveness formerly assumed for them

It is useless to deny that the term 'unconscious' is fundamental and necessary The use of the term is best shown in the study of hypnotic phenomena Some patients do certain things under hypnotic influence, and then seemingly lose all memory traces of such doings upon emerging from hypnosis Careful experiments showed that after prolonged efforts these recollections could be made accessible to the patients' waking consciousness The difficulty in recalling was not ordinary 'forgetting'. What is 'forgotten' can also be spontaneously 'recollected' Here the situation seemed different, in that these lost 'memories' required considerable work and effort for their reconstruction. The psycho-logical

*Teaching of Physiology *Jour Asso Med Colleges* Apr 1929

**The "Complaint" as the Center of Genetic-Dynamic and Nosological Teaching In Psychiatry *New Eng Jour of Med* Aug 23, 1928 state in such cases of perfected 'forgetting' was called 'unconscious', which, as a descriptive term, is very satisfactory

The origin of the freudian theory of the unconscious was strictly scientific The theory was a new generalization in a new structurally appropriate language to account for *experimental facts* Subsequently, a large number of other experimental facts showed that the work of Freud was sound as far as it went Different workers, from different sets of facts, amplified or reshaped the freudian theories At present, there are several schools that differ widely in language, all of them based on the fundamental system-function of Freud, however

In reading the literature of the subject, one finds most diversified material Seemingly, there are facts which prove 'beyond doubt' each and every theory, no matter how widely they differ among themselves It is also easy to find experimental facts which can be accounted for by several different theories

Such a situation appears unsatisfactory This lack of generality conceals a very important and workable semantic mechanism, which, under *A* identification, 'allness', and elementalism, becomes pathological, resulting in arrested or regressive symptoms Once we pass to a $\bar{A}$-system free from the above harmful semantic factors of delusional evaluation, the difficulties do not arise The more my enquiry progressed, the more it became obvious that the underlying mechanism appears similar in all psychoanalytical theories It seems that the general problem may be formulated as the need to discover methods for non-delusional evaluation affecting our *s r*, and so be able to make the 'unconscious' 'conscious'

The term 'consciousness' is an incomplete symbol, as it lacks content If we use the term 'consciousness of abstracting', we ascribe content and also gain empirical means to bring under educational control a vast array of important psycho-logical processes The *negative* term 'unconscious' does *not* imply specific content, and the main difficulty in its practical application is to find its content, or to ascribe content to it Once this is achieved, the 'unconscious' becomes 'conscious' A patient whose unconscious semantic difficulty is made conscious either improves or is entirely relieved For a general theory, we must find general structural means of ascribing semantic content to the 'unconscious' Different schools have elaborated different means of discovering this desired content All schools agree that the behaviour difficulties are due to experiences hidden in the 'unconscious' and that bringing them to 'consciousness' seems the main goal The diverse schools have an unduly bitter attitude toward one another, and have not attempted to analyse the problems at hand from a more general, more workable *non-el* structural, semantic, system-function, and linguistic point of view Whether or not the term 'consciousness' has any content besides 'consciousness of abstracting' may be disregarded for the moment At any rate, the term 'consciousness of abstracting' gives very vital and workable psycho-physiological means of analysis, of an impersonal, and general structural, and semantic character Enquiry into the clinical cases and literature shows that pathological cases, amenable to treatment, appear improved by a similar evaluational treatment, namely, the correction in some form or another of the semantic disturbance of the lack of 'consciousness of abstracting'

'Mental' illnesses (infantilism included) appear as semantic arrested development or a regression to lower levels, to those of the primitive man, the infant, the animal The animal *is not* conscious of abstracting, man can become so. Here we find the precise mechanism of a decisive nature which not only supplies us with *preventive* measures, but which should also become of therapeutic value

All life exhibits conservative characteristics acquired during the long periods of its development In the facts of heredity and embryology, we have an excellent evidence of this In its development, the germ cell of an animal or man repeats in a very abbreviated way the structures of forms from which it descended The ever-changing environmental conditions, although they affect each organism to a large extent, produce extremely few hereditary changes, which again may be considered an indication of the conservative characteristics of life

As we have already explained, life, abstracting, and 'intelligence' started together, and are consequences of the physico-chemical colloidal structure of the protoplasm Psychiatry also assumes that 'the unconscious', 'tendencies', and 'impulses' originated with life itself From this point of view the past piled up structurally upon the past until the highly complex organism called Smith made its appearance In this process of evolution the 'instincts' and 'impulses' have had an important role, not only conservative, but also compensatory and protective In man, *s r* should be based on proper *evaluation* and so play both a stimulating and *protective* role Under *A* conditions of delusional evaluation, the protective role appears practically non-existent, the human organism, under modern conditions, becomes over-stimulated, resulting often in pathological conditions Consciousness of abstracting, or the elimination of delusional evaluation, abolishes man-made, artificial, and harmful irritants.

I shall borrow from Jelliffe an excellent diagram to illustrate the evolution of the periods of growth and shall follow closely his exposition.[1]

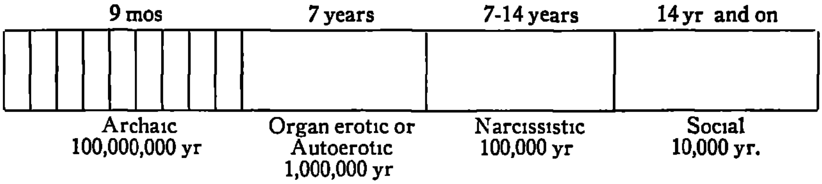


Fig 1

Some such classification of the periods of growth has been forced upon psychiatrists by the study of 'mental' ills, and is justified by embryology and an endless chain of empirical observations

The first period represents the archaic period and is of the greatest antiquity In it the past is roughly recapitulated from, let us say, the beginning of unicellular life to the anthropoid ape At the fertilization of the egg the hereditary constitution is established, and during the gestation period all prenatal influences are laid down The life of the baby before birth may be described as a vegetative existence of full indolence with all needs supplied by the mother's body

At birth, the first 'struggle for existence' begins, the struggle for air, as symbolized by the first cry At this epoch the baby already appears as a self-running organism with some *s r* His vegetative nervous system is integrated and functioning He begins to 'feel' Pleasure and pain begin to be significant semantic factors This period is called the organ erotic or autoerotic, since, as with animals, its main interests are *'sense' gratifications* Many millions of 'sense' receptors suddenly have thrust upon them from the environment a mass of energy with which the organism has to deal somehow At first there is a rivalry between different 'senses' Later, co-ordination appears Each group of receptors establishes its own semantic values for itself, depending upon its own cell growth This period may be divided, schematically, from birth to seven years, and corresponds roughly to the evolution from the higher animals to primitive man This period is extremely important from a semantic and educational point of view At this stage in the human child the nervous system is not fully developed, and different environmental influences (language, doctrines included) may twist this development, so that irreparable harm can easily be done

The narcissistic period is named after the Greek mythical figure, Narcissus, who, seeing his reflection in a pool of water, became so engrossed in self-adoration that he rejected the attentions of Venus and was killed In another version of the myth his punishment was loss of sight This period covers, more or less, from seven years to fourteen years As the name indicates, it represents a semantic period of self-love

The child has not entered, as yet, into a social stage of development. He remains egotistical, egoistical, self-centred, and *asocial.*

At about fourteen, the social semantic period begins, which leads, when 'normal', to the adult socialized individual

We should realize that these semantic stages are 'normal' when they are lived through within the age limits indicated here Even if children show some characteristics which are not desirable (organ erotic or autoerotic, narcissistic), this, in itself, does not constitute a danger, provided they outgrow these undesirable manifestations The serious dangers, and even tragedies, begin when some of the infantile or narcissistic semantic characteristics are carried over into the life of the grown-ups.

Not only 'intellectual' growth but also 'emotional' development may be arrested on some earlier lower level In such cases we speak of idiots, imbeciles, and morons on the 'mental' levels, and of moral imbecility, infantilism, narcissism, and, in general, of 'mental' illness on the 'emotional' levels

Besides arrested growth, or under-development in some respect, cases of so-called *regression* are frequently encountered Regression follows the general scheme as outlined in Fig 1, but in a reversed order. The following diagram, Fig 2, is also taken from Jelliffe, with slight modifications

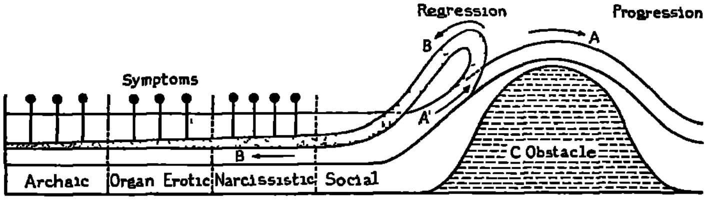


FIG 2

As processes, life, development, or regression are best represented as 'vector quantities' which have direction and magnitude In one type of cases of regression the progressive tendency or energy is strong, yet the obstacle is also very great, so that the progressive tendencies may not be strong enough to carry over the obstacle or to conquer it Again, the progressive tendency or energy may be weak, and the obstacle correspondingly slight, yet strong enough to start the regressive movement

In the healthy individual the progressive tendency is not easily diverted from its forward course He conquers his obstacles (C) and goes on (arrow A) Weaker individuals (A′) may surmount their obstacles with more difficulty or may start regression on smaller obstacles, as indicated by arrows (B) In such cases they may regress to different levels, developing a neurosis or a psychosis, in accordance with the degree of regression It is extremely instructive to study these different phases in regression and to watch how the symptoms arrange themselves in a perfectly orderly manner In some instances the regression goes so far as to bring the patients to the foetal level Such a patient sits in a dark corner in the foetal position with the head covered with a rag His 'mentality' and semantic responses are similar to those of the foetus, practically none

Regressions to the archaic level are usually hopeless of improvement, so that I shall not analyse them in this work We are mostly interested in *under-development,* or in regression not further than to the autoerotic or the narcissistic semantic levels, in which treatment in many instances yields curative results.

Jelliffe gives among others a very instructive diagram as a method of showing how the personal make-up of an individual can be plotted (psychogram) These diagrams afford excellent graphic means for orientation One of them I reproduce on the opposite page (Fig 3) [2]

The circular form of the diagram is particularly appropriate, as it shows clearly how the horizons, activities, and interests widen from the archaic (animal?) through the child and savage, to the adult socialized individual The dips in the eye, stomach, and bladder sectors correspond to definite symptoms In the eye sector the dip goes to the narcissistic semantic level Whenever this patient is riding in an automobile and another car is coming close, so that a collision seems possible, the patient experiences a compulsory shutting of the eyes, a typical narcissistic semantic symptom which symbolizes that something which one cannot see cannot happen The patient did not regress to the organ erotic level and become actually blind or deaf (psychic or rather *semantic* blindness or deafness), so the dips are not plotted to the organ erotic level In the bladder and nutritive sectors we see that the curve sinks as low as the organ erotic level These dips correspond to striking semantic symptoms When the patient drives in her car and is held by the traffic, she has an involuntary passage of urine The dip in the nutritive sector corresponds to the symptom that after eating certain foods the patient is able to bring them back into her mouth (selective rumination) Analysis by Doctor Jelliffe has revealed that in the case of the passage of urine when the patient is held up in traffic, her unconscious organ erotic semantic fantasy triumphs over the need for self-control, and she asserts her mastery through the early and necessary mastery acquired over the control of the bladder Since she is prevented from doing one

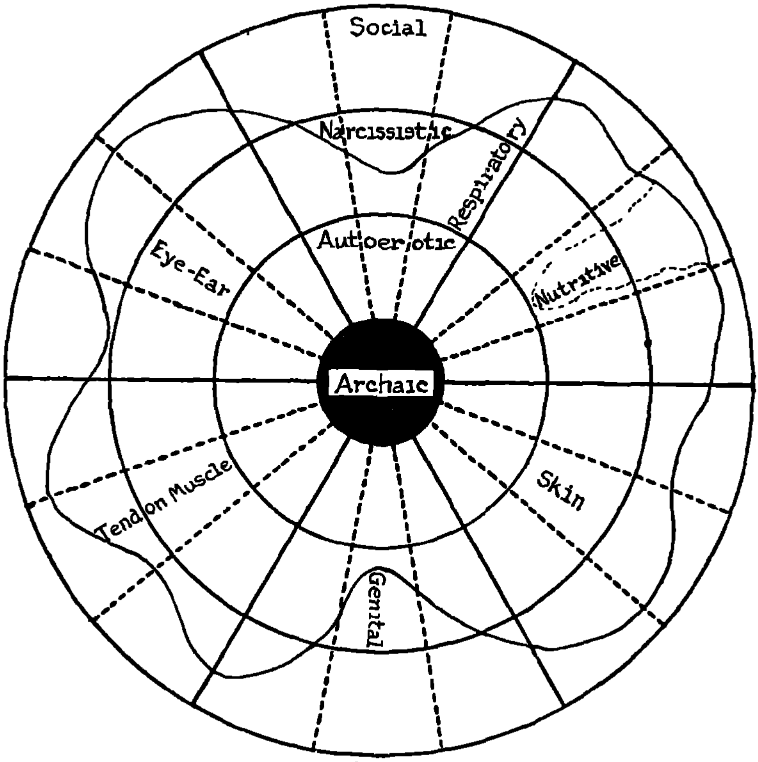


FIG 3

Schematic representation of regressions and fixations in one patient The dip in the respiratory sector represents a psychogenic asthmatic defence *s r*, in the eye and ear sector, refusing to see or hear the 'truth' and 'reality', in the genital, urinary eroticism The deep incision in a fairly well socialized nutritive 'libido' represents an 'emotionally' conditioned *s r* for selective rumination of individual ingredients in the stomach The patient appears as a severely sick individual on the border of a psychotic reaction Present nosological schemes would call this an anxiety-hysteria or a manic-depressive psychosis if the semantic compensation should break and further regression occur (After Jelliffe)

thing, her power of doing gets semantic expression through a substitute act which cannot be prevented by outside interference The selective rumination symptom also goes back to the nursing semantic period. When suckling she would vomit after a full meal, and demand another nursing, to which the mother foolishly acceded Vomiting then became her semantic way of controlling 'reality' She used that as a weapon in getting her wishes with her family She has amplified and refined the methods of semantic expression of this old mastery over 'reality', and the selective rumination seems one of the results

The patient also exhibits other neurotic semantic symptoms She is too impatient to read and does not remember what she has read She can never remain quiet She is very keen-sighted to find fault in others, very acute to hear the last verbal equivocations, and very neat and clean with reference to her bodily secretions Here we see clearly the semantic mechanism of infantilism and the contradictions between the conscious performance and the unconscious fantasies

In an hypothetically healthy individual, his make-up could be represented graphically by a circle on the social level of adjustment He would have outgrown the passing semantic stages of the archaic, organ erotic, and narcissistic periods

Failure in semantic adaptation to 'reality' might be represented by dips in the curve to such a level as the individual fixation or regression has put him By such means we have an excellent method to represent clearly the weak spots and to show the focal semantic points of conflict in evaluation where energy is diverted to useless or harmful fantasy ends

When the dips or deviations are few and slight, we call them idiosyncrasies, for instance, such a habit as the narcissistic tactile fantasy of toying with a button, a moustache, or eyebrow When the number of failures is larger and the semantic symptoms go to lower levels of development, we speak of hysteria,. When the level of regression is still lower (organ erotic or archaic), we are usually entitled to speak of a psychosis [3]

We have already emphasized over and over again that the organism works as-a-whole, and that, therefore, any *el* splittings cannot lead to satisfactory results The verbal division of 'body' and 'mind' remains verbal, and also involves a language whose structure does not correspond to the structure and functioning of the organism A language is like a map, it *is not* the territory represented, but it may be a good map or a bad map If the map shows a different structure from the territory represented—for instance, shows the cities in a wrong *order,* or some places east of others while in the actual territory they are west ,—then the map is worse than useless, as it misinforms and leads astray One who made use of it could never be certain of reaching his destination The use of *el* language to represent events which operate as-a-whole is, at least, equally misguiding and semantically dangerous

With this in mind, let us briefly analyse the 'obstacle' in Fig 2 As we deal with 'obstacles' in a life sense, we can generalize the obstacle to some semantic factors involving meanings and evaluations which may arrest the development as well as result in regression.

From the *non-el* point of view every obstacle and difficulty involves semantic evaluation Any and all reactions to lower order abstractions involve the cyclic chain of higher order abstractions, no matter how imperfectly In ordinary language, a physical occurrence with which we become acquainted through lower nerve centres involves our 'mental' *attitudes,* doctrines , in general, *s r* influenced by the activities of the higher centres From this *non-el* point of view, *surprise,* fear, fright , enter, and usually do the harm Physical pain seldom, if ever, leads to semantic disturbances, but fear, fright , and surprise usually do Anticipation of danger, or proper evaluation of a situation, has a *protective* effect, as it usually tends to diminish or abolish the fear, fright , or *surprise.* The outside world is full of devastating energies, and an organism may only be called adapted to life when it not only receives stimuli but also has protective means against stimuli Now such anticipation or expectation makes an organism *prepared,* and the difference between a prepared and an unprepared organism in the face of danger or pain may turn the scale of the outcome

Section B Consciousness of abstracting

It is obvious that in the human organism the field for stimulations is vastly greater than in animals We are subjected not only to all external stimuli but also to a large number of permanently operating *internal* semantic stimuli, against which we have had, as yet, very little protective psychophysiological means Such structurally powerful semantic stimuli are found in our doctrines, metaphysics, language, attitudes,. These do not belong to the objective external world, and so the animals do not have them in a like degree As our enquiry has shown, in practically all 'mental' ills, a confusion of orders of abstractions appears as a factor When we confuse the orders of abstractions and ascribe objective reality to terms and symbols, or confuse conclusions and inferences with descriptions., a great deal of semantic suffering is produced

Obviously, in such a delusional world, different from the actualities, we are not prepared for *actualities,* and then always something unexpected or 'frightful' may happen The organism cannot adjust itself to such fictions, it is not prepared to face ∞-valued *m o* realities and must suffer from constant surprises and painful semantic shocks, which do the harm

As we have already seen, the general preventive psychophysiological discipline in all such cases of confusion of orders of abstractions is found in 'consciousness of abstracting' When conscious of abstracting we cannot identify the symbol with the thing,. In the case just described, the difficulties of the patient were precisely in intensified *mis-evaluation* —the confusion of the symbol on the infantile semantic level with *m o* reality—and this persisted, in spite of later serious inconvenience and difficulties when the symbol did not any longer produce the desired submission of others and became in itself a nuisance

We could analyse from this semantic point of view all psychiatry, and we would find that the intensified mis-evaluation or confusion of orders of abstractions is always very prominent in 'mental' illnesses This characteristic is very general, and the suffering these confusions produce is very acute The general protective psychophysiological measure, however, is very simple, namely, 'consciousness of abstracting'

A fundamental difference between 'man' and 'animal' is found in the fact that a man can be conscious of abstracting, and an animal cannot This last statement could be reformulated that animals are 'unconscious of abstracting' Now consciousness of abstracting is not inborn as a rule, but becomes a *s r* acquired only by education or through very long, and usually painful, experience in evaluation If we are *un*conscious of abstracting, we obviously copy animals in our 'mental' processes and attitudes and cannot completely adapt ourselves to the structurally more complex human world (with higher order abstractions), so that some arrested or regressive processes are bound to result. In such a more *complex* world we need *protection* against semantic overstimulations, which the animals in their simpler world do not need If, therefore, we copy animals in our 'mental' processes, we could, perhaps, live in their simpler world, but cannot adapt ourselves easily to a structurally more complex human world

We see here the general semantic mechanism of human adaptation. Our human world is more complex, the number of stimuli is enormously increased Against this excessive stimulation we need protection, which is found in the consciousness of abstracting One adjusts oneself by increasing the field of 'consciousness', and by giving it properly evaluated content as against the vast 'unconscious' which covers the animal's life and our own past In 'mental' ills we find the arrested or regressive stages, with a vast and harmful unconscious 'Mental' therapy always has the semantic aim and method, namely, to discover the unconscious material and make it conscious, and so make proper evaluation possible

It is quite remarkable that 'mental' therapy, which actually is a form of semantic, *non-el* re-education, is only successful when it succeeds in making the patient not only 'rationalize' his difficulties but also makes him 'emotionally' revive—live through again, so to say, and evaluate anew—his past experiences This process can be compared with a glass of water in which some chalky sediment lies on the bottom. In semantic difficulties the different 'hurts', may be compared to the water *and* the sediment. 'Rationalization', alone, is like throwing away the clean water and letting the sediment remain. No improvement follows, the semantic sediment of earlier evaluation is still there and does its work. But if we stir up the water *and* the chalk, then we can throw out both and a clearing up will follow. The *non-el* 'living through' of the past experiences is equivalent to this semantic stirring-up of meanings before eliminating the immature evaluations.

This semantic mechanism is well recognized, yet puzzling. It shows that it is more difficult to influence the affective than to affect the 'rationalization'. One may 'rationalize' perfectly well, yet his lower centres will not be affected sufficiently. It is possible that the confusion of orders of abstractions or identification is, in the main, responsible for it. With the use of the Structural Differential and with training in the orders of abstractions and in *silence* on the objective levels, we gain seemingly extremely powerful psychophysiological means of an entirely general character to influence directly the *affective* responses, in which we are aided by the utilization of all available nerve centres. It also shows once more how persistent is the working of the organism-as-a-whole. *The harm was done by organism-as-a-whole methods (affecting higher and lower centres), the protective semantic agencies should employ similar means.*

In the older 'mental' therapy we tried to bring the unconscious or buried material into the conscious, but each psychiatrist proceeded by a *private* method, and according to a special theory. Such procedure is obviously not general enough for simple *preventive* training on a large scale. The present system offers such general and effective semantic psychophysiological means. By making ourselves conscious of abstracting we prevent the animalistic *un*consciousness of abstracting, and so prevent arrested development or regression. We bring into consciousness some of the most fundamental human characteristics, of which animals are *un*conscious, and so prevent arrestment or regression to lower levels. The method is entirely general and simple, based on the elimination of identification, introducing natural and so adaptive evaluation which should not stir up resistance in the child.

In the freudian theory the famous Oedipus complex purported to explain the often unconscious hostility of the son toward the father and his excessive attachment to the mother. The researches of the anthropologist Malinowski show that in primitive matriarchal societies the biological father is not recognized as such and is only a kind of friend and nurse to his son The other functions which the father has in patriarchal societies are here performed by the mother's brother The taboos which apply to the mother in patriarchal societies apply to the sister in such matriarchal societies

The results are quite interesting Malinowski found that seemingly similar unconscious semantic mechanisms are at work But the hostility is toward the uncle, and the excessive attachment is directed toward the sister

Malinowski concludes that the freudian mechanisms are thus proven. According to the present theory these facts are very important and show clearly that 'sex', as such, has nothing or very little to do with these 'complexes', but that the active unconscious agents appear as semantic and *doctrinal* Doctrines and their meanings to the individual, their applications, identifications , make the father in one case, and the uncle in the other, the dreaded , member of the family Because of lack of consciousness of abstracting the child reacts to such application of doctrines with some 'complex', or semantic state, based on identification, non-mature evaluation, involving non-mature *non-el* meanings, in spite of *el* theories and languages

Similarly, some spanking or other pain in childhood may later result in a neurosis A successful analysis can usually trace neuroses back to some such experiences What did the harm? Was it the burning *physical* feeling? Obviously not, for any child has had in its childhood many more painful experiences, and yet no semantic harm has followed So we must look in another direction, and the elimination of identification or of the confusion of order of abstractions at once offers a solution The 'spanking' had many factors, some of them were 'physical', some 'mental' If we consider among the 'mental' factors the objectifications of 'authority', 'hell', 'sin', and other terms of evaluation, these result in fright and other semantic shocks, which ultimately lead to the neurosis We know from our own experience how little affected we are by an accidental hit Such a purely physical experience which does not give a semantic shock cannot product a neurosis

It is not difficult to see that an investigation of 'hurts', 'emotional shocks', 'fear', 'fright', 'surprise' , must lead to a more general enquiry into the structure of 'human knowledge', meanings, evaluation, *s r* , which must include the structure of science and mathematics

In the disregard of the stratification of human knowledge, or in identification or the confusion of orders of abstractions, we find an ever-present and abundant semantic source of human suffering, which increases unnecessarily the internal stimulations and so disturbs the efficient working of the organism-as-a-whole

Psychiatrists in purely pathological fields have also discovered different sources of human difficulties They discovered that the 'unconscious' seems quite a dangerous affair, and that 'mental' ills exhibit symptoms of an arrested or regressive process.

This present investigation, as well as the psychopathological ones here referred to, although conducted on entirely different grounds, one more general than the other, has discovered very similar mechanisms, namely, the benefit in enlarging the field of 'consciousness', by bringing into 'consciousness' important factors of the 'unconscious' and thereby counteracting the semantic possibility for arrested development or regression

With such divergence, both in methods and in material used, the similarity of results indicates strongly the soundness of the conclusions Generality offers, also, a criterion of practical simplicity and workability, and on these grounds the more general semantic $\bar{A}$ discipline commends itself.

Because of this generality the present theory has not only a simple but also an *impersonal* character which makes it available as a preventive measure in elementary education With the older theories we deal in practice with personal responses to meanings, in our case we deal with the *s r* in *general* and with their psychophysiological mechanism in particular

The present enquiry started with the search for a sharp difference between Fido and Smith This was found in the fact that Smith functions as a 'time-binder', while Fido does not Further investigation into the mechanism of the time-binding function revealed that its most important characteristic is found in its peculiar stratification into many orders of abstractions The realization of this stratification eliminates identification and leads to the 'consciousness of abstracting' thus ascribing a permanent *strictly human* content to consciousness', and so automatically eliminating such animalistic, and, therefore, arresting or regressive, '*un*consciousness It is found, also that in the consciousness of abstracting we find a general and simple psychophysiological semantic method for the elimination of the majority of human difficulties In the training in this consciousness of abstracting we find a workable physiological tool with which to integrate the functioning of the human nervous system. We use organism-as-a-whole methods and achieve organism-as-a-whole results We find in the language of 'semantic reactions', 'non-elementalistic meanings'., psychophysiological means to integrate the 'emotional' with the 'intellectual', which was hampered, to say the least, by the old *el* languages and methods and systems The organism by structural necessity acts as-a-whole, but the old elementalism with its psychophysiological effects prepared the semantic background for split personalities, which a *non-el* system helps to re-integrate We find a rather astonishing result, namely, that the structure of human achievements corresponds to the principle of stratification with the resulting consciousness of abstracting, usually limited to the special field The majority of individual and group difficulties are found, also, to be due to the very general disregard of this principle.

By the scientific data of 1933 it seems well established that the enlargement of the field of 'consciousness' is extremely desirable With this aim, a more general enquiry into the character of the 'unconscious' may also be worth while Let us investigate the structure of science (1933), and see if some 'unconscious' factors cannot be found there We find a curious fact, that mathematicians, in addition to their other activities, make a business of unravelling hidden unconscious assumptions Their enquiries have led to a thorough investigation of the structure of their language in two directions one, investigation of the underlying assumptions, the other, working out the 'implications'

Let us give a simple structural example of this Two assumptions are said to be equivalent when each of them can be deduced from the other without the help of additional new assumptions For instance: (*a*) The fifth postulate of Euclid—'If a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles', (*b*) 'Two straight lines parallel to a third are parallel to each other', (*c*) 'Through a point outside a straight line one and only one parallel to it can be drawn' Each assumption silently, unconsciously, presupposes the other, so that they can be deduced from each other They actually are different forms of the same propositional function

Another case is equivalence *relatively* to a fundamental set of assumptions A, B, C, M It might happen that, in diminishing the fundamental set, two assumptions which were formerly equivalent cease to be so For instance, the following assumptions are mutually equivalent and also equivalent to the fifth postulate of Euclid (*a*) 'The internal angles, which two parallels make with a transversal on the same side, are supplementary' (Ptolemy) (*b*) 'Two parallel straight lines are equidistant' (*c*) 'If a straight line intersects one of two parallels, it also intersects the other' (Proclus) (*d*) 'A triangle being given, another triangle can be constructed similar to the given one and of any size whatever.' (Wallis.) (*c*) 'Through three points, not lying on a straight line, a sphere can always be drawn.' (W. Bolyai),

But the following two assumptions are only equivalent to the *E* fifth postulate if we retain the postulate of Archimedes* (*a*) 'The locus of the points which are equidistant from a straight line is a straight line', (*b*) 'The sum of the angles of a triangle is equal to two right angles.' (Saccheri.)[4]

The crucial point of this discussion is that all that has been said here is *not obvious* even to the attentive and intelligent reader, nor to many mathematicians. It took nearly two thousand years and some of the efforts of the best scientists of the world to discover these *connections and implications*. The above examples illustrate a *general* underlying structure of all our languages. They have inherent interconnection, underlying assumptions and implications, the analysis of which, outside of mathematics, is seldom, if ever, carried far enough. Now these structural assumptions and implications are inside of our skin when we accept a language—*any* language. If unravelled, they become conscious; if not, they remain *unconscious*. In the present work we have already had an opportunity to become acquainted with unconscious implications which are concealed in the structure of any language. We saw that we must start with undefined terms, which represent structural assumptions and postulates, as we have no means to explain them or define them at a given date. We found that these *undefined* terms represented our unconscious metaphysics, and that the way to make this unconscious metaphysics conscious was to start explicitly with undefined terms, produce a system of postulates . . , a procedure which is completely fulfilled only in mathematics.

It should be noticed (as this is very important) that the undefined terms, being undefined, are overloaded with *'emotional' values*. As the higher nervous centres cannot handle them the lower nerve centres work upon them overtime. If we do not analyse our languages into their undefined terms and structural postulates, our strongest 'emotional' and semantic components, which made these languages, remain hidden and unconscious.

*The Postulate of Archimedes is stated by Hilbert thus: Let A_1 be any point upon a straight line between the arbitrarily chosen points A and B. Take the points A_2, A_3, . . . so that A_1 lies between A and A_2, A_2 between A_1 and A_3, . . . , moreover, let the segments AA_1, A_1A_2, A_2A_3, . . . be all equal. Then among this series of points, there always exists a certain point A_n, such that B lies between A and A_n.

This hypothesis is used by Saccheri in its intuitive form; namely, a segment, which passes continuously from the length *a* to the length *b*, different from *a*, takes, during its variation, every length intermediate between *a* and *b*.[5]

Here we are face to face for the first 'time' with a wider, more general, and impersonal 'unconscious', which underlies the structure of any language, and so is operative in every one who uses a language We may call this general form the scientific, or public, or linguistic, or semantic, or by preference, as a term, the *structural unconscious* It embodies the underlying *structural* assumptions and implications which are silently hidden behind our languages and their *structures* These assumptions ., may be called 'unconscious', because they are *totally unknown and unsuspected,* unless uncovered after painful research

Any form of representation has its own structural assumptions at its basis, and when we accept a language we unconsciously accept sets of silent structural assumptions of which we become semantic victims For a long while, the white race has been the victim of the unconscious assumptions and metaphysics which underlie the A, E, and N systems It has needed a structural revision of these systems, culminating in $\bar{E}$, $\bar{N}$, and, finally, $\bar{A}$ systems These non-systems are characterized not by the introduction of new assumptions, but by making the older unjustified, primitive, unconscious structural assumptions conscious and so helping us in eliminating the semantically undesirable reactions We have already seen how fallacies and taboos (1933) can be and were manufactured unconsciously by semantic processes, these start with more general more natural and more fundamental structural errors, such as the primitive 'identification', for instance, which are due to pre-human ways of 'thinking' and which result in semantic difficulties and regression even today

Let us assume, as an illustration, that the fifth postulate of Euclid is a false assumption seriously detrimental to human life and comparable to some of the false doctrines that underlie the morbid symptoms with which psychiatrists deal every day Let us assume, further that a doctor ignorant of the structure of 'human knowledge' s r, and the equivalence of assumptions, succeeds, after painful and laborious efforts in eliminating from a patient this special vicious assumption Yet, because of his oversight, he pays no attention to another assumption equivalent to the first, and does not eliminate it In such a case, rationalization about the first false doctrine would probably make the treatment a failure, as the other unconscious and equivalent doctrine would, in virtue of the extremely formal, one-, and two-valued character of the unconscious, perform its task and make the treatment ineffective The tangle of equivalent structural assumptions in daily life is still unanalysed For instance, it is extremely difficult to attempt to impart 'proper evaluation' without eliminating identification,.

The higher and lower order abstractions seem structurally and neurologically, as well as functionally, interconnected in a cyclic chain, and so can never be entirely divided. A language—any language—involves undefined terms which, with the structure of the given language, express the silent and unconscious metaphysics underlying it. A language, for its maximum serviceability, must, at least have the structure of the events it attempts to describe, and so science must first discover the structure of events, for only then can we shape our languages and give them the necessary structure. Any advance in our knowledge of nature is strictly connected with new languages of similar structure which reflect the structure of the world. This last 'knowledge' at each date represents again 'modern metaphysics'. In all such enquiries we have to struggle with the older, mostly primitive structural metaphysics and unconscious linguistic semantic consequences. Enquiry into these subjects must throw new light on the unconscious processes, and so diminish the vast field of the unconscious.

The structural unconscious seems to be more general, more fundamental than the special, or individual, psychiatrical one, because analysis shows that the latter follows from the former. As the reader may recall, life, 'intelligence', and abstracting in different orders started together. Without abstracting, recognition, and, therefore, selection would not be possible. The world of the animal, as well as of man, represents nothing else than the structural results of abstracting, without which life itself would be totally impossible. Man alone has the power of extending the orders of abstractions indefinitely. When Smith has produced an abstraction of some order, perhaps by making a statement, he has the potential capacity of analysing and contemplating this statement, which has become a fact on record, and so he can abstract to a still higher order, without known limits. It is this capacity which crowds the world of Smith with endless 'facts' belonging to very different orders of abstractions. The animal's capacity for abstracting ceases on some level, and is never extended without a change in the nervous structure. So the animal's world is comparatively simple, the world structure of man being, by comparison, indescribably more complex. Man's problems of adjustment, therefore, also become more complex. Human medicine is much more complex than veterinary science, although the higher animals differ very little in their gross anatomical structure from humans. The structural *m.o* facts, which resulted from abstracting in different orders, differ in number as well as in complexity. The human capacity for expanding indefinitely the orders of abstractions brings about this peculiar stratification of 'human knowledge'

It appears as a product of evolution just as stratified as rocks appear. This stratification appears as a crucial, structural *m o* fact, though generally disregarded, except partially in mathematics and psychiatry Its realization necessitates the elimination of the 'is' of identity and results in the consciousness of abstracting, so fundamental for sanity.

Section C Infantilism

As has already been mentioned, the main symptoms of physical and 'mental' illnesses are few and simple This would suggest the possibility of simple and more general theories relating to the fundamental symptoms The colloidal structure of protoplasm accounts for this peculiar simplicity and for the small number of the fundamental symptoms In the 'mental' field these fundamental symptoms are accounted for by a simple structural, functional principle of 'copying animals' in our nervous processes, which must be harmful, and which is characterized by the lack of consciousness of abstracting, implying colloidal disturbances Psychogalvanic experiments show clearly that every 'emotion' or 'thought' is always connected with some electrical currents, and that electricity seems fundamental for colloidal behaviour, and, therefore, for physical symptoms and the behaviour of the organism

In the colloidal processes we find the bridge between the 'physical' and the 'mental', and the mutual link seems mainly electricity It is more than mere coincidence that all illnesses, 'physical' or 'mental', have only a few fundamental symptoms, and we should no longer be surprised to find that physical ills result in 'mental' symptoms, and that 'mental' ills may also involve 'physical' symptoms

If a simple symptom is completely *general,* it indicates that it is structurally fundamental, and we shall be repaid if we devote special attention to it As a rule, in 'mental' ills we observe a striking appearance of symptoms which have a sinister parallel with the behaviour of infants Arrested development or regression in grown-ups also exhibits these infantile characteristics In other words, whenever infantile characteristics appear in grown-ups it indicates that the 'adult' has not grown up fully in some semantic respects, or has already started on the way of regression, implying some colloidal or *m o* structural injury

When we speak of 'infantilism' in 'adults', we include symptoms which belong to the period of childhood in its organ erotic or autoerotic and narcissistic stages It should be recalled that in children these semantic phases are natural, they become pathological only when the individual does not outgrow them and exhibits them as a grown-up The term 'infantilism' is a rather sinister one, and should never be applied to children Children behave like children, and that ends the subject But children have fewer responsibilities, their sex impulses are undeveloped , and so their behaviour cannot be equally dangerous to themselves and others But not so with grown-ups They have responsibilities, duties, often strong sex impulses , which make out of the infantile 'adult' an individual dangerous to himself and others The term 'social' period, or 'socialized' individual, is sometimes wrongly interpreted The fact that human achievements and capacities are accumulative and depend on achievements of others makes us, by necessity, a social time-binding class of life, which again involves more complex modes of adjustment Whether we approve or disapprove the existing legal and police regulations has nothing to do with the fact that in a social class of life some restrictions are necessary Our present commercial 'civilization' can be characterized as of an infantile type, governed mostly by structurally primitive mythologies and language very often involving primitive *s r* One need but read the speeches of different merchants, presidents, and kings to be thoroughly convinced of this The rules and regulations are naturally antiquated, and belong to the period to which the underlying metaphysics and language belong The 'adult' or scientific semantic stage of civilization would be precisely the 'social' stage of complete evaluation of our privileges and *duties*

In speaking about infantilism, it should be remembered that the child has an advantage over the imbeciles, idiots, and 'mentally' ill who have stopped development or have regressed to the age of the infant or the child The 'normal' child profits by experience and outgrows the semantic characteristics that are natural to a given age In cases of arrested development or regression, the undesirable infantile characteristics persist in the grown-ups and are a source of endless difficulties and suffering to them and their associates Thus, in our childhood we all have had experiences similar to that of the patient of Dr Jelliffe, and are no worse off because of them But, if the reader should imagine himself in the position of the patient with those infantile characteristics, he would realize that an enormous amount of suffering, fear, shame, bewilderment , results for the patient The worst feature in such cases is found in the fact that an infantile type usually cannot 'outgrow' or alter such characteristics by himself, and needs very wise and patient outside help in re-training, or medical assistance, if he is ever to overcome earlier inappropriate *s r* But if we *start* the education of an infant with appropriate *s r*, such a procedure must play a most important preventive evaluational role.

We should remember that the nervous system of the human child is not finished at birth. The extension, the growth, and the multiplication of the appendages of the neurones . . ., go on after birth. The nervous system of an adult shows striking differences in the length and complexity of the nerve cells over that of the infant. The researches of Hammarberg showed, in all the cases of idiocy he investigated, an arrest of development in a more or less large part of the cortex, at a stage corresponding to either an embryological period, or to the period of early infancy. Only a small number of cells had reached their full development during the growth of the cortex. The psycho-logical defects were in direct proportion to the defects of the development of the cells, and were the greater the earlier the period of arrest of development.[6] In extreme congenital imbecility the cortex is poorly organized, is thin and deficient in nerve cells. Bolton's second layer of pyramidal cells matures last, and its development in different mammals corresponds to the degree of their 'intelligence'. In humans, its degree of deficiency corresponds to the degree of 'mental' arrest or regression. In organic 'mental' diseases very diffuse cortical lesions, when present, impair 'intelligence'. Affective disturbances depend upon even smaller brain lesions, particularly when the thalamic regions are affected.[7] In general, lesions in the basal ganglia diminish the energy of the impulsive life (sleeping sickness). Lesions at the base of the frontal lobe, and some brain tumours, lead to euphoric excitement, which gives a feeble, stupid expression, facetiousness, and a tendency to teasing. Other lesions in the basal ganglia lead to a labile affectivity. All destructions of parts of the brain usually lead to irritability and moodiness. In different focal lesions of the brain the disturbances lead to anger and rage.[8]

Among other results of organic brain diseases we find semantic disturbances, absence of critical faculty, and a disturbance of judgement; complicated situations can no longer be grasped, the *evaluation of relations* is impaired, . . . In cases of labile affectivity the special 'emotion' dominates the patient completely. Trifles make him either very happy or desperate. Because of the *decrease in association* or deficiency in the process of relating, the patient often appears indifferent, although the defect is not primarily in the affective field. Similar difficulties in association or relating make many patients appear egotistic in their *s.r.* and behaviour. Since the patients have lost their insight in, and evaluation of, different life situations, their actions appear un-ethical. Tenderness, consideration, tact, aesthetic sensibility, sense of duty, sense of right, feeling of shame . . ., may all disappear at any moment, though they would otherwise naturally be present. Any kind of impulse may be translated into action without restraint[9] Hemorrhages in the thalamic region often result in marked lability of affects Diffuse nutritional disturbances of the cortex usually give similar symptoms, as in organic brain diseases, [10]

It should be remembered that in the human nervous system the co-ordinated working of the higher and lower centres is a necessity for the optimum working of the whole In cats and dogs deprived of the association areas of the cortex, the difference is not so marked They still behave in a co-ordinated way, provided the thalamic regions are intact Even in a child without a cortex, we find facial grimaces if we give him something bitter, but higher adjustments are impossible[11] The general 'inhibitory' and regulative action of the higher centres increases with the differentiation of the nervous system, and in man this becomes of paramount importance This has been shown empirically For instance, in man and dogs strong negative action on the flow of gastric juices may be 'psychic' in origin This negative action is weak in the guinea pig, although it is discovered on decerebration With the tortoise, also, there is some acceleration of the movements of the stomach after decerebration, but in a frog we do not find any negative influence at all[12]

The facts given above were established through anatomical and physiological structural examinations If the functioning of the nervous system is examined from the point of view of colloidal chemistry, the gross non-surgical lesions become interpretable as the result of changes in colloidal behaviour Thus Doctors Wilder D Bancroft, J Holmes Richter, H Beckett Lang, John A Paterson, Walter Freeman, and others demonstrated that it is possible to find a correlation between the functional psychoses and the state of dispersion of the nerve colloids For instance, in dementia praecox the nervous system appears in a state of colloidal over-dispersion, and in manic depressive psychoses, in a state of decreased dispersion[13] It is interesting to note that in infants the colloids appear more dispersed than in grown-ups and probably similar conditions will be found in cases of infantilism The above-mentioned scientists have found, also, that the colloidal behaviour of the nervous system can be altered by special chemical treatment with drugs, carbon dioxide, oxygen , with specific reactions on the psycho-logical level As *s r* involve electrical occurrences fundamental in colloidal behaviour, similar symptoms on psycho-logical levels may imply corresponding submicroscopic colloidal states Taking into consideration the structural characteristics of colloidal behaviour and the elaboration of technical means, we may discover that semantic re-education must involve differences in electrical potentials , and result in differences in colloidal behaviour in different regions of the nervous system. Experiments should be made in combining chemical means, which effects are not lasting but which might facilitate the semantic approach with semantic re-education which results, once achieved, often become lasting. Colloidal and psycho-galvanic investigations of a given patient before and after the semantic re-education should also be made.

Let me emphasize once more that from the colloidal point of view free from identification, the 'body-mind' problem ceases to be a puzzle, as we have a well-established electrodynamic, structural colloidal background which can account perfectly for the experimental facts of 'mind'. The subtleties of the sub-microscopic structure involve an endless array of possibilities. At present we lack detailed knowledge of this structure, for the colloidal developments are very recent, and in this special field very little experimentation has been done.

If we accept the *non-el* point of view, and all known evidence seems to demand it, we must conclude that if different macroscopic, microscopic, and sub-microscopic lesions of the nervous system *result* in quite definite psycho-logical symptoms, which on the semantic levels appear as a lack of *evaluation of relations*, then vice versa, the use of linguistic systems, which systematically train the immature nervous system of the child and of the grown-ups in delusional evaluation, must result in *at least* colloidal disturbances of the nervous system. These functional colloidal disturbances become superimposed upon the inborn eventual deficiencies of the nervous system, and the end-results may be quite out of proportion to the seemingly slight induced discrepancy. The actual behaviour, adjustment, sanity, may be considerably impaired.

Before birth, the child can be considered as in ideal conditions. He floats comfortably in a fluid of a temperature equal to his own. All his wants are satisfied, as everything is supplied to him by the maternal body. At birth, the child must begin to breathe, and a little later he must take food, digest, . External influences begin to impinge on him, and he must begin to adjust himself. Very soon the average infant finds that he can get what he wants, within certain limits, by certain movements or by crying. For the infant, a cry or a word becomes semantic magic. In Pavlov's language, a word governs a conditional reflex. In psychiatry, a definite series of such conditional s.r of animalistic low order of conditionality is called a 'complex'. In Pavlov's experiments a dog is shown food and a bell rung simultaneously. At the sight of food, saliva and gastric juice flow. Associations soon *relate* the ringing of the bell and the food, and, later, simply the ringing of the bell will produce the flow. In another animal some other signal, a whistle, for instance, would produce similar effects. In different people, through experience, associations, relations, meanings, and *s.r.* are built around some symbol. Obviously, in grown-up humans the identification of the symbol with the thing must be pathological. But in infancy the confusion of orders of abstractions must be considered as an entirely natural semantic period. The infant 'knows' nothing about science and events. Objects and 'sense perceptions' 'are' the only 'reality' he knows and cares about, so he does not and cannot discriminate between events and objects. By necessity, he identifies unknowingly two entirely different levels. As his symbol usually means a satisfaction of his wants, naturally he identifies the symbols with the objects and events. At this stage, also, he cannot know that the orders of his abstractions can be extended indefinitely, or that his most important terms have the multiordinal character. It is important to notice that objectification, and, in general, identification or confusion of orders of abstractions, are semantically *natural* for the infant. The more the child comes in touch with 'reality', the more he learns, and in a 'normal' child the 'pleasure principle', which was established as a method of adjustment on the infantile level, is slowly displaced by the 'reality principle', which thus becomes the semantic method of adjustment of the complete adult. Science alone gives us full knowledge of current 'reality'. But science represents a social achievement, and, therefore, a complete adult, in growing up to the social level, must become aware of the latest stages of *m.o.* reality. These are given by the current scientific methods and structural notions about this world, and gradually become incorporated in the structure of the language we use, always deeply affecting our *s.r.*

It is important that in the twentieth century we should realize that the work of Einstein and the four-dimensional space-time continuum establishes a language of different structure, closer to the facts we know in 1933, and that it gives us a new semantic method of adjustment to a new 'reality' (see Part IX).

The semantic stages of the development of the child must naturally pass through the stages outlined above. When he begins to differentiate himself from the environment, he is self-centred and concentrated on his 'sensations' (autoerotic). Later he projects his own sensations on the outside events; he *personifies*. This semantic trait is often found in incomplete adults, when in anger they break dishes or furniture.

The child is interested, first, in himself (autoerotic), then in children like himself (homosexual). Slowly his interests turn away to persons less similar to himself, to the opposite sex, and so he enters the semantic period of the race development.

Similar semantic processes are to be seen in the racial developments as given by anthropology, and are reflected in the structure of the languages. In the archaic period of one-valued 'pre-logical thinking', which is found among primitive peoples, the 'consciousness of abstracting' is practically nil. The effect produced by something upon an individual inside his skin is projected outside his skin, thus acquiring a demonic semantic character. The 'idea' of an action or object is identified with the action or the object itself. Identification and confusion of orders of abstractions have full sway.

The paralogical stage is a little more advanced. In it the identification is based on *similarities,* and differences are neglected (not consciously, of course). Lévy-Bruhl describes this primitive semantic period by formulating the 'law of participation', by which all things which have *similar* characteristics *'are the same'.*[14] A primitive syllogism runs somewhat as follows: 'Certain Indians run fast, stags run fast; therefore, some Indians *are* stags.' This semantic process was entirely natural at an early stage and laid a foundation for the *building of language* and higher order abstractions. We proceeded by similarities, much too often considered as *identities,* with the result that differences were neglected. But, in actual life, without some primitive metaphysics, we do not find identities, and differences become as important as similarities. The former primitive emphasis on identity, later enlarged to similarities, must, at some stage of human development, become semantically disastrous and the optimum adjustment an impossibility.

In building a Ā-system, we have to stress *differences,* build a 'non-system' on 'non-allness', and reject identity. The older semantic inclinations and infantile or primitive tendencies were a necessary step in human evolution. For sanity, we must outgrow these infantile semantic fixations. Similarly, for civilization, we must grow out from primitive structural fixations, primitive metaphysics, taboos, and other primitive *s.r.* These primitive habits, languages, and structural metaphysics and reactions have been extremely ingrained in us through the ages, and requires effort and new semantic *training* to overcome them.

In the 'mentally' ill we find sinister and very close parallels to the behaviour of the primitive man and the infant, not only in the 'mental' and 'emotional' responses, but even in physical behaviour, postures, drawings, and other modes of expression. These parallels are today recognized by practically all scientific workers and are analysed in many excellent volumes.

We should notice that in this maze of observational material, one general rule holds; namely, 'consciousness of abstracting' offers a *full* *semantic solution.* In it we find not only a complete foundation for a theory of sanity, but also the semantic, psychophysiological mechanism for the passing from the infantile, or primitive-man, level to the higher level of complete adulthood and civilized social man.

The following drawing is taken from Bleuler's *Textbook of Psychiatry* (p. 402) and was made by a very ill patient (chronic catatonic) *who could formerly draw well,* yet this drawing is obviously *childish.* The literature of psychiatry abounds in such productions, and they confirm fully the processes as described in psychiatry.

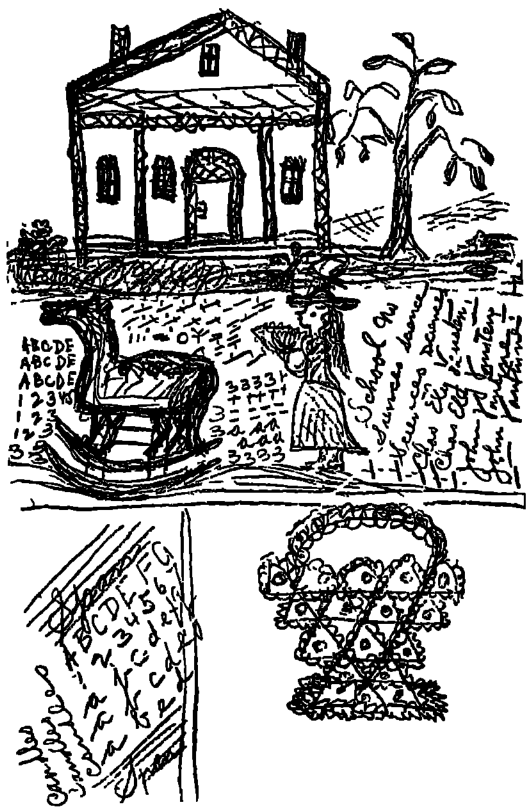


FIG. 4

Adult infantilism becomes usually a potent wrecker of individual lives, and, when viewed from a social, national, or international point of view, accounts, also, for the majority of our semantic difficulties in the social, economic, and political fields.

Although we treat infantile, arrested, and regressive symptoms together, it is important to realize that most of these characteristics are *normal* with the primitive man, and the infant, provided the child outgrows them. The difference between arrested development and regression seems somewhat like that between a poor man and one who has lost his fortune. In the first case, the functioning of the nervous system is unsatisfactory through some deficiency, in the second, the potentialities for efficient working were there, but some semantic obstacle originated the regression, or else some degenerative neural process set in.

In the field of higher abstractions the train of 'ideas' of children, imbeciles, and idiots is restricted Uncommon 'ideas' are left out, and only those which originated in immediate 'sense perception' are easily grasped Until lately, even in science, such an attitude was noticeable as, for instance in gross empiricism, or in the case of the physicist already mentioned who was willing to 'fight' to prove that he 'saw' the 'electron', He did not realize that inferential entities are just as good abstractions as those he 'sees' The attitude of the 'practical man' who pooh-poohs science and the 'highbrows' may serve also as an example

Children, idiots, and imbeciles cannot comprehend anything complicated, they see some elements, but miss the relative wholes. We have elaborated a racial language of 'senses' and elementalism. Similarly, in schizophrenics the relative whole is disregarded, while, on the other hand a single semantically effective characteristic is sufficient to connect the most heterogeneous abstractions in an unnatural whole Word-relations have a predominance over actualities (identification). Thus a patient looks anxiously at a moving door and exclaims 'Da fressen mich die Thuren'* and refuses to pass through the door-way Here we see the identification of words with objects carried to the limit In general, the s.r of the schizophrenic seem such that he identifies intensely his higher abstractions with the lower [15]

Much excellent material on infantilism can be found in Dr Joseph Collins' *The Doctor Looks at Love and Life*, particularly in his chapter on Adult Infantilism, from which much of the following material is taken, and which I acknowledge gratefully

Children and idiots live in the present only and do not concern themselves with the past and the future beyond their immediate gratification Infantile types also want all the 'sense' enjoyment of the moment, never enquiring about the sufferings of others or of the consequences for themselves in the future Indeed, their attitude is often hostile toward those who take into consideration a larger field 'Après nous le déluge' represents their royal semantic motto On national and commercial grounds, they devastate their natural resources, since they are interested only in some immediate and selfish advantage They love praise and hate blame, not realizing that a *critical* attitude gives the foundation for proper evaluation and becomes a semantic characteristic of maturity and that, generally, it is *more beneficial* in the *long run* They thrive and thrill on commendations and compliments, and shiver and shrink at disapproval Such characteristics are found even in whole nations They

***Animal = Thier, Door = Thür, so that the unconscious play upon words gives the meaning · 'Doors devour me', for 'Animals devour me'.**

are self-satisfied, and keep aloof from others in international affairs, not realizing that this is impossible, and that the attempt is ultimately harmful to them. They assume, as an excuse, the superiority of their institutions , and the 'righteousness' of their own conduct

Children and superior idiots appreciate resemblances more readily than differences Simple generalizations are possible, but often they are hasty and faulty A child's pride and self-respect are hurt if he is considered different from other children, or is dressed differently Originality and individuality are tabooed among children Because of semantic undevelopment, differences become a disturbing factor to them, they want everything standardized On national grounds, the adult infants standardize all they can and have even a kind of hostility to anything which has an individual flavour For instance, those who wear straw hats after an arbitrary date are attacked on the streets Not wanting to 'think', or to bother about differences, they fancy that they can regulate life by legislation and they keep busy manufacturing 'laws', which are very often impracticable and self-contradictory. When they pass several thousand 'laws' a year, these become a maze and a joke The ultimate semantic result of such over-legislation is a complete lack of justice or of any respect for 'law' Not being able to 'think' for themselves, they leave that bothersome function to politicians, priests, newspapermen,. Under such conditions life is impossible without expensive lawyers

Not having the critical semantic capacity for proper evaluation, their likes and dislikes are very intense They cannot differentiate the essential from the unimportant The immediate 'sense' perception or 'emotion' unduly influences their actions Impulses to copy others dominate them They are often prejudiced This results in weak judgement, oversuggestiveness, 'emotional' outbreaks, exaggerated sensibility, variability of affective states , and, finally, in an attitude toward life devoid of proper evaluation Their moods are changeable, their attention readily gained and as readily diverted They become easily intimidated and frightened, and easily influenced by others

The above semantic characteristics are sponsored by commercialism, and build up the kind of methods, advertisements, and business policies which we see about us This also introduces a semantic factor of disintegration into human relationships, as it leads to methods of trickery, to 'putting something over' on the other fellow, and appeals to self-indulgence,. When such commercial tactics are national, their sinister educational effect is pronounced Children, from the age when they begin to read, are impressed by such practices as *normal* and take them as semantic standards for their own further orientations. Unfortunately even psychiatrists have not, as yet, analysed the semantic influence of such advertisements on the building and preserving of infantile characteristics.

Children lack moderation and a semantic sense of proper evaluation. Tolerance is not one of their characteristics. To them persons and 'ideas' are evaluated in extremes, either good, 'wonderful', or bad, 'terrible'. Their *s.r.* appear dogmatic and stubborn, as in all the unexperienced. They talk too much or are silent; they praise too much or blame too much; they work too hard or play too hard, and know no middle ground. The whole life of a nation may be coloured by such semantic attitudes. Nations become boastful of their own possessions and achievements, and happily borrow and forget the achievements of others. They pride themselves on having the largest airships, the largest cities, the highest buildings, the longest bridges. They know no moderation in food or drink; they eat or drink too much or become total 'prohibitionists'. They exhibit quick friendships and quick dislikes. They are solemn in their games, like children who are playing father and mother, and make out of games a national event. The childish pleasure of defeating an adversary accounts for national crazes, like racing, boxing, football, baseball, and similar sports, which often overshadow in public attention all really important issues.

Children and many idiots are incapable of any choice which involves meanings and evaluation. When confronted with a situation in which they have to choose between two alternatives, they have difficulties, and often want both. Similarly with 'ideas', they often keep sets of entirely self-contradictory 'ideas'. Even scientists of an infantile type do so, and then publish 'manifestos' in which they try to justify such behaviour and semantic attitudes. Merchants train salesmen especially to induce customers with such infantile *s.r.* to buy what they do not need. This attitude is often extended to marriage. Any man and woman may marry simply because they come across each other; then, when they meet somebody else, they soon change the object of their sentiments.

All classes of feeble-minded and children show marked credulity; they like fairy tales and fantastic stories. Free inventions, by a process of objectification, are taken as experience. Children and schizophrenics *pun and play on words*. They build up languages of their own. Perseveration and stereotypy in speech are also found among them. National commercialism utilizes this principle in advertisements and tries to run a country by verbal slogans and play on words.

Many children and feeble-minded show distinct acquisitiveness. Like some animals, they show a tendency for collection of objects, and value their collections highly. It is a well-known childish game to claim the best morsel of some food because one has put one's hand on it first. Acquisitiveness is made a national slogan and proclaimed a highest aim, which, of course, becomes a semantic source of endless wars and miseries. Infantile legalistic 'putting hands first', on a piece of paper as a title to land, or some such similar form of a 'claim', becomes a source of ridiculous fortunes for the few and of unbearable life-conditions for the many.

Children are gregarious and afraid to be alone. Similar tendencies are carried on by Rotary and other clubs and lodges. Infantile grown-ups are too empty in their heads to desire to be alone. Children seldom stick to anything for long. They hunt for new excitements, and the old toys are often soon forgotten. Similarly, grown-up infants hunt for new excitements, for new toys, whether they be a house or an automobile, a wife or a lover.

In children and the feeble-minded, we seldom find such feelings as shame, aesthetic sentiments, or appreciation of beauty. They like things bizarre, grotesque, glittering, and enormous, things which attract and hold their attention. Similar characteristics are found in incomplete adults. Children and the feeble-minded are usually untidy and noisy. Visiting a public park, or witnessing a 'celebration', will show an observer clearly how infantile grown-ups behave.

Children like to domineer over their younger brothers and sisters and to play the leading part in a game. Similar semantic characteristics are carried into adult life, sometimes taking the form of sadism. We often see infantile docility or resentment, as expressed in sentimental approval or bitter disillusionment, both generally unjustified.

Self-respect is little developed in the idiot, but plays an important semantic role in the life of imbeciles and children. The infantile adult also shows an exaggerated self-respect. Bus conductors and university professors label themselves with a title—even if it is only '*Mr.*' John Smith, as if being called simply 'John Smith' would be offensive to him. An adult evaluates a man by what he has in his head or character, but the infantile type largely judges him by the *symbols* (money) which he has, or the kind of hat or clothes he wears. Since commercialism cannot sell brains, but can sell trousers or a dress, it establishes semantic standards whereby a man is evaluated by his clothes and hats.

In speaking of exaggerated self-regard based on improper self-evaluation, we touch the problems of infantile self-love and self-importance Infantile grown-ups carry these even further, and are unable to make dependable attachments to other persons The love of parents toward their child is largely because it is *their* child, and infantile A 'loves' B only because B 'adores' A and gives up his individuality The moment something changes in B, all the 'love' A had disappears The unbelievable bitterness which appears in divorce-court scenes shows clearly the value of infantile 'love' Such 'love' is often based on purely egoistic grounds They 'love' what they represent to themselves, what they once represented, what they would like to represent Infantile parents see all kinds of perfections in their babies, although a sober outsider does not share these opinions An infantile mother treats her child like a doll, plays and is thrilled with it, but soon gets tired as the responsibilities become irksome An infantile father sees in the child, first, a toy, and, later, a nuisance

Infantile adults have little regard for, or endurance of, life responsibilities They tire quickly, are easily discouraged and frightened They are thus irresponsible, unreliable, and a source of suffering for those connected with or dependent on, them This permanent suspense for others produces, perhaps, one of the most serious sources of worries and unhappiness Since it is persistent, it gives continual, painful nervous shocks, the cumulative effect of which is bound to be harmful

The infantile individual himself cannot fail to notice that something is wrong, for life makes him quickly aware of it But, in his self-love, exaggerated self-esteem , he overlooks his own shortcomings, and blames everybody and everything but himself In the face of 'injustice', he becomes discouraged, timid, or bitter and pessimistic He is unable to discharge his duties and becomes a disappointment as a father, husband, friend, and, ultimately, as a human being and citizen Bitterness, disappointment, and painful semantic shocks pile up on all sides under such conditions

One of the important characteristics of infantilism of all degrees takes the form of exhibitionism, an impulse for showing off, even by crude display of himself, his body,. This tendency is very common, and leads to many results of a very undesirable social character Infantile men and women are primarily in love with *themselves* and care only how pretty they are They spend large portions of their income and life on dresses and grooming, which, of course, have no social value Such types live in an infantile world and are socially useless, often parasitic on the social body Often those who support them ruin their lives to satisfy these infantile semantic characteristics

Infantile exhibitionism leads, also, very often to a selection of a career. Most diplomats, politicians, professional military men, preachers, actors, boxers, wrestlers, athletes, many lawyers and public speakers, to list only the more important professions, select their professions because of this infantile tendency. We should notice that in this list we find the most important professions which, as yet, have shaped our destinies. Royalty, hereditary potentates, and many plutocrats live under such infantile fairy-tale conditions that they necessarily become semantically twisted.

This pathological tendency probably accounts for our so-called civilization being at an infantile *a*social level, based, as it is, on selfishness, 'sense' gratification, might, brutal competition, acquisitiveness, We should notice that whole 'philosophies', such as theism, the older ontology, teleology, materialism, solipsism, the Anglo-Saxon philosophy of selfishness, and different military and commercial philosophies, clearly display these infantile characteristics. Commercialism, the 'law of supply and demand', as a by-product, also follows from infantile world-outlooks. Those who are interested in problems of politics, economics, sociology, war, and peace, should investigate their problems from this semantic point of view. As Burrow well said, the problems of war are more the problems of psychiatry than of diplomacy.[16]

Many women at present are still infantile, very little developed as human beings, they are themselves exhibitionists and also *sponsor exhibitionism*. It should not surprise us to find that these characteristics, the lavishing of 'love' on shiny buttons and on regiments marching to their destruction, have favoured wars. During the Russian revolution of 1905, the czar's soldiers were on the streets. But women did not 'love' them then. Little children spat on them from behind corners. The result was that very soon the soldiers *refused to carry on this unapproved service*. I know many cases connected with the World War, where, in spite of unspeakable horrors, many regretted the ending of the war because of the infantile approval shown by their women for their 'glory' and the infantile thrill the soldiers themselves experienced because of the shiny buttons, martial music, and parades. In the old system, militarism, religionism, legalism, and commercialism are strictly interconnected by similar *s.r.* Eliminate any one of them for good, and the others would become obsolete or would disintegrate. Our infantile women, no doubt, have sponsored through the ages these infantile social cancers.

The future war will, perhaps, automatically bring these problems to the foreground. It will be an extremely devastating (and less picturesque) aerial war, in which women and children will not be spared.

'Then, perhaps, some of these infantile women will begin to *face into reality,* and so will help to start a new era of human adulthood Men will always depend in their standards on the wishes of women

In infantile nations we witness, also, a great deal of exhibitionism, a craze for athletics, clothes, sumptuousness, noisy behaviour, parades, uniforms, 'military academies', military drills, 'Serious', yet infantile, 'business men' love to parade on the streets dressed up like little boys or circus performers, give themselves some 'mysterious' high-sounding and empty high titles, play with swords which they do not know how to handle, In international affairs, of course, a nation of a more pronounced infantile semantic tendency will seek to keep away from adult international associations The attitude of the United States towards the League of Nations and that of Great Britain towards the project of a confederated Europe suggest themselves at once in this connection

Infantilism has another serious and detrimental connection with race problems, namely, through the sex glands or gonads in their effect upon 'love' and other activities We should realize and emphasize that the sex glands do not function only as 'sex' glands in the common meaning of the word, but even more as *internal secretion glands* with an enormous bearing on all life and 'mental' processes, a $\bar{A}$, *non-el* orientation should never forget that

The various consequences of castration are well known, and need not be repeated here But the interrelation of the gonads with the thymus and with thyroid glands is of interest to us The term gonad means reproductive gland, which produces the egg cells or the sperms The thymus is a term applied to a light pink gland situated in the superior and anterior part of the thorax It extends up into the roots of the neck and comes close to the thyroid gland The thyroid gland is a term applied to a deep red glandular mass consisting of two lobes which lie, one on each side of the upper part of the trachea and lower part of the larynx In women and children the thymus is relatively larger than in the adult male

In humans the thymus grows up to the second year of life and then rapidly diminishes, so that only traces of it are found at puberty In certain cases of *arrested development* or of general weakness in young people the thymus has been found to be persistent Castration at an early age leads to the persistence of the thymus gland Normally, the gland atrophies before the gonads come to maturity and begin to function In some of the lower mammals the gland does not disappear as early as it does in humans The thymus of the calf is popularly called the 'chest sweetbread'.[17]

Atrophy of the thyroid in the adult is usually followed by blunting of 'mental' capacity Speech is slow, and cerebration delayed If the secretions are lacking in childhood we have what is known as cretinism Excessive activity of the thyroid produces a condition known as exophthalmic goitre In many females at each menstruation the thyroid is perceptibly enlarged Extirpation of the thyroid before puberty brings about, among others, signs of cretinism, failure of development of the ovaries , so that puberty is delayed partially or completely [18]

Even these few particulars are sufficient to make us understand that when we begin to deal with 'infantilism', 'arrested development', or 'regression', or 'adulthood', we deal with fundamental *non-el* semantic life-problems which are connected structurally with the organism-as-a-whole Bleuler describes disturbances of affectivity thus 'The so-called psychopaths are really nearly all exclusively or mainly *thymopaths* Furthermore, since affectivity dominates all other functions, it assumes a prominent rôle in psychopathology generally, even in slight deviations, not only on account of its own morbid manifestations, but even more because in disturbances in any sphere, it is the affective mechanisms that first create the manifest symptoms What we call psychogenic is mostly thymogenic. The influence of the affects on the associations produces delusions, systematic splittings of personality, and hysteroid twilight states, repressed pain is the source of most neurotic symptoms, while displacements and irradiations produce compulsive ideas, obsessive acts, and similar mechanisms ' [19]

The thymus appears not only as a childhood gland, but the adult gonads begin to function when the thymus ceases to function When the thymus persists, we often find arrested development and psychopathological disturbances connected with infantilism It should be remembered that in the organism not all 'cause and effect' sequences appear as *one-to-one,* but mostly as *many-to-one* relations Therefore, no standard mechanism can be readily assigned to a semantic disturbance But there are enough structural, functional, and colloidal mechanisms known to account for most disturbances, although the precise working is not known, 1933, in most cases

Psychopathology and experience show that the 'self-love', 'self-sufficiency' , of infantilism are usually accompanied by marked sex disturbances, which, from a racial point of view, are just as important as the semantic disturbances

Infantile types often have 'charming' qualities The women 'are sweet', 'nice', the men seem 'good mixers' and 'popular' The opposite sex often likes these characteristics In men a feeling of sympathy is aroused for the 'helpless little girl', or else pedophilic tendencies are released (Pedophilia is used as the name of a 'mental' disturbance or desire for relations with children, often found among senile dementia and imbeciles) In women, often a feeling of motherhood leads them to like infantile males The charm of the child lies, to a great extent, in his narcissism, his self-sufficiency, and inaccessibility Certain animals, such as cats and larger beasts of prey, fascinate us, as they do not concern themselves with us and are inaccessible But this 'charm' has another and very tragic side Such infantile types cannot stand responsibilities, their affections are shallow and unreliable, they know how to take, but do not know how to give, In life, such connections lead invariably to great unhappiness, and often to disasters The children produced by such infantile types are usually completely ruined by the lack of parental understanding or lack of care Instead of liking such types, men and women of semantic maturity should either avoid them or suggest psychiatrical consultation

Infantile types invariably show some sex disturbances, which also add greatly to family and social difficulties The men are often impotent, the women, frigid Onanistic and homosexual habits or tendencies persist, although such an adult infant is married and has an opportunity for normal life A very important $\bar{A}$, *non-el* fact should be noticed Since the organism works as-a-whole, 'mental' components should be considered in connection with sex life An infantile type appears still in an organ erotic stage He wants only sense gratification From a theory of sanity point of view, *prostitution appears as a substitute for onanism* In adult infants, we very often find either impotence, frigidity, onanism, homosexualism, as simple forms of arrested development or regression, or more extreme forms, like many cases of prostitution Infantiles not only indulge in promiscuity, but build up fanciful rationalizations and represent their own infantile tendencies by 'theories' as 'normal' conduct Many criminals, professional 'vamps', and professed 'heartbreakers' belong to this type It is interesting to note that many 'mental' illnesses are connected with different onanistic rationalizations Often excessive cleanliness, continual washing of hands , appears If a schizophrenic onanist has a melancholic make-up, he rationalizes his problems that he is 'rotting because of his sins' If he has a manic make-up, he feels that he 'is a saviour of mankind' [20]

In all such cases family life is very unhappy, and the future of the children bred under such conditions is usually gloomy Children need healthy family and semantic conditions to develop into healthy individuals

The majority of professional criminals and prostitutes have an infantile make-up. No matter how cunning, they usually show little foresight. They appear egotistic, boastful, exhibitionistic; Gangsters love pomp; their funerals are, as a rule, very expensive,—they want, even after death, to 'show off'. Criminals seldom become good fathers or mothers. They treat each other brutally and are generally promiscuous. Ethically, they behave usually as 'moral imbeciles', not realizing fully what they do. I am not advocating the abolishment of the death penalty on sentimental grounds, but an *enlightened* society should abolish any *penalty* on sick individuals. The 'mentally' ill criminal type should be either taken care of or else eliminated with some scientific benefit, but *not* as a *penalty*. Professional criminals can hardly ever become 'morally reformed' or useful members of society, unless the application of medical science can alter their pathological *s.r.* Without scientific attendance, they would practically always remain socially dangerous individuals. If we want to grow out of the present infantilism, experimentation on humans should be encouraged. Modern experimentation on animals is very humane, and suffering is eliminated. Criminals who are condemned to death should be given to science for experimenting. They would not suffer. Ultimately, they would probably die, but the benefits to the rest of mankind through scientific discoveries would be very important. Under present conditions, we 'take revenge on', 'punish', mostly *sick* individuals, with seriously brutalizing semantic effects on the rest of us. There is not the slightest doubt that experimentation confined solely to animals, no matter how useful, will not solve many problems of Smith. Experimentation on humans is essential, and must be permitted. Most of the notorious criminals who go to the gallows appear at least infantile. How instructive it would be to make experiments on such individuals in respect to their thymus; The list of experiments which science ought to make is very long, but material is lacking for such experimentation. Let me repeat that modern science can conduct its experiments without suffering to the individual, in spite of the fact that some of these experiments would be dangerous and might easily end in the painless death of the subject. The killing off of criminals (sick individuals) as a 'revenge' or 'punishment' or 'justice' is really too antiquated and too barbaric and *wasteful* for an enlightened society. If society wants to *eliminate* them, society can do it; but, at least, let us do it without such brutalizing morbidity, and with as great benefit to knowledge as possible.

The elimination of infantilism must be considered more than a personal issue with individuals; it becomes an ***international semantic*** *problem*, and such an international body as the League of Nations might originate a new era by starting a fundamental enquiry into this subject

Infantilism in its national aspects is not equally distributed Some countries are more infantile than others In some countries even the university students show marked under-development for their age Burrow reports that a questionnaire among students of a prominent university in the United States of America shows a surprisingly large percentage of onanism and homosexualism [21]

We should notice that not even all scientists are free from infantilism. Many of them are childlike in that they do not really care for science, or civilization, or society, but are *asocial* and merely like to play with their toys As an excuse (rationalization of tendencies and 'emotions'), they usually profess 'science for science' sake', not realizing that a complete adult must become a *socialized* individual and cannot keep aloof from general human interests, and that science represents a *public*, time-binding activity and concern, not the private pleasure or benefit of some one person.

Section D Constructive suggestions

As we have already seen, a young child cannot be 'conscious of abstracting', but he can acquire it gradually with experience. Racial, ordered experience is called science Every one of us has the tendencies, and, to some extent, the capacities, for developing science The main aim of such racial, ordered experience is to save effort and unnecessary experiences, so that a child may start where the father leaves off (time-binding) The problems of consciousness of abstracting should be formulated by science and made available for semantic training This would fulfill the main requirements of science, to save experience and effort, and to predict the future, to help in the mastery over external and internal 'nature', and so to produce semantic and physical adjustment

If we teach and train the children in the consciousness of abstracting, we save them an enormous amount of the effort which would be necessary to acquire it eventually by themselves, and we also eliminate a great deal of unnecessary sufferings and disappointments There is no danger of taking 'the joy out of life', the opposite is true With the consciousness of abstracting, the joy of living is considerably increased We have no more 'frights', bewilderments, or similar undesirable semantic experiences We grow up to full adulthood, and when the body is matured for the taking up of life and its responsibilities, we accomplish that, and find joy in it, as our 'mind' and 'emotions' have also matured Such a consciousness of abstracting leads to an integrated, semantically balanced and adapted adult personality. Joys, pleasures, and 'emotions' are not abolished, as this cannot be done, given the structure of our nervous system and 'mental' health, but they are 'sublimated' to higher adult human semantic levels. Life becomes fuller, and the individual ceases to act as a nuisance and a danger to himself and others.

In the racial aspects, if the development of the individual became normal, we should grow beyond infantile organ erotic fixations and *el* languages and infantile systems in all fields. A $\bar{A}$-system, in accordance with science 1933 ($\bar{E}$, $\bar{N}$ systems), would be the human link supplying scientific standards of evaluation to the affairs of Smith.

With the older infantilism and the practically general lack of full consciousness of abstracting, the fears, frights, painful 'emotional' shocks under which mankind lived were bound to have a marked, lasting, and sinister semantic and neurological effect upon the race. The race has never had an opportunity to develop in an adult way. What will be the results for the race of such a transformation it is impossible, at present, to foresee, but one thing is certain, that the results are bound to be very far-reaching.

To afford a better appreciation of what the consciousness of abstracting can accomplish, two more points should be explained. Most young fish do not know their parents, and, from the beginning, their life is independent of parental influences. The human child is helpless, and, for a comparatively long period, is under parental influence. His *s.r* are consequently moulded, 'mentally', 'emotionally', by the doctrines, taboos, structure of language , of the parents, When we speak of a human child, we should never consider him in a fictitious isolation, which has nothing to do with *m.o* reality. *Both* parents and child should be made 'conscious of abstracting'. Only under such semantic conditions can the full benefit be reached. If parents are conscious of abstracting, and realize that their child represents, also, an abstracting in higher orders organism, which consciously or unconsciously registers in one form or another all happenings, the majority of the present unfortunate conditions, 'complexes' , could not possibly arise.

An important, yet usually disregarded, characteristic should be mentioned here. It is known that *repeated* 'emotional shocks' in childhood do harm. As the experiments of Watson show, the child is usually born without 'fears' and without 'frights'. Now 'fears' and 'frights' are not *simply additive* (a linear function) but follow some other more complex function of higher degree. If we denote the constitutional potentialities of the child by f, and the given event by x, the result of the impact of x on the life of the child would be a reaction $f(x)=F_1$. This F_1 designates what his make-up would make out of, or abstract from, x When another event y happens, the reaction of the child is no longer $f(y)$, because this new event is usually taken by the child in the light of the former experience $f(x) = F_1$ Thus, the effect on the child would be different, namely, $F_1(y) = F_2$ If a new event z should happen, the child would react in his 'feelings' , as $F_2(z) = F_3$, . We see, then, that 'hurts' and, in general, *s r* are *not simply additive* but may follow some other higher degree function This process appears general, perhaps necessary, and yet it involves many dangers which can be completely eliminated *only* by the acquired consciousness of abstracting.

In practice, when we train a child in the consciousness of abstracting, we begin to check this devastating semantic process of piling up 'hurts' on 'hurts'. Let us assume that before we begin to train the child, the child has already had painful experiences His memories are still fresh, still fluid, he has little difficulty in dwelling on them With the consciousness of abstracting, and so proper evaluation, dawning upon him, further 'hurts' would 'hurt' less and less until the hurting process would stop altogether In case some semantic harm had been done to the child before he became conscious of abstracting, the memories would be still fresh and he could apply his newly acquired semantic evaluational immunity to the harmful 'hurts' New 'hurts' in practice are usually related or similar to the old ones, they would 'revive' the older hurts Accordingly, he could not only 'live through' the older experiences but at once revise them, and after re-evaluation eliminate the harmful effects

Semantic 'emotional pains' absorb nervous energy and prevent a full development of our capacities Directly the consciousness of abstracting is acquired, the vast field of '*un*consciousness' is diminished, and the nervous energy which was engaged in fighting semantic phantoms is released We should expect keener and sustained attention, strengthened interest, and other creative manifestations Consciousness of abstracting, as it leads to proper evaluation, not only eliminates many unnecessary sufferings and semantic disturbances, but, by doing so, actually releases stores of energy for useful and creative purposes

The human brain has vast areas which, at present, have no definitely known functions Perhaps, with the older *lack* of consciousness of abstracting, the flow of nervous energy was misdirected or absorbed by the older ways of 'feeling' and 'thinking' in the lower centres Thus, the available energy left was not sufficient to utilize the higher centres to the full extent

Personal semantic difficulties always seem very personal, and no outsider can ever fully grasp the situation One of the benefits of the present method of training in sanity consists in the fact that we do *not* dwell upon the personal affairs of the individual, but that we give, instead, a general structural semantic *method,* by the aid of which every one can solve his problems by *himself*.

We have established sharp differences between 'man' and 'animal'. These differences must be considered of higher order, as the terms 'man' and 'animal' are applied to abstractions of higher orders. We found that 'man' through ignorance and inappropriate *s.r.* can copy animals in his nervous reactions. Such copying appears either as arrested development or as regression. In dealing with the terms 'conscious' and 'unconscious', we discovered a general and human content for *human* 'consciousness', namely, the 'consciousness of abstracting'. The ascribing of a *general content* abolishes a vast field of 'unconsciousness', and so tends to prevent arrested development, infantilism, regression, whenever this is possible. The problem of making the *structure* of language similar at a given date to the structure of the events it symbolizes, is introduced. The conquests of science become incorporated into daily life by the use of the new *language*. The *structure* common to both science and language appears to be the intimate bond between science and *human* life. The masses gain simple structural and semantic means for adjustment.

A theory of sanity must draw attention to problems involving 'truth', 'falsehood', 'repressions'. Since the main usefulness of the theory is to help in attaining the most efficient working of the nervous system by the elimination of disturbing semantic factors, 'attitudes', doctrines, we must investigate the effect false (or repressed) statements may have on the working of the nervous system.

For instance, if we *see* that A, B, and C are given in the order A,B,C, such lower abstractions start cycles of nervous currents, which correspond to the *seen* order. If we *see* the order A,B,C, and *say untruthfully* that the order appears as C,B,A, this *statement* results, also, from some cyclic nerve currents. Obviously, we have *some conflict and disturbance in the working of the system.* If we make a mistake, the situation is different. Let us say that many observers definitely establish the given order as A,B,C. A new observer *sees by mistake* the order as C,B,A. His nervous currents correspond to his error, and when he makes a *truthful statement* that he has seen C,B,A, this statement also is connected with the appropriate nerve currents and there is *no conflict* or disturbance between the corresponding nerve currents. The seen and reported correspond to each other.

It is easy to conclude that mistakes and deliberate falsehoods have a different mechanism A mistake, which leads to a subjectively true but objectively false statement, has no nervously disturbing factors Deliberate false statements about facts involve semantic conflicts and disturbances in the functioning of the nervous system Similarly, with 'repressed' material, permanently conflicting nervous currents are present The nervous energy is spent on conflicts and struggles, while all of it is needed for constructive purposes.

In scientific work we have similar problems We gather different abstractions of lower orders and then make higher abstractions about them When these two different orders of abstractions fit nicely structurally, we are satisfied and enjoy the resultant harmony If they conflict we feel restless Often scientists spend years, or even a lifetime, formulating higher order abstractions which do not conflict structurally with the lower abstractions Then they feel satisfied Scientists know well the feelings of 'mental' pain and discomfort Creative work is carried out because of such discomfort Those who are *not creative* do not experience this, but they also do not produce important work

The problems of structure, of correct symbolism, of evaluation, of the production of higher order abstractions which are structurally similar to the lower order abstractions , must have a neurological significance, and should be investigated from this point of view. Scientists should try to eliminate these unnecessary conflicts Those who feel no conflicts may, nevertheless, be so involved in it that they have no free nervous energy left to overcome it Some such attempt is being made empirically in semantic therapy The psychiatrist tries to discover and eliminate the semantic conflict, thus freeing nervous energy which may then be spent on useful work

In racial and national levels, systems of politics, economics , which are *based on falsehoods and repression of truth*, must unbalance the working of the nervous systems of the people Since they are the result of the infantile *s r* of the race, they propagate the arrested or regressive development in the part of the race whose *s r* they influence As usual, the vicious circle is working here also A $\bar{A}$-system throws an entirely new light on the significance of science in *human* life Radios, with their attendant possibilities of hearing jazz or a delusional 'revivalist', and the invention of bigger and better means for killing people, do not represent the main *practical* importance of science *Generalized* science means scientific method and the discovery of the structure of events, to which the structure of our language must be adjusted if this daily tool of everybody is not to play dangerous semantic tricks

A 'science of man' must follow science (1933) in its structure and method. Only by accepting the current 'scientific metaphysics' as given by science at a given date *is sanity possible*. The passing from an infantile 'civilization' to an *adult civilization* of fuller human life and happiness will come with the development of a scientific civilization which has scientific standards of evaluation. But the passing will not be so easy. As we have already seen, science contains affective factors, and many scientists still appear infantile. In order to enter upon an adult civilization, we must first have non-infantile leaders, who must be produced by appropriate training. This involves much research work along the lines sketched in the present work, and the establishment of chairs of general semantics and psychophysiology in universities. Educational methods must be radically revised, and experimentation encouraged in the widest sense.

In 1933 we know positively that in the physico-chemical and colloidal structures we find conditions of practically endless possibilities corresponding to the very large numbers of semantic states and reactions. Medical practice shows experimentally that a great many physical symptoms involving some colloidal states are produced by semantic disturbances, because, once these disturbances are eliminated, the physical symptoms vanish. The enormous numbers of observed and possible different *s.r.* could not be accounted for by the older, still prevailing, *el*, *A*, and two-, or three-valued outlook, and the cumbersome, extremely limited, and necessarily slow chemical 'passing of different substances' through the nervous system.

It is true that every student of medical science is acquainted with colloidal behaviour, but this knowledge has been neither emphasized nor consistently applied, because colloidal behaviour represents physico-chemical processes involving electromagnetic, high pressure. . ., manifestations which cannot be dealt with at all by *el*, *A* means. Thus, a physician who is not trained in $\bar{A}$ general semantics, cannot 'think' in colloidal and physico-chemical terms, which in 1933 are the only modern ways of dealing with the organism-as-a-whole. This is much more serious than the layman or even the physicians realize, and accounts for the fact that, in spite of different special achievements and different discoveries, the practice of medicine is becoming more and more unsatisfactory. It also explains why the average physician cannot grasp the importance of psychiatry for general medicine, and why some psychiatrists indulge in very unscientific and doubtful metaphysics. Thus, a general physician who 'thinks' uniquely in seriously antiquated chemical and physiological terms, deals with a non-existent, fictitiously isolated *A* and *el* 'body', and cannot grasp the necessity for a *non-el*, $\bar{A}$, and a physico-chemical, colloidal outlook, which integrates 'body' and 'mind' The majority of psychiatrists in their turn, and for similar reasons, often have a highly metaphysical outlook, repulsive to the general physician They do not seem to realize that they have at their disposal colloidal and physiological mechanisms as well as physico-mathematical formulations based on four-dimensional order, and that they, therefore, do not need any doubtful metaphysics With $\bar{A}$ modern semantics, the only possible scientific outlook (1933) must be colloidal, physico-chemical and physico-mathematical, in which the long sought for *non-el* solution of the 'body-mind' is found The difficulties I am dealing with are general and depend on fundamental principles, the disregard of which introduces semantic blocking factors, at present imposed on the medical students, and from which only a few exceptional, scientifically inclined individuals are capable of breaking away From the present point of view the older reflexology is also unsatisfactory and requires a $\bar{A}$ reformulation

The present system, although far from complete, already suggests many most important structural issues which should be *verified empirically* Experiments *alone* can decide which verbal structures are similar to empirical structures, and experimentation should be encouraged in the widest sense Some further theoretical work should also be done Clinical literature describes many new and unexpected facts These facts should be described anew in the new language, to see what relations survive the transformation of forms of representation Thus, if it is found that *all* 'mental' ills in *all* different formulations indicate *improper evaluation*, we should be justified in concluding that *evaluation* represents an invariant general characteristic of the activities of the human organism-as-a-whole, and, consequently, must be of extraordinary importance for adjustment and sanity When we reach this conclusion, we should investigate the *mechanism of evaluation,* starting with the simplest issues; namely, investigating those factors which make proper *evaluation impossible* We should discover that identification in *all cases* makes proper evaluation impossible, and should then conclude that identification must be entirely eliminated before we can go one step further In fact, once we have reached these rather obvious results, the rest of the $\bar{A}$-system follows But this would not be enough, we must verify the conclusions *empirically,* and this suggests directly that a definite series of experiments should be undertaken

In hospitals for 'mentally' ill two equally large groups of accessible patients exhibiting similar clinical symptoms should be selected, and isolated A physician who himself has undergone a $\bar{A}$ training should attempt to re-train the *s r* of one group The other group should not be re-trained, but treated in the average passive and standard way,—it would be the control group One physician should be in charge of both wards and keep a detailed record of the cases and treatment It is to be expected that at the end of the year, in the ward trained in the $\bar{A}$ standards of evaluation, a larger number of unexpected and spontaneous recoveries would happen than in the untrained ward It would be extremely instructive to have more than two groups, and to attempt a different group method, following some other medical school based on another system-function The passive attitude toward the patients should be changed, as under the older methods physicians in 'mental' hospitals are more glorified keepers than medical men This is what theory suggests. Experiments alone can show if these conclusions are correct In special individual cases, the theory has already been confirmed, but it should be tried as a group method, and, if successful, only then would 'mental' hospitals become hospitals, and not mere places of detention

A few words concerning psychotherapy will not be amiss In a time-binding class of life, we must take into account the historical four-dimensional experiences of the race, which, even in individual cases, have sound neurological foundations, as it is known that the nervous reactions are influenced by past experiences History teaches us that the work of some men has influenced great masses of mankind for many years, and that the works of others have had but little general, lasting effect The considerations of doctrinal functions and system-functions explain this fact quite simply The older an individual or a race grows, the more structural observations they gather, and the more they notice the structural dissimilarity of their forms of representation with the first order facts they encounter As adjustment is generally useful, individuals, as well as groups, and particularly scientists, always attempt to discover more structural data about the world and themselves This process requires, among others, the comparison of the structure of the forms of representation with the structure of the world and ourselves All so-called 'progress', 'civilization', and science depend on this

In this particular field, achievements are of two kinds

1) Some individuals produce a *new* system-function, with a *new* structure, more similar to the world , (see Chapter XI) In the great majority of cases, the new system-function is *not* formulated *explicitly*, but is hidden implicitly behind some explicit particular and individual interpretation or particular system of the discoverer The production of a new system-function is usually a most important event and is independent of the special value given to the variables in this function by the

originator. In such cases, the given originator has many followers, and there is a possibility of many doctrines or systems which have *one* doctrinal function or system-function The content of the doctrines may be changed, but they all have *one structure* The importance of the new doctrine or system was not in its particular interpretation, or in the assigning of a particular value to the variables, but in the underlying doctrinal function or system-function, *which alone has explicit structure,* and is given by the postulates which establish the function

2) Some individuals do *not* produce explicitly or implicitly new doctrinal function or system-functions, with new structure, but simply assign a new and individual value to the variables in the *one* doctrinal function or system-function produced by others These workers very often bitterly defend the private individual value they have assigned to a variable, and are often entirely innocent of the serious debt they owe to the originator of the new function which they utilize But these works never mark a milestone in the progress of mankind and are usually soon forgotten

Because of the disregard of the considerations explained here, the proper evaluation of different doctrines and systems is very difficult, and even in scientific circles the lack of orientation in this field is astonishing It seems that it is not enough to produce a 'new theory' to have made an important contribution to knowledge, but it is essential to produce a new doctrinal function or system-function, because this only has a structural significance This point of view, perhaps, solves the tremendous and, as yet, unsolved difficulties we have in reducing doctrines to sets of postulates, which is admittedly desirable, and yet so hard to produce Thus, to find the doctrinal function or system-function which underlies a theory, *we must strip it of all accidental values privately ascribed to the variables, and formulate only the invariant relations which are posited between the variables* The finding of this function is also equivalent to finding the *structure* of a given theory

We may consider that the psychoanalytical and psychotherapeutic movement originated with the work of Freud The epoch-making value of his work consists in the fact that, underlying his special theory, there can be discovered a new system-function All other schools simply ascribe a different value to the variables, but do not produce, structurally, new system-functions, they represent different systems which have one freudian system-function

It is here impossible to analyse this problem systematically or in detail, but a few structural hints may be useful

First, we must discriminate permanently between the freudian particular system, which represents a particular interpretation of the freudian *system-function* without specific interpretations, say 'complex x', on semantic levels, which corresponds, let us say, to 'cluster X' on colloidal levels. Second, we must realize that the freudian system-function (not system) was scientific at the date of its production, but to be scientific 1933 it must be revised and reformulated, taking into consideration the newer physico-mathematical, physico-chemical, colloidal, $\bar{A}$, points of view. The struggle for a special 'complex A' or invention of a new 'complex B' is useless because in 1933 the colloidal, structural 'cluster X' which underlies the semantic 'complex x', which alone can legitimately be considered in a system-function, includes all of the 'complexes' in literary existence, and there are no assignable limits to their numbers. If we analyse in such ∞-valued terms as, for instance, 'cluster X', we evade an enormous amount of unnecessary and confusing metaphysics, and become scientific in the 1933 sense. From a modern, $\bar{A}$ system-function point of view, which means, when we recognize the necessity of ∞-valued semantics, structure, necessitating the reduction of a system to a postulate base, we readily see that the freudian *system-function* (not system) is a necessary and natural passing step between the A and $\bar{A}$ systems.

The postulates which are discovered in the freudian system-function can be divided into two main groups.

1) The observations of human behaviour and, in my language, of s.r, have to be formulated in a special language to fit the more structurally fundamental parts of the system.

2) The fundamental and revolutionary new postulates were, at the date of their introduction, quite scientific. In 1933 these postulates have to be reformulated and made to comply to modern physico-mathematical, physico-chemical, and general semantic $\bar{A}$ standards.

A satisfactory analysis of the above problems would require a special volume, therefore, I shall entirely disregard No. 1, and from No. 2 shall only suggest a few most important and new postulates. These can be expressed, roughly, as follows. (*a*) The postulation of an *active* 'dynamic' unconscious. This postulate departs widely from the older notions, although the word 'dynamic' is used in this connection in the vernacular, but not strictly scientific, sense. The methods of translating the dynamic into static and vice versa, are disregarded, owing to the innocence of modern science and the assumption by physicians, in general, of the permanent validity of A principles. (*b*) Once the *active* unconscious is postulated, some determinism follows according to the date Freud, at his date, accepted the (in 1933 antiquated) two-valued determinism Unfortunately, the great majority of physicians and the medical education still follow the antiquated notions (*c*) As the past is taken into account and man is treated as a process in which the past experiences play an important role, we might say that the outlook is four-dimensional, but this statement is not entirely justified, because the notion of a consistent four-dimensional orientation carries us much further than physicians, who neglect physico-mathematical aspects, can possibly produce

In a $\bar{A}$-system, the fundamental postulates of the system-function which underlies all psychotherapy have been accepted, although they have been vastly enlarged to comply with the known facts and scientific requirements of 1933 It is important to investigate independently, systematically, and in detail, the corresponding system-functions and to find to what extent they mutually intertranslate, but such an investigation cannot be carried out successfully if we confuse, through the habits of speech, the two different terminologies

It seems that the older psychotherapeutic schools have been formulated as systems, and that the system-function, which underlies them, has not been explicitly stated, greatly hampering future creative work

The special benefit of a generalized theory is in its fundamental simplicity and entirely general linguistic applicability, which, for prevention, plays a decisive role Accidentally, we acquire psychophysiological means to influence the so difficult 'narcissistic' cases

The present author has attempted to indicate the most important structural and semantic factors which would facilitate the future workers in the impending, necessary revision and co-ordination.

CHAPTER XXXI

CONCLUDING REMARKS

Pitiless indeed are the processes of Time and Creative Thought and Logic, they respect the convenience of none nor the love of things held sacred, agony attends their course Yet their work is the increasing glory of a world,—the production of psychic light,—the growth of knowledge,—the advancement of understanding,—the enlargement of human life,—the emancipation of Man (264) CASSIUS J KEYSER

Yet the barbarians, who are not divided by rival traditions, fight all the more incessantly for food and space Peoples cannot love one another unless they love the same ideas (461) G SANTAYANA

The individual whose brutish desire for personal profit is unrestrained by the needs and rights of his fellows reverts to barbarism If a bandit he is outlawed, if a politician he is—usually reelected, with resulting retrogression of the entire social organization (221) C JUDSON HERRICK

a "League of Sound Logic" is the best "League of Nations" because effective under the subtle inevitable laws of Logical Fate—Unified Doctrines Will Unify Man (280) A K

A little less worry over the child and a bit more concern about the world we make for the child to live in, an inclusion of the child in a life of which the aim is not merely to earn money so as to become independent of the job, more love for whole-hearted, creative work and progress that will make possible what we all can share in, with these conditions, the adult and the young both will have a better chance * ADOLF MEYER

The present remarks were originally written for the last chapter of the whole volume, but a final critical survey of the material suggested the newly ordered sequence of the present three main divisions Book I gives a general preparatory introduction which will help the reader to differentiate between the A and $\bar{A}$ systems, and to evaluate properly the differences Book II formulates the main $\bar{A}$ principles which constitute an organic interrelated whole, to which the present concluding remarks belong Book III gives some additional structural data about mathematics and physics which usually are not treated from the present point of view, but which furnish the essential structural material needed

The writing of Book I, and particularly Book III, was very laborious and difficult I often had the temptation to omit Book III entirely, and to refer the reader to other authors After months of search, however, I found, to my sorrow, that, in spite of many excellent volumes, there were no books written from a structural and semantic

*What Can the Psychiatrist Contribute to Character Education? *Rel Educ* May, 1930 point of view To refer the reader to other writers would necessitate the reading of a fair-sized library, because often from a whole book he would need only a few scattered paragraphs This would involve a very expensive and laborious process of hunting, which very few would undertake, besides, it would not give a connected structural or semantic picture I tried to induce some specialists to write a book on the structure and semantic aspects of mathematics, and another similar book on physics I was told that it would be very laborious and difficult, if at all possible, and so I had no other choice than to try to write it myself

I earnestly suggest the reading of Book III, so that the reader may, at least, become acquainted with the existence of such problems I hope that even specialists may find some suggestions helpful, because the structural and semantic aspects of science and mathematics are usually neglected, the neglect of which introduces needless difficulties in the teaching The elimination of identification helps to solve many scientific puzzles, besides eliminating semantic blockages and so helping creative activities

The world affairs have seemingly come to an impasse and probably, without the help of scientists, mathematicians, and psychiatrists included, we shall not be able to solve our urgent problems soon enough to prevent a complete collapse Now those who are professionally engaged in human affairs, economists, sociologists, politicians, bankers, priests of every kind, teachers , 'mental' hygiene workers, and psychiatrists included, do not even suspect that material and methods of great general semantic value can be found in mathematics and the exact sciences The drawing of their attention to this fact, no matter how clumsily done at first, will stimulate further researches, produce better formulations and understanding, and ultimately create conditions where sanity will be possible

Some of those who have seen my manuscript or with whom I have discussed the problems seemed to dislike the term 'copying animals in our nervous reactions' and also the explicit introduction of 'Fido' As identification is found among animals, primitives, infants, and 'mentally' ill, it could be said that the introduction of 'Fido' was not necessary I have given serious consideration to the eventual desirability of completely eliminating 'Fido' from my work and substituting the term 'primitive' , but, after mature deliberation, I decided that it will be helpful to accentuate the distinction between the reactions of animal and man The main justifications of this are as follows·

1) My whole work and the formulation of a $\bar{A}$-system started with an attempt to produce a science of man, this necessitating a mod-

ern, scientific *functional, non-elementalistic, sharp* definition of man. Such a definition was given in my *Manhood of Humanity* to the effect that man differs from the animals in the capacity of each human generation to begin where the former generation left off This capacity I called the time-binding function This definition cannot be denied, and it fulfills the modern requirements

2) The present enquiry originated in the investigation of the mechanism of time-binding, and is a further analysis of the sharp differences between the reactions of animals and humans, which became the psychophysiological foundation of a $\bar{A}$-system and a theory of sanity

3) The further the investigation advanced, it became increasingly evident that the issues involved are extremely complex, and that in this field, from a structural and *non-el* point of view, practically nothing has been done In general, all existing 'logics' and 'psychologies' are structurally misleading, since they are still thoroughly *el* and pre-*A* or *A*, these conditions necessitate the elimination of them, as well as other dependent disciplines, to prevent their being accepted as structurally fundamental It was then desirable, in my pioneering enterprise, to keep a simpler and more obvious contrast between 'Fido', whom we nearly all know quite well and usually like, and 'Smith', whom no one seems to know properly This method has proved very useful to the writer and I am convinced that many readers will find it equally helpful I frankly admit that if I had not followed this simplified method, I could not have produced the $\bar{A}$-system and discovered in this psycho-logical maze the blockages introduced into our *s r* by identification, elementalism, lack of consciousness of abstracting, improper evaluation , and, in general, infantilism

For these three main reasons it seemed advisable to retain 'Fido' as a most useful factor in my analysis, with all due apologies to 'Fido'

I also admit that I did not realize the difficulties of the task and the magnitude of the undertaking The last revision alone of the manuscript required more than a year I am all too well aware to what extent the presentation falls short of my expectations and how much better it could have been written by some one more gifted, but the following rather unexpected developments sustained my courage

1) Curiously enough, the principles involved are often childishly simple, often 'generally known', to the point that on several occasions some older scientists felt 'offended' that such 'obvious' principles should be so emphasized Yet my experience, without any exception, was that no matter how much these simple principles were approved of verbally, *in no case* were they *fully applied in practice* Slowly I understood that

we cannot train mankind in identification by all available means, which must prevent adjustment, and then live by non-identification Thus, when non-identity is pointed out, even a moron will 'agree', or wonder at the silliness of an author who fusses about it, yet, because all of us were *trained* in a linguistic and semantic system based on identity, that infantile identification will unconsciously play havoc with all our *s r* the rest of our lives, unless this semantic blockage is counteracted Naturally, the 'simpler' a principle appears, to which we may pay lip service, but which we *never fully apply*, the more I became convinced that the discovery of new methods for the application of this simple yet neglected principle must be considered most important. Any reader may verify by himself to what extent identification introduces difficulties in his own life In fact, the main difficulties we have can always be traced to some identification somewhere

2) The experimental data of Doctor Philip S Graven with the 'mentally' ill and those cases of semantic disturbances which, in the orthodox way, were not supposed to be un-sane, showed that a change from the A standards of evaluations involving identification to the $\bar{A}$ standards without identification often either brought about a complete semantic reconstruction of an individual, or semantic, expedient, and lasting 'cures' This fact again impressed me with the genuine workability and so human importance of a $\bar{A}$-system If the old 'impossible to change', 'human nature' can be 'changed' by the new simple psychophysiological methods, this again suggests that this new system, no matter how imperfect, may be useful

3) I was also very much impressed by the far-reaching power of the $\bar{A}$ methods As a rule, only mathematicians and epistemologists fully appreciate what the power of a method means Thus, the differential methods were invented, and later we found that these methods were structurally applicable to all processes A tensor calculus was invented, and we found that it gave us absolute, invariant formulations applicable to all physics Many other methodological innovations could be cited, and always the generality of the applications gave the value to those new formulations The present $\bar{A}$-system was formulated in a way independent of other diciplines, as it was the direct result of structural semantic researches *free from identification* This led to the formulation of fundamental *general* principles which underlie all human 'knowledge', such as non-identity, requiring the recognition of structure as the only possible content of 'knowledge' and so leading to the formulation of 'similarity of structure', non-elementalism as a general principle, the general principle of uncertainty, ∞-valued general semantics,. It is

naturally very reassuring to find that the newest most important achievements of science have followed these principles unconsciously and have applied them *before* they were explicitly formulated

From another point of view, a $\bar{A}$-system which could claim to be 'modern', should formulate general principles that all scientists in every field could follow. This was practically the case with the A-system until Francis Bacon It is also the case with the present system except that different scientists have applied these new principles without having produced a *general formulation* The fact that these principles had no general formulation was a retarding factor even in science and made the application of science to human affairs impossible In the following examples, the different $\bar{A}$ aspects overlap, and I am emphasizing only the most marked features Thus Einstein-Minkowski's space-time, Einstein-Mayer's new unified field theory, the newer quantum mechanics, the new physics of high pressure, piezochemistry , the tropism theory of the late Jacques Loeb, the physiological gradients of C. M Child., ., exhibit clearly the application of non-elementalism Heisenberg's restricted principle of uncertainty is also the result of the application of non-elementalism, based on the observation that the 'observer' and the 'observed' cannot be sharply divided This principle becomes a particular instance of the general $\bar{A}$ principle of uncertainty, which again is based on the observation that we deal actually with absolute individuals and speak in more or less general terms, with the result that all statements are only probable in different degrees.

The absolute individuality of four-dimensional events, objects, situations, *s r* , necessitates an indefinitely flexible evaluation requiring ∞-valued $\bar{A}$ semantics Outside of daily life, the best examples are given in science by the newer developments in vitamins, the effects of radiant energy on heredity, but particularly by the bewildering possibilities disclosed by the developments of physics, physics of high pressure, piezochemistry, polymorphism, colloidal behaviour, and the application of colloidal knowledge to psychiatry The Polish school of mathematicians has produced the extension of the traditional two-valued A 'logic' to three-, and many-valued 'logic', Chwistek has based a new foundation of mathematics and a new theory of aggregates on his semantic methods, but even these writers disregarded the *general* problems of non-elementalism, non-identity, and the necessity for a full-fledged $\bar{A}$-system before their formulations can become free from paradoxes, valid, and applied to life

All these issues combined are of particular interest to mankind in general, and to the medical profession in particular, because obviously, if mankind is to pass from an infantile stage of its development into an era of general sanity, this would require a serious collaboration of medical science Unfortunately, medical science is one of the most laborious and difficult disciplines, and, of late, in spite of some specific advances, it is rapidly ceasing to be in general a modern science Any one who attends medical congresses, scientific meetings, or follows up medical literature often wonders whether he listens to, or reads, scientific arguments, or sixteenth century religious disputes Dr F G Crookshank, in his chapter on *The Importance of a Theory of Signs and a Critique of Language in the Study of Medicine* in Ogden and Richard's *The Meaning of Meaning,* gives an excellent picture of the present sad state of affairs, but a $\bar{A}$ analysis discovers deeper foundations underlying the difficulties in medicine, which would have to be remedied by the revision of medical education. In this connection, $\bar{A}$ issues become very important Organisms in general, and humans in particular, represent colloidal processes which involve tremendous pressure because of colloidal attraction for water Dr Neda Marinesco[1] has recently suggested that Ice VI constitutes an important factor in the human organism Ice VI represents a new form of ice discovered by P W Bridgman[2] who found that water in bulk and at the temperature of the body may be found to crystallize by the application of high pressure It is the notion of Dr Marinesco that the forces of adsorption may be as high as the pressure used by Bridgman, so that in thin surface films the arrangement of water molecules may be much like that found in Ice VI It may interest the reader to know that, among others, Professor Bridgman discovered that acetone becomes solidified at room temperature, albumen coagulates , under high pressure

Although physicians in their university days are well acquainted with colloidal chemistry, yet somehow, in practice, they have great difficulties in 'thinking' in colloidal terms With the newest discoveries of physics of high pressure and piezochemistry, with their bewildering variety of physical manifestations, which, under different pressure, change with every individual material, a modern physician will have to 'think' not only in terms of colloids, but of colloids in combination with the data of high-pressure physics and piezochemistry Now such 'thinking' is humanly impossible under the traditional two-, or three-valued *A* disciplines and becomes only possible with ∞-valued $\bar{A}$ general semantics One of the immediate results of the use of $\bar{A}$ disciplines is the elimination of the elementalism of 'body' *and* 'mind', 'intellect' *and* 'emotions'., and the introduction of the *non-el* point of view as given in the present work This requires every physician to be acquainted with psychiatry, which acquaintance would eliminate many harmful cults. It should be fully realized that the older chemistry which dealt with different 'substances', having different 'properties', could have been treated by *A* subject-predicate and two-, or three-valued means. But not so in 1933, the older chemistry is gone, and today we deal only with a special branch of physics based on structure, the newer physics of high pressure show clearly that many of the older characteristics of 'substances' are only accidental functions of pressure, temperature, and what not, which vary in a bewildering way, requiring new semantic principles, new epistemologies, in short, a new *non-el* and ∞-valued $\bar{A}$-system. In other words, whoever retains the *A s r* is entirely unable to 'think' scientifically in the modern sense. If we want to have a science of man or a 1933 science of medicine, the first step is to revise thoroughly the *A-system*.

In fact, many more interconnections and interrelations could be shown which would make still more obvious how a $\bar{A}$-system results from, and leads to, modern scientific results, which can be extended and *applied to all human concerns* only after a *general* formulation *as a system*.

4) If the difference between the animal and man consists in the capacity of the latter to start where the former generation left off, obviously humans, to be humans, should exercise this capacity to the fullest extent. If we fail to do this, we again 'copy animals in our nervous reactions', which copying is the very thing we should struggle against. This 'where the former generation left off' would not only include all science, but also epistemology and the 'wisdom' which through painful experiences each former generation has accumulated, which, *in principle, should be given to every child*. Under the *A* conditions of our present education, systems, and evaluational systems, this is completely impossible and may sound visionary. Thus, to acquire scientific knowledge in all fields, one would have to spend a lifetime devoted to science, entirely free from financial worries, and even then he would only be able to acquire a small part of it. Before any older epistemological insight could be imparted, one should not only have special gifts, interests, but should also have an enormous amount of knowledge before such an education could be attempted. Similarly with 'wisdom'. The older and the younger generations, by colloidal necessity, cannot fully understand each other and, to a large extent, have mutual mistrust, which, as yet, is an entirely normal *A s r*,

In a *non-el*, $\bar{A}$-system this whole situation becomes radically changed. The impossible is made possible, I may say more, it is made simple and easy, and becomes a necessary and unavoidable factor in the life of any child A *non-el*, $\bar{A}$-system is based on the complete elimination of identification, from which it directly follows that the only link between the un-speakable objective levels and the verbal levels is found in *structure* Structure, then, becomes the only possible content of all knowledge , and all scientific technicalities, admittedly laborious and difficult, become only a necessary tool in the search for structure, with little, if any, intrinsic value, and are unnecessary for 'knowledge' as soon as in a given case the structure is discovered. This structure is always simple and can be given to children

It is meaningless and utterly useless to argue whether or not the world is 'simple', as the world *is not* our understanding of it, but as our 'understanding' happens to be structural, our nervous system, through its abstracting capacities, makes it simple, once its structural content is discovered As the search for structure involves similarity of linguistic and empirical *structures,* we readily understand that any language, which we cannot evade teaching our children, has structure and involves structural assumptions In the *structural revision* of our language and the teaching of a few structurally appropriate terms, entirely abandoning a few structurally misleading ones, we directly impart all up-to-a-date fundamental knowlege to any child We train him automatically in the appropriate linguistic structure, which builds up in him appropriate *s r* Mankind at large does not need scientific technicalities to absorb and thereby obtain semantic benefits from the structural results of science These results are the only ones which really matter, and which can be given in an extremely simple way, automatically abolishing the primitive metaphysics, structural assumptions, and infantile *s r*

By abolishing the structurally false to facts one-valued identification, we automatically train in ∞-valued differentiation leading toward consciousness of abstracting, which results in all the wisdom that epistemology and private experience can give us, being structurally a total result of racial experience As structure is based on relations and *order,* structural training, when done consciously, becomes a physiological method, working simply and automatically

In the A-system these semantic mechanisms were not consciously recognized, although they worked fatalistically with us We were imparting primitive psychophysiological reactions to our children, who had to spend a lifetime to learn by very painful experience that something was wrong somewhere Now we understand that the origin of the difficulty was in the lack of scientific investigations which would have analysed, non-elementalistically, the structural aspects of language and connected *s r* All of which, let me repeat, works automatically, as experience and experiments abundantly show Thus, an analysis of the mechanism of time-binding depends on the discovery of a sharp *non-el* difference between 'Fido' and 'Smith', and the formulation of means to make the time-binding characteristics of man fully effective with all except heavily pathological individuals

By abolishing identification we generalize differentiation and so impart consciousness of abstracting, an indispensable factor in *proper evaluation,* and an absolute condition for adaptive and so survival behaviour Thus a $\bar{A}$-system becomes a general theory of sanity and the general theory of time-binding, from which general semantics follow

5) One of the most important features of the present $\bar{A}$-system consists of its *non-el* structural character We may analyse problems in a scientific 'intellectual' way, yet this analysis, because *non-el,* structural, and semantic, appeals to, and affects, our 'feelings', 'intuitions' , involving psycho*physiological* factors based on order Thus, the structurally necessary translations of one level of abstractions into the others and vice versa, is enormously facilitated, while in *el* systems these translations were hampered by unavoidable semantic blockages Accordingly, 'intellect', 'emotions', 'body', and 'mind' , are not divided The organism is affected *as-a-whole,* because structurally correct *non-el* means are employed, making many benefits of the system accessible to children, morons, and, perhaps, even superior idiots. The last results are to be foreseen, although they have not, as yet, been verified empirically.

6) But the most workable feature of the system consists in the fact that, being based on such fundamental principles as non-identification non-elementalism , it has an organic unity The main issues are all strictly interrelated and apply to 'body', 'mind', 'emotions' , in a *non-el* way, all working *automatically,* no matter from what angle we approach the training

Thus, if we start with order, we are led to relations and structure, these establish differentiation and stratification, eliminating identification and 'allness', which result in consciousness of abstracting, necessitating ∞-valued general semantics, indispensable to proper evaluation and adjustment If we start with non-identity, we are led to order, relations, structure, differentiation, stratification, non-allness, consciousness of abstracting, ∞-valued semantics, proper evaluation, and adjustment If we start with differentiation or stratification, we are led to order, relations, structure, non-identification, non-allness, consciousness of abstracting, and proper evaluation

It should be noticed that consciousness of abstracting and proper evaluation are complex end-results which cannot be imparted directly, but which become automatically lasting semantic states only after we have eliminated one-valued identification, or introduced order, ∞-valued differentiation, stratification , The *non-el* benefit of the system consists in engaging the organism-as-a-whole Thus, four-dimensional *order* plays the role of a potent *physiological* factor in the process and becomes the foundation for psychophysiology Non-identity is a term applied on the verbal levels, which, on visual and intuitive levels, involves differentiation, ordering, and stratification This system thus involves all necessary nerve centres and operates in a *non-el* way, as reactions on one level are easily and organically translated into the terms of other levels, making psychophysiology possible

7) Finally, it is significant that many publications in the last ten years have shown efforts in a similar direction, which have received scientific and public approval As I am more interested in creative work, rather than critical, I shall not analyse these strivings except to make one general remark that, because they are not based on order, structure, *non-el s r*, the complete elimination of identification , they are valuable and useful to the selected few, but under no conditions could a psychophysiology or a theory of sanity be based on these works which could be applicable in general elementary linguistic and semantic education If I am not mistaken, in this respect the present work differs radically from the others with which I am acquainted

From a *non-el* point of view we can never disregard the effect the 'body' or 'emotions' have on the 'mind', and vice versa the effect that the 'mind' has on the 'emotions' and the 'body', Identification and all its consequences involve seriously disturbing semantic factors with corresponding colloidal disturbances, and it seems that, as yet, the human race, outside of very exceptional cases, has never been free from these disturbances What effect the elimination of such disturbances will have on the human race it is impossible to foretell at this stage, beyond expressing the expectation that the consequences must be highly beneficial

We have already become acquainted with the terms 'conditional' and 'unconditional' reactions In the example of the patient and the paper roses, we have seen that the pathological symptoms were 'unconditional' They were compulsory, as in the case of the dogs mentioned in Part VI In a healthy individual they would have been fully conditional reactions, under semantic control The above terminology may be extended so as to apply to all 'mental' ills, for here that which in the 'normal' person is a fully conditional reaction becomes unconditional, or a *reaction of* *lower order conditionality* (compulsory) beyond conscious control Here we differ from animals and hospital cases When our conditional reactions are not fully regulated by proper *s r* and become unconditional, we copy animals, and so are in a state of arrested development or of regression.

The general therapeutic and *preventive* measures are clearly indicated by such considerations Conditional reactions in man should become *fully* conditional and not fixed as *un*conditional, or conditional of lower orders In other words, instead of 'fixation', we should have means and methods to preserve and foster *semantic flexibility This last is accomplished by acquiring the semantic reactions connected with the consciousness of abstracting* I recommend this last point to the attention of specialists, as it is impossible in the present work to go further into details Flexibility is an important semantic characteristic of healthy youth Fixation is a semantic characteristic of old age With the colloidal background, the imparting of permanent semantic flexibility which every one acquires who becomes conscious of abstracting might prove to be a crucial neuro-physico-chemical colloidal factor of, at present, unrealized power The colloidal behaviour of our 'bodies' is dependent on electromagnetic , manifestations, which, in their turn, are connected with 'mental' states of every description If the colloidal ageing, which brings on old age, 'physical' and 'mental' symptoms, and, ultimately, death, is connected with such 'mental' fixity, we may expect some rather startling results if we impart a permanent semantic flexibility The 'ageing' involves electrical changes in the colloidal background, which must be connected with the older semantic states The new fluid semantic states should have different electrical influences, which, in their turn, would bring about a difference in the colloidal behaviour on which our 'physical' states depend

From a $\bar{A}$ point of view, a new era of human development seems possible, in which, by mere structural analysis and a linguistic revision, we will discover disregarded semantic mechanisms operating in all of us, which can be easily influenced and controlled, and we will discover, also, that at least a great deal of prevention can be accomplished

It seems, also, that we will discover more about the dependence of 'human nature' on the structure of our languages, doctrines, institutions , and will conclude that for adjustment, stability., we must adjust these man-made and man-invented semantic and other conditions in conformity with that newly discovered 'human nature' This, of course, would require a thorough scientific 1933, physico-mathematical, epistemological, structural, and semantic revision of all existing human interests, inclinations, institutions , to be made by those specialized in a 'science of man' If such a revision is produced soon enough, it will, perhaps, help to adjust peacefully the standards of evaluation and prevent the repetition of bloody protests of unenlightened blind forces against *equally blind* forces of existing powers and reactions

The forces of life, humanity, and time-binding are at odds, in modern slang, a 'show-down' is imminent, it *will happen,* and no one can prevent it To a $\bar{A}$ understanding the only problem of importance is whether this 'show-down' will be scientific, enlightened, orderly, and peaceful, with minimum suffering; or whether it will take a blind, chaotic, silly, bloody, and wasteful turn with maximum suffering

The problems of structure, language, and 'consciousness of abstracting' play a crucial semantic role To be modern, one must accept modern metaphysics and a structurally revised modern language As yet, these semantic problems have been *completely* disregarded as far as general education is concerned This is probably due to the fact that in an infantile and commercial civilization we encourage engineering and applied sciences, medicine, biology , to increase private profits , and preserve or increase the ranks of buyers But we do not encourage to an equal extent branches of science like mathematics, mathematical philosophy, linguistic, structural, and semantic researches , which would not directly increase profits or the numbers of customers, but which would, nevertheless, discover structural means for more happiness for all

Accidentally—and this is recommended to the attention of economists—the classical law of 'supply and demand' is structurally and semantically an *animalistic law,* which in an adult human civilization must be reformulated In fact, an adult human civilization cannot be produced at all if we preserve such fundamental animalistic 'laws' In the animal world the numbers of individuals cannot increase beyond what the given conditions allow The animals do not produce artificially.

Not so with our human world We produce artificially because we are time-binders, and all of us stand on the shoulders of others and on the labours of the dead We can over-populate this globe as we have done Our numbers are not controlled by unaided nature, but can be increased considerably In the animal world the numbers are regulated by the supply of food , and not by conditions imposed by the animals on that food supply The animal law of 'supply and demand' is strict. In a human class of life, which does produce artificially, production should satisfy the wants of all. or their number should be controlled until the wants can be filled The application of animalistic laws to ourselves makes conditions very complicated, and detrimental to most, if not all of us It is also easily understood why it should be so Ignorant and *A* handling of powerful symbols has proved to be dangerous when we do not realize the overwhelming semantic role and the importance of symbols in a symbolic class of life

Another interesting application of the consciousness of abstracting is given in our attitude toward money, bonds, titles to property, Money represents a symbol for all human time-binding characteristics Animals do not have it No doubt bees produce honey, but these products of the bees do not constitute wealth until man puts his hands on them Money is not edible or habitable It is worthless if the other fellow refuses to take it The *m o reality* behind the symbol is found in *human agreement* The *value* behind the symbol is *doctrinal* Fido does not discriminate between the different orders of abstractions. If we copy him, we worship the symbol alone 'In gold we trust' becomes the motto, with all its identifications and destructive consequences Smith should not identify the *m o* reality behind the symbol with the symbol It is amusing, when not tragic, to see how the so-called 'practical man' deals mostly with fictitious values, for which he is willing to live and die When he has the upper hand and ignorantly plays with symbols, disregarding the *m o* realities behind them, of course, he drives civilization to disasters History is full of examples of this

We see the utter folly of racing to accumulate symbols, worthless in themselves, while destroying the 'mental' and 'moral' values which are behind the symbols For it is useless to 'own' a semantically unbalanced world Such ownership is a fiction, no matter how stable it may look on paper Commercialism, as a creed, is a folly of this type Some day even economists, bankers, and merchants will understand that such 'impractical' works as this present one on structure, *s r* , lead to the revision of standards of evaluation and are directly helping the stabilization of an economic system. Meanwhile, in their ignorance, they do their best to keep the economic system unscientific, and, therefore, unbalanced History shows clearly how the rulers have generally made life unbearable for the rest of mankind, and what bloody results have followed Since the World War certain conditions are becoming increasingly more difficult, and the infantile and animalistic systems drive us fatalistically toward further catastrophes Whether these disasters will occur, the unknown future shall decide, but out of this unknown, one fact remains a certainty, namely, that this will depend on whether or not science can take hold of human affairs, I hope it can, but the blind forces of identification are so strong and powerful that perhaps such hopes are premature Perhaps a new race can accomplish it after this one is extinct, with the exception of a few remnants in museums

The problems of determinism and indeterminism are not purely 'academic' but influence, to a large extent, our theories and behaviour, and so are fundamental for adjustment. Historically, science has utilized determinism of the two-, and three-valued variety, which has lately, in the case of the newer quantum mechanics, proved insufficient The lack of the formulation of ∞-valued semantics, necessary for ∞-valued determinism, seemed to indicate that even science tends to drift toward indeterminism, a tendency which was rather baffling and disturbing to many scientists

Different 'ethics' and 'morals' have fought determinism throughout all our past on the ground that in a deterministic world all 'morals' and 'ethics' would be impossible If a man is compelled to do something, then, we are told, he is not responsible They state that the result would be undesirable licence, forgetting that determinism implies quite the opposite of licence

We have already become acquainted with infantile self-love and self-importance These infantile characteristics have not only shaped our semantic attitudes, but also our 'scientific' theories Smith and this little earth were in many ways postulated as the centre of the universe Scientific discoveries showed that such statements did not cover the facts at hand, and Smith was displaced from this primitive and infantile self-centred position The Polish astronomer Copernicus was the first to give this rude shock The little earth was no longer *the* 'centre of *the* universe' Next came Darwin with another shock to such infantile pride Smith was no longer a 'special creation', but belonged to the general series of living forms, none of which were 'special creations' Finally, Freud developed the notion that even in semantic processes, determinism prevails All our actions, psycho-logical and semantic states , have very definite conscious and unconscious psychophysiological 'causes' which activate us

An infantile society had difficulties in abandoning their pleasing delusions, and these three men were duly persecuted, criticized, and bitterly attacked and hated by many

The present situation may appear baffling because science discovers facts which would seem to lead to an 'undesirable' indeterminism in science, and to a determinism in 'mental' processes The reader, by now, I hope, realizes that both 'undesirable' results are only undesirable because of identification and the confusion of orders of abstractions, which resulted in the ascribing of *undue generality and uniqueness* to the *A* two and three-valued 'logic' But once we realize that in a $\bar{A}$, ∞-valued, more general system, the two-, and three-valued aspects are only particular instances, which apply to some instances but not to others, all our difficulties vanish From a $\bar{A}$ structural point of view we also understand that ∞-valued determinism becomes a necessity of our *s r* in the search and comparison of structures

The result seems to be that the problem of determinism or indeterminism is not primarily a problem of the outside world, but simply one of our *s r* and ignorance versus 'knowledge' Abandoning elementalism and identification, we stop arguing '*is* the *world* deterministic or not' , but, by analysis, we find which semantics better fit, structurally, the facts and our abstracting capacities The results we reach are not entirely new, but the semantic conflict is eliminated

Science employs determinism because of the structure and function of our nervous system We cannot do otherwise than preserve ∞-valued determinism and step by step supply the missing links in our structural adjustments of language to the structure of empirical data

Let us again repeat that the older problems of 'determinism' *in general* were the results of elementalism and identification and of a complete misunderstanding of the role of structure Once these undesirable afflictions are eliminated, the artificial problems which they create are also eliminated Structural considerations show clearly that determinism is a neurological necessity If empirical facts lead to linguistic indeterminism, it is an unmistakable sign that the language used is not similar in structure to the structure of the world around us, and that we should simply produce a language of different structure *Such determinism is a vital condition in the search for structure, and cannot be abandoned*

Shall we, then, preserve the deterministic attitude in our 'mental' processes? Are the objections on 'moral' and 'ethical' grounds serious enough to induce us to reinstate in our semantic attitudes the older structurally misleading 'indeterminism'?

Let us, first, recall the facts In our old *el* and infantile attitudes with identification we analysed a child or an adult in *isolation* Determinism was applied to such a fictitious *non-existent* individual, and the old objectified and *el* speculations followed If any one is inclined to challenge the above statement, let him perform an experiment and immediately after birth isolate a child 'completely' He will find that this cannot be done with a human baby without destroying the child Therefore, the old speculations deal with structurally *fictitious* conditions The facts are that a baby is, from the first, subjected to a treatment based on the semantics, structure of language, doctrines, understanding, knowledge, attitudes, metaphysics , of his parents or their substitutes, which *shape his semantic reactions*

If we abandon the problem of the two-valued 'determinism' in connection with such a fictitious, isolated individual, and apply ∞-valued determinism to an actual, non-isolated individual, we see at once that the whole situation is different If parents and society accept ∞-valued determinism, they realize their own *responsibilities* toward the individual, and understand that the actions of parents, society.., are, to a large extent, responsible for the future development of the child on quite deterministic psychophysiological grounds If an individual behaves in a way detrimental to others and to himself, and an enlightened society decides to do this or that with him, that is a different proposition. The main point is that, if we were to accept an indeterministic attitude, a great deal of harm would be done by parents, teachers, preachers, and society in general; harm which could be prevented This is, to a large extent, unrealized, and in the old way no one was supposed to be *responsible* except the poor victim of 'free will'. Under such *A* conditions, we sponsor bitterness, cruelty , under the labels of 'sin', 'justice', 'revenge', 'punishment', or whatever it may be On deterministic grounds, when society and educators realize fully their own responsibilities, we should blame the individual less, and should more and more investigate structure, language, our systems, metaphysics, education, conditions of living, Instead of a holy frenzy for 'justice', 'punishment', 'revenge' , we would try to improve conditions of life and education, so that a newborn individual would not be handicapped from the day of his birth

Since the organism operates as-a-whole and *no one* is free from higher order abstractions and structural assumptions, we see that the *keeping of savage-made metaphysics* must involve us individually and collectively in an arrested or regressive development From the organism-as-a-whole point of view structural ignorance must result in some semantic defectiveness

The objection that there are cases of great 'mental' brilliancy accompanied by very vicious tendencies is easily answered by the fact that the problem is formulated in an *el* way 'Mental' brilliancy does not tell the whole story of the organism-as-a-whole One may be 'mentally' brilliant, yet infantile or a 'moral imbecile' In life, we deal with the whole non-isolated individual, who may be pathological in a great many ways If it is objected that science is so complicated that it would be impossible to impart such knowledge to the masses, the answer is that, as this enquiry shows, science involves some structural metaphysics and semantic components which, once discovered, are childishly simple, and can be given in elementary education.

Science represents the highest structural abstractions that have been produced at each date. It is a supreme abstraction from all the experiences of countless individuals and generations. Since the lower centres produce the raw material from which the higher abstractions are made, and these higher abstractions again influence the working of the lower centres, obviously *some means can be devised to put back into the nervous circuits the beneficial effects of those highest abstractions.*

The above statement may appear visionary, and many are likely to say, 'It cannot be done'. Now, the main contention of the present theory, verified empirically, is that it *can* be done in an extremely simple way, provided we study the neglected *non-el* aspects of mathematics and science, namely, their structural and semantic aspects. Such study has helped us to discover in a $\bar{A}$-system the means for affecting lower centres by the products of the higher centres of the best men we had. We have already discovered that all advances in science and mathematics supply us with an unbelievable amount of purely psycho-logical and semantic data of extreme simplicity, which, without any technicalities, can be imparted to the masses in elementary structural education. Such education allows us to give very simply to children the 'cultural results', or to impart the *s.r.*, which are the aim of university training, in a relatively short period and without any technicalities. These benefits, under an *A* education, are too rarely acquired even by university graduates, and impossible to impart to the masses, who are left helpless with archaic, delusional structural assumptions.

From one point of view the $\bar{A}$ issues are childishly simple and obvious, but from another, because of the power of old established habits and *s.r.* are quite difficult for the grown-ups to apply. It seems evident that an infant must be under the influence of the standards of evaluation of those who take care of him, automatically connected with the structure of the language he is taught. Under such unavoidable conditions, it is obvious that to give the full benefit of a $\bar{A}$-system in the training of children, parents and teachers should, themselves, have entirely absorbed these new standards.

A $\bar{A}$ civilization will require a unification of all existing human disciplines on the base of exact sciences. This unification will require all scientists, mathematicians, physicists, and psychiatrists included, to become acquainted and *fully* to practice $\bar{A}$ standards of evaluation. A $\bar{A}$ revision would have an international and interracial application, requiring a very thorough revision of all doctrines, a better acquaintance of specialists in one field with the accomplishments in other fields and an *up-to-date epistemology*. If we try to disregard epistemology consciously, we delude ourselves, as we cannot eliminate *some* epistemology as a foundation for our methods of evaluation, and, therefore, unconsciously retain some primitive epistemology which through inappropriate standards of evaluation introduces semantic blockages.

Mach said long ago: 'Not every physicist is an epistemologist, and not every one must or can be. Special investigation claims a whole man, so also does the theory of knowledge.' The influence of Mach on modern science is well known; men such as the late Jacques Loeb, Einstein, the younger quantum pioneers . . ., were deeply influenced by the writings of Mach, because Mach was a deep student of epistemology. But in a $\bar{A}$ society his statement must be slightly reworded; namely: 'Not every individual knows or realizes the importance of, or seemingly consciously cares for, epistemology, yet every one unconsciously has one and acts and lives by it. Each individual has his own special problems, the solution of which always claims the whole man, and no man is complete, unless he consciously realizes the permanent presence in his life of some standards of evaluation. Every one has thus *some* epistemology. There is no way of parting with it,—nor with air, nor with water,—and live. The only problem is whether his standards of evaluation are polluted with primitive remains of bygone ages, in a variety of ways; or sanified by science and modern epistemology.'

The present work shows how any system involves a special epistemology which we accept unconsciously, once we accept the system. To evaluate a system is practically equivalent to formulating its epistemology. This is strictly connected with linguistic and structural investigations.

To centralize and co-ordinate the $\bar{A}$ efforts, an *International Non-aristotelian Library* has been originated, which field embraces, ultimately, all known doctrines and human interests, the first publication being the present handbook. To facilitate the application of $\bar{A}$ disciplines and to stimulate further researches, an *International Non-aristotelian Society* has been incorporated with headquarters in New York City and branches to be established in all cities of the world which have educational institutions. The main aims of the Society are scientific and educational for the study, by means of papers and lectures followed by discussions, of the $\bar{A}$ aspects necessary for a revision and, therefore, a co-ordination of all existing sciences and concerns of man. As the aspects of science which are of interest to the Society would be *structural* and *semantic*, from the point of view of a *general theory of values*, the lectures would be of a general non-technical character on the level of intelligent laymen

Science would not be 'popularized' but analysed from a fundamental $\bar{A}$ epistemological point of view, compelling the speakers and authors of papers to analyse the deeper *non-cl*, structural, and semantic foundations of experience, as well as of theories The layman would benefit because he would be given a structural education readily understood, without being led astray by the older A 'popularization' Later, if economically feasible, it is intended to issue a monthly *International Non-aristotelian Review*, and also to organize *International Non-aristotelian Congresses*.

The A-system was the result of the *s r* of the white race of more than two thousand years ago, it built up the doctrines, institutions , appropriate to this system In those days, knowledge was very scanty, the interconnection of different peoples, vague, the means of communications, very primitive, It may be considered that science, and particularly mathematics, began a $\bar{A}$ revolution by explicitly searching for structure and adjusting the structure of the scientific languages, which we usually call 'terminology', 'theories', Modern conditions of life are, to a large extent, affected by $\bar{A}$ science but exploited by the thoroughly A doctrines of the commercialists, militarists, politicians, priests, lawyers , which results in a bewildering chaos, resulting in needless, great, and imposed suffering for the great masses of mankind, as exemplified by such cataclysms as wars, revolutions, unemployment, different economic crises,

$\bar{A}$ disciplines, or science *as such*, are thoroughly beneficial to mankind at large, but an A exploitation and use of these $\bar{A}$ products are, and must be, a source of endless sufferings to the enormous majority of mankind, leading automatically to every kind of break-down It is impossible to give a fuller analysis of this complex interrelation, as this would require a separate volume, I shall, therefore, tabulate only a few overlapping suggestions

Non-aristotelian, Scientific, Adult Standards of Evaluation	Aristotelian, Infantile Standards of Evaluation of Commercialism, Militarism ,
Biological Sciences	
Particularly medicine has discovered means of how to keep or restore health, how to eliminate suffering, how to save and to prolong life , but medicine also gives means whereby we may prevent the overpopulation of this globe and so teaches us how to avoid great sufferings through overcrowding, which results in bitter struggle for food, shelter, employment , in turn, leading to wars, revolutions, unemployment,.	Commercialized medicine unavailable to great masses of poor people Commercialism, militarism, infantile dreams of 'world empires' sponsor unintelligent breeding in vast numbers which runs up the prices of land and houses, lowers the price of labour, and supplies cannon-fodder, The intelligent control of numbers of population is prevented by jailing and otherwise persecuting scientific workers, which persecution affects only the poor and uneducated

NON-ARISTOTELIAN, SCIENTIFIC, ADULT STANDARDS OF EVALUATION	ARISTOTELIAN, INFANTILE STANDARDS OF EVALUATION OF COMMERCIALISM, MILITARISM ,
Chemistry.	
Antiseptics, fundamental for medicine and the control of the increase or decrease in the population	Sponsoring of overpopulation by forcibly withholding knowledge from the masses
Drugs, fundamental for medicine	Use of drugs in war to allow a soldier to suffer twice Commercialized drug pedlars
High explosives, necessary for agriculture, mining , .	High explosives used for killing in the struggle for ultimately futile 'world empires'
Production of food products Alcohol, wine, beer,	Food destroyed to keep up prices Commercialization of drinking The saloon, crimes, 'prohibitions' the financing of gangsters, corruption of government and justice , .
Poisonous gas, necessary for the elimination of insects	Poisonous gas in wars
Linguistics	
Law, as expressing some standards of evaluation	Interpretations by commercialized lawyers to evade law, and the influencing of the wording of the law so as to make evasion possible Lobbyists
Newspapers, magazines , giving most powerful educational means	Commercialized newspapers , controlled by profits and advertisements, supplying stultifying, controlled material, stimulating the morbid potentialities of the mob, to increase circulation ,
Other public prints, giving necessary or useful informations	Commercialized advertisements Schizophrenic play on words, the promoting of infantilism ,
Religions represent primitive structural rationalizations, or primitive 'science', intended, also, as guides for conduct and adjustment, under the structural assumptions of the epoch of their primitive origin	Commercialized religions Religions having outlived their usefulness, often become priestcraft as a source of income and control The sponsoring of primitive and delusional standards of evaluation often for private gains The imposition of primitive standards of evaluation involves pathological factors
Physics and Related Sciences	
Aeroplanes, as means of communication and scientific exploring	Militarized aeroplanes as means of destruction and murder
Automobiles, as means of transportation and pleasure	Militarized automobiles, as means of destruction and suffering

Non-aristotelian, Scientific, Adult Standards of Evaluation	Aristotelian, Infantile Standards of Evaluation of Commercialism, Militarism,
Physics and Related Sciences (continued)	
Machinery and tools, as means to eliminate avoidable efforts and to benefit fully through natural resources, to give greater comfort and sanitary conditions, to allow greater leisure for cultural pursuits,	Commercialized machines, as means for larger individual profits and suffering for the increasing numbers of unemployed and starving masses Mass production of guns, ammunitions , for mass extermination and destruction, and larger profits for the manufacturers and investors
Moving pictures, as powerful educational means	Commercialized moving pictures, stimulating and satisfying the erotic and morbid crowds, and for private propaganda
Radio, as powerful means of communication and education	Commercialized radio, advertisements, private propaganda, often stimulating morbid inclinations of the mob
Railroads, as means of public transportation	Commercialized railroads, as means for private gains and the control by a few of large areas and many people
Tractors, as means of transportation and sources of power in agriculture	Tractors, as tanks and means of destruction and murder in wars
Public Servants	
Judges, as guardians of some standards of evaluation	Commercialized politicians, as judges, corruption, lack of justice,
Lawyers, as assistants in the administration of justice	Commercialization and corruption of the legal profession Means to evade or pervert justice
Police, as an executive, regulating, safety-force	Commercialization of police by politicians, corruption, combinations with the underworld,
Sports, as means for preserving and building up health, co-ordinated orientation, fair play, and in a lesser degree as recreation	Commercialization of sports, elimination of benefits Gambling Lowering of educational standards,

From a $\bar{A}$ point of view, which eliminates primitive *s r*, it becomes obvious that mankind represents an interdependent time-binding class of life, and any group of people who possess physical means for destruction and still preserve infantile standards of evaluation become a menace to the culture of the whole race Under such conditions we must have agencies for an international exchange and evaluation of our standards, as well as methods which would help us in adjusting these standards

At present, we must admit that with the modern, rapid, and international advancement of science we have fairly well-established international $\bar{A}$ standards of scientific values International Scientific Congresses are not only necessary for the advancement of science, but they also explicitly prove science to be entirely international

The latest, most important $\bar{A}$ institution is found in the League of Nations, which embraces practically the whole civilized world, with the exception of a very few nations who display infantile and A aloofness, using different self-deceptive excuses, and, to a large extent, handicapping the power and usefulness of the League

As we have learned lately, not only human achievements, but also human disasters, are mostly interrelated and international, and are becoming more so every year Obviously, with A narrowness, selfishness, shortsightedness, infantilism, commercialism, militarism, nationalism , rampant, mankind, to prevent further major A disasters, would have to produce a special international body which would co-ordinate various structural achievements, strivings , formulate and inform the great masses of mankind of the modern scientific $\bar{A}$, adult standards of evaluation

At present, we already have the necessary agencies, but, as yet, they are inefficient and non-co-ordinated These are to be found in International Scientific Congresses, and the League of Nations The weak spots of these organizations are found in the fact that the Scientific Congresses are too cumbersome, expensive, non-co-ordinated and only periodic The League of Nations, although a $\bar{A}$ body in structure, is mostly made up of men who do not know any other standards of evaluation than A, and so they often lack the means to present a scientific, or $\bar{A}$ argument, and usually do not realize the tremendous power they would have in a $\bar{A}$-system In human affairs, for instance, there cannot be a neutral and innocent absentee One such absentee with guns and battleships becomes a powerful blocking and so, ultimately, disrupting factor for the rest of civilization Such an absentee is not, then, guilty by omission, but, from a $\bar{A}$ point of view, becomes guilty by commission The League, when definitely and fully allied with international science, will some day have the pluck to make such a declaration and act accordingly A consciously $\bar{A}$ League of Nations will not limit itself to the thankless and very often useless task of adjusting inevitable clashes of A standards of evaluation, but will, with the full co-operation of scientists, undertake the much more important, constructive, and unique duty of a guardian and leader of human culture Such a League would become a scientific, professional, international, co-ordinating, cultural, time-binding advisory organization for all nations National A governments, instead of only *instructing* their A representatives, would first *consult* with the new $\bar{A}$ *specialists*

Many politicians and their followers often become all but hysterical at the mention of the League of Nations, which, in some mysterious way, they associate with a 'super-state' or 'control',. Let me say, at once, that a symbolic, or human class of life, is very largely controlled by ignorant, hidden, often pathological , factors *beyond public control,* of which the majority are entirely innocent In the human symbolic class of life no one is entirely free, but all our lives are entangled in an interdependence of human relations The dependence on those powers which are now hidden. and *beyond public control,* constitutes a grave danger to all Not so with a scientific, enlightened public opinion with adult standards of evaluation as formulated by a future co-ordination of science and the League of Nations Such great majority-opinions will remain opinions, or statements of standards of evaluation, which any one member of the League may accept or reject, but, then, it would be necessary for him to state publicly his standards of evaluation and to decide consciously to act with or against, or to enlighten further the opinion of the human race There is, of course, no question of 'super-states' or 'control' except the unified demand for a conscious and explicit stand on any important subject by any nation Public opinion will do the rest, once it is convoked to act

I am not a pacifist in the accepted sense In an animalistic, infantile, or A society, this would be not only impossible but downright silly. Quite the contrary, I am disgusted with the infantile standards under which wars are conducted Thus, our rulers and war lords, sponsored by commercialism, like little boys make a sort of game out of wars, and thus help to preserve them as an institution In a consistent society, wars should be as ruthless as possible *to all* If any one wants a war, he should consistently take all the consequences But this would not suit our infantile rulers, they know that when little boys play at war, and one group becomes too rough, the other group refuses to play, and so the play at war ends [3] All these perverted 'humanitarianisms' only sponsor wars because, in an unlimited modern warfare, the people would soon come to their senses and would refuse to suffer for the benefit of the very few So I am far from being an A pacifist

But why our destinies should be dependent on the accidental and primitive structure of the language we use , is beyond my comprehension I grant that if we accept such and such postulates, two-valued, *el,* structurally false-to-fact A 'logics', 'psychologies'., all the old, too famillar consequences follow, which we, in our ignorance, have forced upon human life.

But if we put all systems and all 'logics' . on new $\bar{A}$ foundations, which are structurally closer to the facts of life (1933), all the older conclusions may even be reversed. The problem now before mankind is whether or not the new $\bar{A}$-system is more similar in structure to the world and our nervous system than the old On the answer to this question, the future of civilization depends

From the present point of view, we should establish with the League of Nations a permanent $\bar{A}$ or scientific department, composed of a few of the best scholars from all countries who would keep in touch not only with the developments of their specialties, but would also co-ordinate them on general structural and epistemological foundations This department would be the international authority on modern revised and co-ordinated standards of evaluation which would be published in special proceedings The present discrepancy and lack of co-ordination between different branches of knowledge becomes genuinely alarming and detrimental to mankind, because in 1933 it is humanly impossible for a single individual to attempt such a co-ordination Members of this group would be selected by the universities of each country. In their researches, joint studies, and results, mankind at large would find the most reliable scientific and $\bar{A}$ opinions produced at each date, and would have some definite and conscious standards of evaluation by which to orient themselves

The modern 'voting' has some benefits in local affairs, but when its very limited validity is not understood, it becomes a serious danger to mankind Thus, when we are ill, or when we want a bridge built, we ask specialists for their scientific co-operation, we would hardly depend on ignorant voters. Similarly, in a $\bar{A}$ scientific civilization, the major problems of mankind would be analysed by scientific specialists, recommendations offered to be accepted or rejected, as the case may be; but the ignorant voter would have at his disposal an unbiased, impersonal, and responsible opinion of international scientific specialists to compare with the equivocations of some local ignorant politician

To facilitate such future $\bar{A}$ activities, the International Non-aristotelian Society has been established It is hoped that soon the scientific, educational, 'mental' hygiene., workers will begin to unite on a local and national $\bar{A}$ basis Later, International Congresses will unite the local societies, which ultimately will be embodied as a permanent institution, most probably in the League of Nations

In the process of formulating the above system a curious observation has been forced upon me, namely, that statements which are, for instance, quite legitimate for the English language, even though they probably apply in general to all Indo-european languages, do not apply in a similar degree.

I am intimately acquainted with six languages, two Slavic, two Latin, and two Teutonic, and also with the psycho-logical trends of these groups. I have been led to suspect strongly that the finer differences in the structure of these languages and their use are connected with the semantics of these national groups. An enquiry into this problem, in my opinion, presents great semantic possibilities and might be the foundation for the understanding of international psycho-logical differences. Once formulated, this would lead us to a better mutual understanding, particularly if a $\bar{A}$ semantic revision of these different languages is undertaken. To the best of my knowledge, this field of enquiry is entirely new and very promising.

It must be obvious to the reader that such a vast program is beyond the power of a single man to carry out, and the present author hopes for public interest in this enterprise.

If the $\bar{A}$-system has accomplished nothing more than to draw the attention of mankind to some disregarded problems; if it has done nothing more than point the way, not to panaceas, but to suggestions toward an expedient, constructive, and unified scientific program whereby future disasters may be avoided or lessened—the writer will be satisfied.

BOOK III

ADDITIONAL STRUCTURAL DATA ABOUT LANGUAGES AND THE EMPIRICAL WORLD

The every-day language reeks with philosophies   It shatters at every touch of advancing knowledge   At its heart lies paradox
The language of mathematics, on the contrary, stands and grows in firmness   It gives service to men beyond all other language (25)

ARTHUR F BENTLEY

Nothing is more interesting to the true theorist than a fact which directly contradicts a theory generally accepted up to that time, for this is his particular work (415)

M PLANCK

It is not surprising that our language should be incapable of describing the processes occurring within the atoms, for, as has been remarked, it was invented to describe the experiences of daily life, and these consist only of processes involving exceedingly large numbers of atoms   Furthermore, it is very difficult to modify our language so that it will be able to describe these atomic processes, for words can only describe things of which we can form mental pictures, and this ability, too, is a result of daily experience (215)

W HEISENBERG

PREFATORY REMARKS

In re mathematica ars proponendi quaestionem pluris facienda est quam solvendi (74) GEORG CANTOR

We cannot describe substance, we can only give a name to it Any attempt to do more than give a name leads at once to an attribution of structure But structure can be described to some extent, and when reduced to ultimate terms it appears to resolve itself into a complex of relations A law of nature resolves itself into a constant relation, , of the two world-conditions to which the different classes of observed quantities forming the two sides of the equation are traceable Such a constant relation independent of measure-code is only to be expressed by a tensor equation (148) A S EDDINGTON

We have found reason to believe that this creative action of the mind follows closely the mathematical process of Hamiltonian differentiation of an invariant (148) A S EDDINGTON

The only justification for our concepts and system of concepts is that they serve to represent the complex of our experiences, beyond this they have no legitimacy. I am convinced that the philosophers have had a harmful effect upon the progress of scientific thinking in removing certain fundamental concepts from the domain of empiricism, where they are under our control, to the intangible heights of the *a priori* (152) A EINSTEIN

In writing the following *semantic* survey of a rather wide field of mathematics and physics, I was confronted with a difficult task of selecting source-books. *Any* mathematical treatise involves conscious and many unconscious notions concerning 'infinity', the nature of numbers, mathematics, 'proof', 'rigour' , which underlie the definitions of further fundamental terms, such as 'continuity', 'limits', . It seems that when we discover a universally *constant empirical relation*, such as 'non-identity', and apply it, then all other assumptions have to be revised, from this new point of view, irrespective of what startling consequences may follow

At present, neither the laymen nor the majority of scientists realize that human mathematical behaviour has many aspects which should never be identified Thus, (1) to be somehow aware that 'one and one combine in some way into two', is a notion which is common even among children, 'mentally' deficients, and most primitive peoples (2) The mathematical '$1+1=2$' already represents a very advanced stage (in theory, and in method ,) of development, although in *practice* both of these *s r* may lead to one result It should be noticed that the above (1) represents an individual *s r*, as it is not a general formulation, and (2) represents and involves a generalized *s r* Does that exhaust the problem of '$1+1=2$'? It does not seem to Thus, (3), in the *Principia Mathematica* of Whitehead and Russell which deals with the meanings and foundations of mathematics, written in a special shorthand, abbreviating statements perhaps tenfold, it takes more than 350 large 'shorthand' pages to arrive at the notion of 'number one'

It becomes obvious that we should not identify the manipulation of mathematical symbols with the semantic aspects of mathematics. History and investigations show that both aspects are necessary and important, although of the two, the semantic discoveries are strictly connected with the revolutionary advances in science, and have invariably marked a new period of human development. In Chapter XXXIX, the reader will find a very impressive example of this *general* fact. Thus, what is known as the 'Lorentz transformation', *looks like* the 'Einstein transformation'. When manipulated numerically both give equal numerical results, yet the meanings, and the semantic aspects are different. Although Lorentz produced the 'Lorentz transformation' he did not, and *could not* have produced the revolutionary Einstein theory.

It is well known that when it comes to the manipulation of symbols mathematicians agree, but when it comes to the semantic aspects or meanings, they are *admittedly* hopelessly at variance. In a prevailingly A world we have had no satisfactory theory of 'infinity', or a $\bar{A}$ definition of numbers and mathematics. This necessarily resulted in the fact that the semantic aspects of practically all important mathematical works by different authors often involve *individual semantic presuppositions*, or orientations concerning fundamentals. My presentation intends to be primarily semantic and elementary and is only remotely concerned with the manipulation of symbols. A $\bar{A}$-system which rejects 'identity', differs very widely from A attitudes and introduces distinct $\bar{A}$ requirements. I had, therefore, to select from many works, with their *individual* presuppositions, those which were less in conflict with $\bar{A}$ principles than the others.

A survey of important mathematical treatises shows that although the majority of modern mathematicians explicitly abjure the 'infinitesimal', yet in some presentations, this notion persists. In my presentation I reject the 'infinitesimal' explicitly and implicitly, although the formulae are not altered. 'Modern' calculus is based *officially* on the theory of limits, but as the theory of limits involves the unclarified theory of 'infinity', nothing would be gained semantically and for my purpose, had I stressed these formal possibilities of the calculus. Quite the opposite, if I had done so, I would have failed to stress the most fundamental $\bar{A}$ principle and task of establishing the *similarity of structure between languages and the un-speakable levels and happenings as the first and crucial consequence of the elimination of identity*. For these weighty reasons, in my presentation, I followed some older textbooks, particularly Osgood's, which, from a $\bar{A}$ point of view, are sounder than the newer, largely A rationalizations and apologetics.

However, it should be realized that practically all outstanding and creative mathematicians have had, and still have, $\bar{A}$ attitudes, yet, these private beneficial attitudes, not being formulated in a $\bar{A}$-system, could not become *conscious*, simple, workable, public, and educational assets. We can be simple about this point. With the elimination of identity, structure becomes the *only possible* content of 'knowledge'—and structure of the un-speakable levels has to be *discovered*. Discovery depends on the finding of *new*, and therefore *different* characteristics In the *formulation* of the last sentence, we cannot make the 'training in discovery' an *educational discipline* The opposite is true in a $\bar{A}$-system, based on non-identity, as *we can train simply and effectively in non-identity*, which ultimately leads to differentiation, and so discovery

Because of the elementary, and purely semantic character of the following pages, I have often restrained myself from giving technical, supposedly 'rigorous', and often A rationalizations, which we occasionally call definitions In a semantic and $\bar{A}$ treatment, at this pioneering stage, stressing old definitions would be seriously confusing, and I wished to avoid such witty wittgensteinian 'definitions' as 'A point in space is a place for an argument' In a number of instances, and for my purpose, I often avoided unsatisfactory formal definitions, preferring to depend upon the ordinary meanings of words

For the reader who wishes to acquaint himself with an elementary theory of limits and corresponding sets of definitions, I would suggest the book of the late Professor J. G Leathem, *Elements of the Mathematical Theory of Limits* (London and Chicago, 1925). This theory is based on Pascal's *Calcolo Infinitesimale*, Borel's *Théorie des fonctions*, and Godefroy's *Théorie des séries* Leathem's book has been printed under the supervision of Professor H F Baker, F.R S , of the University of Cambridge, and Professor E T Whittaker, F R S , of Edinburgh I give these names for professional mathematicians, to indicate the semantic trend which underlies this particular treatment of limits and which does not greatly conflict with a $\bar{A}$ outlook This outlook may be summarized in part, in the words of Borel somewhat as follows 'To the evolution of physics should correspond an evolution of mathematics, which, without abandoning the classical and well-tried theories, should develop however, with the results of experiments in view' This statement implies vaguely the 'similarity of structure' , and so requires as a *modus operandi* the rejection of identity

There seems to be little doubt that a complete and radical revision of the semantic aspects of human mathematical behaviour is pending Such a revision appears to be laborious and difficult, and should be undertaken from the point of view of the theory of the unique and specific relations, called numbers I doubt if a single man could accomplish this revision Such an undertaking will probably be the result of group activities, and may, in the beginning, be *unified* by the formulation of one fundamental $\bar{A}$ principle of non-identity, the disregard of which, from science down to 'mental' ills, can be found at the bottom of practically all avoidable human difficulties

The problems are very complicated and extremely difficult, and need to be treated from many angles At present, we have many scientific societies, grouped by their specialties, but we do not have a scientific society composed of *many different specialists* whose work could be unified by some *common and general principle* There can be no doubt that the principle of 'identity', or 'absolute sameness in *all* aspects', is invariably false to facts The main problem is *to trace* this semantic disturbance of improper evaluation in all fields of science and life, and this requires a new *co-ordinating scientific body* of many specialists. with branches in all universities Each group would meet, say monthly, to discuss their problems, and give mutual technical assistance in tracing *this first general semantic disturbance.* Such meetings would stimulate enormously scientific productivity. In fact, without such a co-ordinating body, the present enormous technical developments in each branch of science preclude the revision of general principles, on which, in the last analysis, all other of our activities greatly depend The first task then, is to find a co-ordinating principle, and present it to the scientific world

Psychiatry, and common experience, teach us, that in heavy cases of dementia praecox we find the most highly developed 'identification' $\bar{A}$ considerations suggest that *any* identification, no matter how slight, represents a dementia praecox factor in our semantic reactions The rest is only a question of degrees of this maladjustment From this point of view, we will find dementia praecox factors even in mathematics In physics, only since Einstein has this factor of un-sanity been eliminated, and this elimination has already produced an ever-growing crop of 'geniuses', which merely means, that some inhibitions of mis-evaluation have been eliminated from these younger men, and that they are humanly more 'normal' than the others

In mathematics, from a $\bar{A}$ point of view, we must first of all *not identify* different aspects of our mathematical behaviour, nor try to cover up these identifications of endless aspects by the one very old term 'mathematics'. This word, 'mathematics', in its *accepted sense* covers a non-existing fiction. What does exist, and the only thing we actually deal with, is *human mathematical behaviour*, human *s r*, and the *results* of human *mathematical behaviour* and *s r* A treatise, say, on a new quantum mechanics, has no value to a monkey or a corpse, and only human *mathematical behaviour* and *s r*, have any actual *non-el* existence, and is the *only* thing which actually matters So we see that 'mathematics' covers a non-existent fiction *if elementalistically* separated from human mathematical behaviour and *s r* I use the term 'mathematics' in the *non-el* sense, and attempt to signalize some of the difficulties non-elementalism involves at this transitory stage

From a $\bar{A}$ non-identity, structural, *non-el* point of view, human mathematical behaviour must be treated uniquely as a physico-mathematical discipline, and postulational methods to be used exclusively as a most valuable *checking* method To *base* mathematical behaviour and *s r* on postulational methods exclusively, is to introduce dementia praecox factors into science, which only induces the spread of semantic maladjustment in life

Our main task in producing a $\bar{A}$ revision of mathematical *s r*, is in the elimination of identification from our *s r* about 'infinity' and in the formulation of a $\bar{A}$ definition of numbers in terms of relations This would enable us to rebuild human mathematical *s r* from a theory of numbers point of view, as a *physico-mathematical* discipline The intrinsic, or internal theory of surfaces, and the tensor, or absolute calculus, are methodologically our most secure epistemological guides.

I would suggest that mathematical and scientific readers who are interested in a $\bar{A}$ revision should, at first, in their special fields, sketch in technical papers, presented before the local International Non-aristotelian Societies, *A* pitfalls and $\bar{A}$ problems and outlooks Only after this is done, shall we be able to begin a co-ordination of their findings, and thereby initiate a revised and unified $\bar{A}$ science, mathematics, and perhaps ultimately a saner *scientific civilization.*

The scientific achievements dealt with in Book III, are developing so rapidly, and the technical points of view alter so often, that on a static printed page it is impossible to do them justice The writer has spared no efforts to keep informed of these scientific developments until two weeks before the appearance of this book, yet because these new developments do not represent new and fundamental semantic factors, I deliberately do not include them here In some instances, a given author may seem to change his opinions, but, from a $\bar{A}$ point of view, it sometimes appears that the original notions were more justified, and so I preserved them without change

The following pages are written exclusively from a semantic point of view, an undertaking which is far more difficult than dealing with a restricted technical physico-mathematical problem, because it involves *second order* observations, of the first order observations, of the first order observer, and of the relations between them , . When it came to a final revision of the manuscript, and reading of the proofs, I found that dealing with so many varied fields, languages, and symbolisms at one period, was no small task, and I only hope that I have not over-looked too many errors or misprints

If we must have slogans, a $\bar{A}$ motto readily suggests itself—'Scientists of the world unite' Perhaps this motto may prove more constructive and workable than the familiar *A elementalistic* slogans which have mostly led to the dismembering of human society Protests against any misrule should not be confused with the proclaiming of disrupting *general principles* Let me repeat once more, that the most lowly manual worker is useful *only* because of his human nervous system, which produced all science, and which differentiates him from an animal, and not primarily for his hands alone, otherwise we would breed apes to do the world's work

In the explanations of some geometrical notions, and some parts of the theory of Einstein, I have followed often very closely the *Einstein's Theory of Relativity* by Max Born, which is easily the best elementary exposition I have read, and also the books of Eddington In the quantum field I have followed mostly the books by Biggs, Birtwistle, Bôcher, Haas, and Sommerfeld, and I wish to acknowledge my indebtedness to the above authors

I am also under heavy obligations to Professors E T Bell, P W Bridgman, B F. Dostal, R J Kennedy, and G Y Rainich, who were so kind as to read the MS and/or proofs, and whose criticism and suggestions were invaluable to me However, I assume entire responsibility for the following pages, especially since I have not always followed the suggestions made.

PART VIII

ON THE STRUCTURE OF MATHEMATICS

Being myself a remarkably stupid fellow, I have had to unteach myself the difficulties, and now beg to present to my fellow fools the parts that are not hard Master these thoroughly, and the rest will follow What one fool can do, another can (510) SILVANUS P THOMPSON

Besides the theory of surfaces is the model on which all the higher theories are built and must be built, and it is well to master it completely before attempting generalizations (425) G Y RAINICH

To find such relations Einstein has applied a mathematical method of great power—the calculus of tensors—with extraordinary success The calculus threshes out the laws of nature, separating the observer's eccentricities from what is independent of him, with the superb efficiency of a modern harvester (21) E T BELL

CHAPTER XXXII

ON THE SEMANTICS OF THE DIFFERENTIAL CALCULUS

> **The principle of gaining knowledge of the external world from the behaviour of its infinitesimal parts** is the mainspring of the theory of knowledge in infinitesimal physics as in Riemann's geometry, and, indeed, the mainspring of all the eminent work of Riemann, in particular, that dealing with the theory of complex function (547) HERMANN WEYL

> The conception of tensors is possible owing to the circumstance that the transition from one co-ordinate system to another expresses itself as a **linear** transformation in the differentials One here uses the exceedingly fruitful mathematical device of making a problem "linear" by reverting to infinitely small quantities (547) HERMANN WEYL

Section A Introductory

In the first draft of this book written in 1928, the following pages preceded Part VII In a final revision in 1932, it seemed advisable to transfer pages which to laymen look 'mathematical', to the end of the volume, because the majority of even intelligent readers have a sort of 'inferiority complex' about anything 'mathematical'

The patient reader knows by now, I hope, that on neurological grounds, he must for the sake of sanity, be able to translate the dynamic into the static, and the static into the dynamic, and also that he must know at least about the modern structure of 'space', 'time', 'matter' These conditions seem essential for sanity, and so I had no choice but to give the minimum of a structural and semantic outline, and to acquaint the reader with the existence of modern scientific problems and vocabularies It is not my aim to teach the reader mathematics or modern physics I must limit myself to structural and semantic issues, for there are excellent elementary books which will give him the necessary informations

The following pages should in no way intimidate the intelligent reader Elementary structural statements and definitions are given in simple language, followed by illustrations to render their meanings more understandable The pages are less technical than they look, as each example is carried through in the most elementary way in all of its details, so as to make easy reading A real difficulty for some readers may come from the semantic blockage created by the use of apparently strange, and, to them, unknown terms, or from a feeling of fright or abhorrence of anything mathematical, due to deplorably faulty introduction to some branch of mathematics at the hands of some teacher innocent of the broader epistemological aspects of science. I am acquainted with scientists of very considerable mathematical gifts, who have had to overcome this phobia of mathematics. Once the word 'mathematics' was mentioned to them, they became 'mentally' paralysed An 'emotional' fright seized them and it took some months to overcome this undesirable childish *s r* I use the subject of mathematics as an illustration of this difficulty, because I want to contrast the comparative simplicity of mathematical notions with the complexity of human problems and language For when we have understood the *simplest* notions, which happen to be mathematical, then only shall we be able better to understand our human problems, which are in comparison so difficult and so confused

Any reader who has a distaste for mathematics will benefit most if he overcomes his semantic phobia and struggles through these pages, even several times. As a result of so doing he will find it simple although not always easy. It is always semantically useful to overcome one's phobias, it liberates one from unjustified fears, feelings of inferiority, The main point of this whole discussion is to evoke the semantic components of a living Smith, when he habitually uses the method which will be explained herewith This method is so simple and so fundamental that in the form given by a $\bar{A}$-system and further simplified according to the gifts of the teacher, it will some day be introduced into *elementary* schools without technicalities, as a *preventive* semantic method against 'insanity', un-sanity and other nervous and semantic difficulties, and as a foundation for a training in *sanity* and adjustment

Section B On the Differential Calculus

1 *GENERAL CONSIDERATIONS*

As we have already seen, the structural notion of a function is strictly connected with that of the variable The variable on one level does not 'vary', it is a selection by Smith of a definite value from a given set As these processes are going on inside of the skin of Smith he might experience on a different level a feeling of 'change' The method of dealing with such problems is given by the mathematical differential and integral calculus

The beginnings of methods dealing with 'change' are to be found even among the ancients Galileo, Roberval, Napier, Barrow, and others were interested in 'fluxional' methods, before Newton and Leibnitz.[1] The epoch-making discoveries of the last two mathematicians consisted not only in perfecting the knowledge they had and in inventing new methods, but also—and this is perhaps the most important—they formulated a *general* theory of these structural methods and invented a new notation suitable for their purpose The definite abandonment of the old tentative methods of integration in favour of methods in which integration is regarded as the inverse of differentiation was especially the work of Newton Leibnitz' main work was in the field of precise formulation of simple rules for differentiation in special cases and the introduction of a very useful notation

It is not an exaggeration to say that the calculus is one of the most inspiring, creative, structural methods in mathematics There is little doubt that the analysis of the foundations of mathematics, and their revision, was suggested by a study of the methods of the calculus *It is structurally and semantically the 'logic' of sanity* and, as such, can be given ultimately without technicalities by the present $\bar{A}$-system and semantic training, with the aid of the Structural Differential

The application of the differential calculus to geometry produced differential geometry This prepared the way for the notions of Einstein and Minkowski

The whole of modern physics becomes possible through the calculus, and it will probably be correct to say that the achievements of the future also will be dependent on it

The present work is also to a large extent inspired by it, and develops simple non-technical methods by which the psycho-logical structural s r necessitated by the calculus can be given to the masses in elementary education without any technical knowledge of it This statement does not include teachers, who should be acquainted with at least the rudiments of the calculus [2]

It is true that in the beginning we did not suspect that the semantics of the calculus are indispensable in education for *sanity* It is the *only* structural method which can reconcile the as yet irreconcilable higher and lower order abstractions Without such a reconciliation, at our present level of development, sanity is a matter of good luck quite beyond our conscious or educational control

Let us recall the rough definition of a function y is said to be a function of x if, when x is given, y is determined In symbols we write $y=f(x)$ which we read 'y is equal to a function of x' or 'y is equal to f of x' If y is a function of x, or $y=f(x)$, then x is called the independent variable, being the one to which we arbitrarily assign *any* value we choose out of a given set of values The y is called the dependent variable as its value depends on the value we assign to x

A function may have more than one independent variable, in which case we have a function of several variables It happens frequently that to one value of the independent variable there may correspond several values of the dependent variable Then y is said to be a multiple-valued function of x

Roughly speaking, a function is said to be continuous if a small increment in the variable gives rise to a small increment of the function

A theory of functions can be developed without any references to graphs and geometrical notions of co-ordinates and lengths, but in practice (and in this work), it is extremely useful to introduce these geometrical notions, as they help intuition A modern definition of an analytic function is technical and unnecessary for our purpose Suffice it to say that it is connected with derivatives and power series, *which means structure*

Geometry is a very remarkable science It may be treated as pure mathematics, or it may be treated as physics It may therefore be used as a link between the two or as a link between the higher and lower order of abstractions. This fact is of tremendous psycho-logical and semantic importance It is not by pure 'chance' that the most important writers on mathematical philosophy, authors who have generalized their knowledge of mathematics to include human results, were mostly geometers

Indeed, Whitehead, in his *Universal Algebra* (p 32), says, and justly so, that a treatise on universal algebra is also a treatise on certain generalized notions of 'space' 'Space' should be understood as 'fulness', 'fulness of something', a plenum. Naturally coherent speech, like universal algebra, must be coherent speech about *something*. 'Generalized space' becomes generalized plenum, and so it belongs to *two* realms. One is contentless and formal, hence generalized algebra, the other, in that it refers to a generalized plenum, becomes generalized geometry, or generalized physics.

The main importance, perhaps, of geometry is in the fact that it can be interpreted *both ways*. One way appears as pure mathematics, and therefore as the study of sets of numbers representing co-ordinates. The other takes the form of an interpretation, in which its terms imply a connection with the empirical entities of our world. Obviously if speech is not the things spoken about, we must have a special discipline which will translate the coherent language of pure mathematics, which is contentless by definition, into another way of speaking which uses a different vocabulary capable of *both* interpretations.

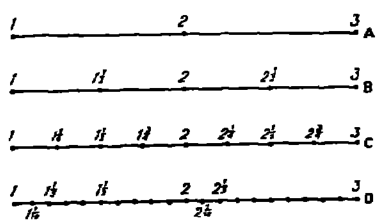
Again, the different orders of abstractions, which our nervous structure produces, are perfectly reflected in the very structure and methods of mathematics. The possibility of the use of the 'intuitions' of lower order abstractions, is extremely useful in pure mathematics. This fact makes geometry also *unique*. It allows us to apply to the development of geometry both orders of abstractions—the 'intuitions', 'feelings', of the lower order of abstractions, and the static, 'quantum' jump methods of pure analysis. This is also why the einsteinian physics becomes four-dimensional geometry, which, because it can be treated on both levels of abstraction, gives tremendously powerful and important psycho-logical means for sanity and nervous co-ordination of the individual. Since Einstein, many far-sighted scientists have said that although they do not know in what respect the Einstein theory will affect our lives, yet they feel that it will have a tremendous influence. I venture to suggest that the bearing of the Einstein theory and its development on the problems of sanity, as explained in this work, is a new and unexpected semantic result of the application of modern science to our lives. As the Einstein theory could have been formulated more than two hundred years ago when the finite velocity of light was discovered, so the present theory is also several hundred years overdue. The only consolation we have left is that it is better late than never.

The scope of this work allows us to go but a little beyond these simple remarks, and permits only a very brief explanation of the most fundamental and elementary beginnings of the calculus. In this presentation I shall appeal very often to intuition (lower order abstractions), as this will help the reader.

The notion of differentiation of a continuous function is the process for measuring the rate of growth, that is to say, the evaluation of the increment of the function as compared with the growth or increment of the variable. We may describe this process as follows. If y is a function of x, it is helpful not to consider x as having one or another special value but as flowing or growing, just as we feel 'time' or follow the ripples made by a stone thrown into a pond.

The function y varies with x, sometimes increasing, sometimes decreasing. We have already defined the variable as *any* value selected from a given range. Let us consider our x as given in the interval between 1 and 5. We are now interested in all values which our x may take between these two values, or, as we say, in this interval. Obviously we can select a few values, or, in other words, take big steps, as, for instance, assigning to x the successive values $x_1=1$, $x_2=2$, $x_3=3$, $x_4=4$, $x_5=5$. In such a case we would have few values and the difference between two successive values would be rather large, for instance, $x_3-x_2=1$. But such large differences are not of much interest to us here. We may, if we choose, select smaller differences, in other words, assign more values to our variable in the given range.

Let us take, for instance, for our x the series of values 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5. Here we see that the difference between two successive values is smaller than 1, it is $\frac{1}{2}$. So we already have nine, instead of the former five, values which we may assign to our x. Thus we have selected smaller steps by which to proceed. Let us select still smaller steps, for instance, $\frac{1}{4}$. Our extensional manifold of values for x in the interval between 1 and 5 would then be 1, $1\frac{1}{4}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2, $2\frac{1}{4}$, $2\frac{1}{2}$, $2\frac{3}{4}$, 3, $3\frac{1}{4}$, $3\frac{1}{2}$, $3\frac{3}{4}$, 4, $4\frac{1}{4}$, $4\frac{1}{2}$, $4\frac{3}{4}$, 5. We see that in the interval between 1 and 5, we have already 17 values which we may assign to our variable, but we have followed the 'growth' of our x by smaller steps, namely, by steps of $\frac{1}{4}$. If we choose to diminish the steps to $\frac{1}{10}$, we would have for our extensional manifold of values 1, 1.1, ..., 1.9, 2, 2.1, ..., 2.9, 3, 3.1, ..., 3.9, 4, 4.1, ..., 4.9, 5; in all, 41 values for x, any two succeeding values differing by $\frac{1}{10}$. If we select still smaller steps—let us say, $\frac{1}{100}$—we have 401 values for x and the difference between two successive values is still smaller, namely, $\frac{1}{100}$. This process may be carried on until we have as *many* numbers between 1 and 5 as we choose, since we may make the difference between successive numbers in the sequence as small as we please. In the limit, between any two numbers, let us say, 1 and 2, or any two fractions, there are infinite numbers of other numbers or fractions. It is obvious that in a given interval, let us say, between 1 and 5, we can have an indefinitely large number of intermediary numbers arranged in an increasing progression, such that the difference between two successive numbers can be made smaller than any assigned value, which is itself greater than zero.



The above may be made clearer by a geometrical illustration. Let us take a segment of a line of definite length, let us say 2 inches. Let us designate the ends by numbers 1 and 3. In figure (A) we divide the segment into 2 equal parts of one inch each, and see that to reach 3 starting with 1 we have to proceed by two large jumps from 1 to 2, and from 2 to 3. In figure (B) we have more steps in the interval and therefore the steps are smaller. In figures (C) and (D) the steps are still smaller and their number greater. If the number of steps is very large, the steps are very small. In the limit, if the numbers of steps become infinite, the length of the steps tends toward zero and the aggregate of such points of division represents (in the rough only) a *continuous* line

It is important that the reader should become thoroughly acquainted with the above simple considerations as they will be very useful in *any* line of endeavour Here we already have learned how, somehow, to translate discontinuous jumps into 'continuous' smooth entities Because of the structure of our nervous system we 'feel' 'continuity', yet we can analyse it into a smaller or larger number of definite jumps, according to our needs The secret of this process lies in assigning an increasing number of jumps, which as they become vanishingly small, or tend to zero, as we say, cease to be felt as jumps and are felt as a 'continuous' motion, or change, or growth or anything of this sort

An excellent example is given by the motion pictures When we look at them we see a very good representation of life with all its continuity of transitions between joy and sorrow If we look at an arrested film we find a definite number of *static* pictures, each differing from the next by a measurable difference or jump, and the joy or sorrow which moved us so in the play of the actors on the *moving* film, becomes a static manifold of static pictures each differing measurably from its neighbour by a slightly more or less accentuated grimace If we increase the number of pictures in a unit of 'time' by using a faster camera and then release this film at the ordinary speed, we get what is called slow motion pictures with which we are all familiar In them we notice a much greater smoothness of movements which in life are jerky, as, for instance, the movements of a running horse They appear smooth and non-jerky, the horse looks as if it were swimming Indeed we do swim no less than fishes, except that our medium, namely, air, is less dense than water, and so our movements have to be more energetic to overcome gravitation The above example is indeed the best analogy in existence of the working of our nervous system and of the difference between orders of abstractions Let us imagine that some one wants to *study* some event as presented by the moving picture camera What would he do? He would first see the picture, in its moving, dynamic form, and later he would arrest the movement and devote himself to the contemplation of the static extensional manifold, or series, of the static pictures of the film It should be noticed that the differences between the static pictures are finite, definite and *measurable*

The power of analysis which we humans possess in our higher order abstractions is due precisely to the fact that they are *static* and so we can take our 'time' to investigate, analyse, The lower order abstractions, such as our *looking* at the moving picture, are shifting and non-permanent and thus evade any serious analysis. On the level of *looking* at the *moving* film, we get a general *feeling* of the events, with a very imperfect memory of what we have seen, coloured to a large extent by our moods and other 'emotional' or organic states We are on the shifting level of lower order abstractions, 'feelings', 'motions', and 'emotions'. The first lower centres do the best they can in a given case but the value of their results is highly doubtful, as they are not especially reliable Now the higher order abstractions are produced by the higher centres, further removed, and not in direct contact with the world around us. With the finite velocity of nerve currents it takes 'time' for impulses to reach these centres, as the cortical pathways offer higher neural resistances than the other pathways.[3] So there has to be a survival mechanism in the production of nervous means for arresting the stream of events and producing *static* pictures of permanent character, which may allow us to investigate, verify, analyse, . It must be noticed that because of this higher neural resistance of higher centres and the static character of the higher abstractions, these abstractions are less distorted by affective moods. For, since the higher abstractions persist, if we care to remember them, and the moods vary, we can contemplate the abstractions under different moods and so come to some *average* outlook on a given problem. It is true that we seldom do this, but we *may* do it, and this is of importance to us.

As one of the aims of the calculus is to study relative rates of change we will consider a series of successive values of our variable which differ by little from each other. If we have $y=f(x)$ we can consider the change in x for a short interval, let us say, from x_0 to x_1, so that we assign to our x two values, $x=x_0$ and $x=x_1$. The corresponding values of our function or y will be $y_0=f(x_0)$ and $y_1=f(x_1)$. In general, small changes in y will be almost proportional to the corresponding changes in x, provided $f(x)$ is 'continuous'.

Denoting the small increment of x by Δx, so that $x_1-x_0=\Delta x$ or $x_1=x_0+\Delta x$, function y receives the increment $y_1-y_0=\Delta y$ or $y_1=y_0+\Delta y$. Since $y_1=f(x_1)$ and $x_1=x_0+\Delta x$ we have

$$y_0+\Delta y=f(x_0+\Delta x),$$

if we subtract from both sides

$$y_0=f(x_0)$$

we would have

$$\Delta y=f(x_0+\Delta x)-f(x_0),$$

dividing both sides by Δx we have

$$\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \tag{1}$$

The above ratio represents the ratio of the increment of the function to the increment of the variable. In the limit when the increment in the variable becomes vanishingly small or when Δx tends toward zero, and our function is continuous, the limit of this ratio gives us the law of change or growth of our function.

The limit which the ratio (1) approaches when Δx approaches 0,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x} \tag{2}$$

is called the derivative of y with respect to x and is denoted by D_xy, which we read 'D_x of y', in symbols,

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=D_xy \tag{3}$$

Let us illustrate this by a simple numerical example. Take the equation $y=x^2$ and assume that $x=100$, whence $y=10{,}000$. Suppose the increment of x, namely, $\Delta x=1/10$. Then $x+\Delta x=100.1$ and $(x+\Delta x)^2=100.1\times 100.1=10020.01$. The last 1 is $1/100$ and only one millionth part of the 10,000, and so, we can neglect it and consider $y+\Delta y=10{,}020$, whence $\Delta y=20$ and $\Delta y/\Delta x=20/0.1=200$. In the general case, if $y=x^2$ and instead of x we take a slightly larger value, $x+\Delta x$, then our function y also becomes slightly larger, thus,

$$y+\Delta y=(x+\Delta x)^2=x^2+2x\Delta x+(\Delta x)^2$$

If we subtract $y=x^2$ from the last expression we have

$$\Delta y=2x\Delta x+(\Delta x)^2, \text{ dividing by } \Delta x, \text{ we have}$$
$$\Delta y/\Delta x=2x+\Delta x$$

In the limit as Δx approaches zero, the value of the above ratio, or the rate of change of our function, would be $2x$, as Δx would disappear. If our $x=100$, the above ratio would be 200, as determined above in the case of the numerical example. Another way of symbolizing the derivative is $D_x y=dy/dx$, but this requires a short explanation.

In Chapter XV we have already discussed the problem of the 'infinitesimal' and we have seen that 'infinitesimal' is a misnomer and that there is no such thing at all. Yet this word is very often uncritically used by mathematicians and is therefore often confusing. By an 'infinitesimal' mathematicians mean a *variable* which approaches zero as a limit. The condition that it should be a variable is essential. It would probably be better to call an 'infinitesimal' an *indefinitely* small quantity or 'indefinitesimal', and that is what the reader should understand when he sees anywhere the word 'infinitesimal' or 'infinitely small quantity'.

These indefinitely small quantities are in general neither equal, nor even of one order. Some by comparison are indefinitely smaller than others, and hence are said to be 'of higher order'. Usually several quantities are considered which approach zero simultaneously. In such a case one of them is chosen as the principal indefinitely small quantity. Let us recall that if we take any number, for example, 1, and divide it by 2 we have $1/2$. If we divide 1 by 4 we have $1/4$ which is smaller than $1/2$, if we divide 1 by 10 we have $1/10$ which is still smaller. If we carry this process on indefinitely, taking larger and larger denominators, the results are fractions of smaller and smaller values. In the limit, as the value of the denominator becomes indefinitely large the value of the fraction approaches zero. This simple consideration will help us in the classification of indefinitely small quantities.

Let us take a as the principal indefinitely small quantity and b another indefinitely small quantity. If the ratio b/a approaches zero with a we say that b is an indefinitely small quantity of higher order with respect to a. In other words, although a approaches zero in the limit yet it is infinitely larger than b and so the ratio b/a also approaches zero.

If the ratio b/a approaches a limit k different from zero as a approaches zero, then b is said to be of the 'same order' as a and $b/a=k+\epsilon$ where ϵ is indefinitely small with respect to a. In such a case $b=a(k+\epsilon)=ka+a\epsilon$, and ka is called the principal part of b. The term $a\epsilon$ is obviously of a higher order than a.

We may say in general that if we have a power of a, for instance a^n, such that the ratio b/a^n approaches a limit different from zero, b is called an 'infinitesimal' (indefinitesimal) of order n with respect to a.

Let us give a numerical illustration. We know that there are 60 minutes in one hour, 24 hours in a day, or that there are 1440 minutes in a day, and by multiplying 1440 by 7, that there are 10,080 minutes in a week. Our forefathers called this 1/10,080 part of a week a 'minute' because of its minuteness. It is obvious that a minute is very small as compared with a week. But if we subdivide a minute into 60 equal parts we have a still smaller quantity, a quantity of second order smallness and so we called it a second. Indeed there are 3600 seconds in one hour, 86,400 seconds in a day and 604,800 seconds in a week. If we decide that for some purpose a minute is as short a period of 'time' as we need to consider, then the second, 1/60 of a minute, is relatively so small that it could be neglected. In a calculation where 1/100 of some unit is the smallest value which needs to be considered, we may define this 1/100 as of first order

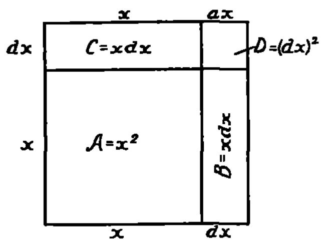


FIG 1-A

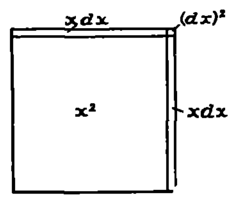


FIG 1-B

smallness. Then 1/100 of 1/100, or 1/10,000, of that unit, which is relatively of second order smallness, is entirely negligible. The fractions whose smallness we are considering here are comparatively large, and we usually deal with much smaller quantities, but the smaller a quantity is, the more negligible the correspondingly smaller quantity of higher order becomes.

Let us consider a geometrical interpretation of the above. If we represent a quantity x by a line segment, and a slightly greater quantity, $x+dx$, by a slightly longer line segment, then the quantities x^2 and $(x+dx)^2=x^2+2xdx+(dx)^2$ may be represented by squares where sides are the line segments which represent the quantities x and $x+dx$ respectively.

If we denote the areas by A, B, C, D, we see that $A=x^2$ and that $A+B+C+D=x^2+2xdx+(dx)^2$. If we select our dx smaller and smaller the areas $B=C=xdx$ diminishing in one dimension only, become also smaller and smaller, but $D=(dx)^2$ is vanishing much more rapidly as it is diminishing in each of two dimensions, whence it is said to be a quantity of second order smallness, which for all purposes at hand may be neglected.

If we take $\qquad y=f(x) \qquad$ and its derivative

$$\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=D_x y$$

Then $\qquad \dfrac{\Delta y}{\Delta x}=D_x y+\epsilon, \qquad$ where ϵ is an indefinitesimal,

and $\qquad \Delta y=D_x y\Delta x+\epsilon\Delta x$

In the above expression $D_x y \Delta x$ represents the principal part and $\epsilon \Delta x$ appears as an indefinitesimal of higher order This principal part is called the differential of y and is denoted by dy If we choose $f(x)=x$ we have $dx=\Delta x$ and so, $dy=D_x y\, dx$.

So we see that the differential of the independent variable x is equal to the increment of that variable This statement is not generally true about the dependent variable, as ϵ does not generally vanish

The derivative is also sometimes denoted as $f'(x)$ or y' and this notation is due to Lagrange, all three notations are used and it is well to be acquainted with them

The derivative of a function $f(x)$ is in general another function of x, let us say $f'(x)$. If $f'(x)$ has a derivative, the new function is the derivative of the derivative or the *second derivative* of $f(x)$ and is denoted by y'' or $f''(x)$. Similarly the third derivative y''' or $f'''(x)$ is defined as the derivative of the second derivative and so on In the other notations we have

$$D_x(D_x y)=D_x^2 y \text{ or } \frac{d}{dx}\left(\frac{dy}{dx}\right)=\frac{d^2y}{dx^2}$$

Having introduced these few definitions it must be emphasized that the main importance of the calculus is in its central idea, namely, the study of a *continuous function* by following its history by *indefinitely small steps*, as the function *changes* when we give indefinitely small increments to the independent variable As was emphasized before, the whole psycho-logics of this process is intimately connected with the activities of the *nervous structure* and also with the structure of science In this work we are not interested in calculations, complications, or analytical niceties. Mathematicians have taken excellent care of all that We need only to know about the structure and *method* which help to translate dynamic into static, and vice versa, to translate 'continuity' on one level, or order of abstraction, into 'steps' on another

To illustrate what has been said and to give the reader the *feel* of the process, let us take for instance a simple equation $y=2x^3-x+5$ where y represents the function of the variable x expressed by a group of symbols to the right of the sign of equality

To determine the relative rate of growth of this function, that is, to differentiate it, we replace x by a slightly larger value, namely, $x+\Delta x$, and see what happens to the expression $2x^3$ becomes $2(x+\Delta x)^3=2x^3+6x^2\Delta x+6x(\Delta x)^2+2(\Delta x)^3$, $-x$ becomes $-x-\Delta x$ and the constant 5 remains unchanged. In symbols, $y+\Delta y=2x^3+6x^2\Delta x+6x(\Delta x)^2+2(\Delta x)^3-x-\Delta x+5$, where Δy represents the increment of the function and Δx represents the increment of the independent variable

Subtracting the original expression $y=2x^3-x+5$ we get the amount by which the function has been increased, namely

$$\Delta y=6x^2\Delta x+6x(\Delta x)^2+2(\Delta x)^3-\Delta x$$

To determine the *relation*, or *ratio*, of Δy, the increment of the function, to Δx the increment of the independent variable which produced Δy, we divide Δy by Δx, and obtain the equation

$$\frac{\Delta y}{\Delta x}=6x^2+6x\Delta x+2(\Delta x)^2-1$$

Then as Δx approaches 0 the terms in the right-hand side of the equation which contain Δx as a factor also approach 0 and replacing the left-hand side by $\frac{dy}{dx}$ we obtain the equation $\frac{dy}{dx}=6x^2-1$ which means, that as Δx approaches 0, the ratio of the increment of the function to the increment of the independent variable approaches $6x^2-1$, true for any value we may arbitrarily assign to x

It should be noticed that in our function the left-hand side represents the 'whole' as composed of interrelated elements which are represented by the right-hand side When instead of x we selected a slightly larger value, namely, $x+\Delta x$, we performed upon this altered value *all* the operations indicated by our expression We thus have in mathematics, because of the self-imposed limitations, the first and only example of *complete* analysis, impossible in physical problems as in these there are always characteristics left out

An important structural and methodological issue should also be emphasized In the calculus we introduce a 'small increment' of the variable, we performed upon it certain indicated operations, and in the final results this arbitrary increment disappeared leaving important information as to the rate of change of our function This device is structurally extremely useful and can be generalized and applied to language with similar results

It has been noticed already that the calculus can be developed without any reference to graphs, co-ordinates or any appeal to geometrical notions, but as geometry is an all-important link between pure analysis and the outside world of physics, we find in geometry also the psycho-logical link between the higher and lower orders of abstraction But the appeal to geometrical notions helps *intuition* and so is extremely useful For this reason we will explain briefly a system of co-ordinates and show what geometrical significance the derivative has

We take in a plane two straight lines $X'X$ and $Y'Y$, intersecting at O at right angles, so that $X'OX$ is horizontal extending to the left and right of O, and YOY', is vertical, extending above and below O, as a frame of reference for the locations of point, lines, and other geometrical figures in the plane We call this a two-dimensional rectangular system of co-ordinates This method may be extended to three dimensions, and our points, lines, and other geometrical figures referred to a three-dimensional rectangular system of co-ordinates consisting of three mutually perpendicular and intersecting planes

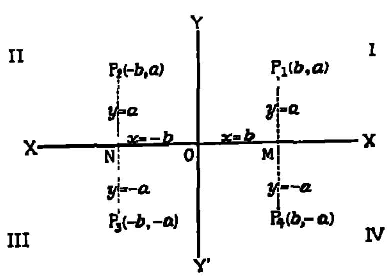


FIG 2

As we see in Fig. 2, we have four quadrants I, II, III, IV, formed by the intersecting axes $X'X$ and $Y'Y$ The co-ordinates of a point P, by which we mean the distances from the axes determine the position of the point uniquely. We call $X'X$ and $Y'Y$ the *axis* of X and the *axis* of Y respectively, and O the ***origin***. If we select a point P_1 in the plane of $X'X$ and $Y'Y$ and draw a line P_1M perpendicular to $X'X$ then OM and MP_1 are called the co-ordinates of P_1, OM is called the *abscissa* and is denoted by $x=b$, and MP_1 is called the ordinate and denoted by $y=a$. We speak of P_1 as the point (b,a), or, in general, of any point as the point (x,y).

Let us draw $ON=OM=b$ and draw lines P_1P_4 and P_2P_3 through M and N respectively perpendicular to $X'X$, making $MP_4=NP_2=NP_3=MP_1=a$. We then have four points P_1, P_2, P_3, P_4, in each one of the four quadrants and all of them by construction would have equal numerical values for their abscissas and ordinates. To be able to discriminate between the four quadrants, and so avoid ambiguity, we make the convention that all values of y above $X'X$ are to be positive and below $X'X$ negative, and all values of x to the right of $Y'Y$ positive, to the left negative. Thus we see that by such conventions the point P_1 would have both b and a positive, P_2 would have b negative and a positive, P_3 both b and a negative, and finally P_4 would have b positive and a negative, or in symbols $P_1(b,a)$, $P_2(-b,a)$, $P_3(-b,-a)$; and finally $P_4(b,-a)$.

It is obvious that for any point on the X axis (for instance M) the ordinate $y=0$. If our point is on the Y axis the abscissa $x=0$ and the co-ordinates of the origin O are both zero $(0,0)$.

From the above definitions we see at once how to plot, or locate, a point. To plot the point $(-4,3)$, since the abscissa x is negative and the ordinate y is positive we locate N on $X'X$, 4 units to the left of O. At N we erect a perpendicular upon which we locate the point $(-4,3)$, 3 units above N. The symbol $(-4,3)$ represents a particular case of the general symbol (x,y) and is accordingly plotted as a particular point as just shown. If instead of the pair or relations expressed by two equations $x=-4$, $y=3$, we have a single relation expressed by one equation, for example, $y=x-2$, we have y expressed as a function of x, whence by assigning to x different values, corresponding values of y are determined, and a set of points may be plotted where abscissas and ordinates are corresponding values of x and y respectively. Thus, when $x=0$, $y=-2$, when $x=1$, $y=-1$, when $x=2$, $y=0$, when $x=3$, $y=1$, when $x=4$, $y=2$, . .

We may now plot the points $A(0,-2)$, $B(1,-1)$, $C(2,0)$, $D(3,1)$, $E(4,2)$, or as many more points as we may choose by giving x additional different values.

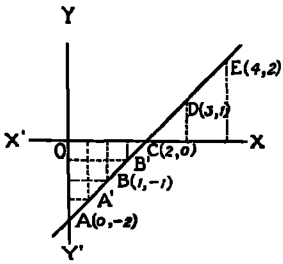


FIG. 3

If we give to x successive values with smaller differences our points would be closer together, for instance for

$x=0$	$y=-2$	(A)
$x=0.5$	$y=-1.5$	(A')
$x=1$	$y=-1$	(B)
$x=1.5$	$y=-0.5$	(B')
$x=2$	$y=0$	(C)
...	...	

As we plot larger and larger numbers of points closer and closer together, in the limit, if we take indefinitely many such points, we approach a smooth line It can be proved that an equation of the type given in this example, namely, where both variables are of the first order, always represents a *straight* line Such equations are called therefore *linear* equations, as they represent straight lines

The problem of linearity and non-linearity is of extreme importance, and we will return to it later on Here we are interested only in the definition and meaning of linearity of equations

Let us consider next a simple equation of second degree, $y=\frac{x^2}{2}$ In assigning arbitrary values to x, we note that x^2 is always positive (by the rule of signs) whether x is positive or negative Hence, we may tabulate values of x with the double sign $\pm$ meaning either $+$ or $-$.

$x=0$	$y=0$	(O)
$x=\pm 1$	$y=\frac{1}{2}$	(A)
$x=\pm 2$	$y=2$	(B)
$x=\pm 3$	$y=4\frac{1}{2}$	(C)
$x=\pm 4$	$y=8$	(D)
. .	. .	

We see for each value of y we have two values for x which differ only in sign This means that we have points on two sides of the Y axis with numerically equal abscissas and, since for $x=0$, $y=0$, the beginning of our curve is at the origin of co-ordinates and the curve is symmetrical with respect to the Y axis

If we connect the points D', C', B', A', O, A, B, C, D, with straight lines we have a broken line But if we choose smaller and smaller differences between the successive values of x, the broken line becomes smoother and smoother, and, in the limit, as we take increasingly smaller steps, or, in other words, plot indefinitely larger numbers of points in one interval, we approach a smooth, or continuous curve

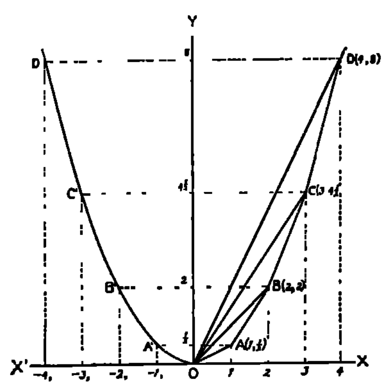


FIG 4

It must be noticed that in equations of higher orders the ratio of changes in the function y to corresponding changes in the variable x vary from point to point, and so we have a *curve* instead of a straight line It is necessary to become quite clear on this point so we may better compare the two different types of equations as to the law of their growth

Let us write down in two columns the successive values for the two types of equations Let us take the equation $y=\frac{x^2}{2}$ with the graph shown in the preceding diagram (Fig 4) and the equation $y=2x$ as shown in Fig. 5.

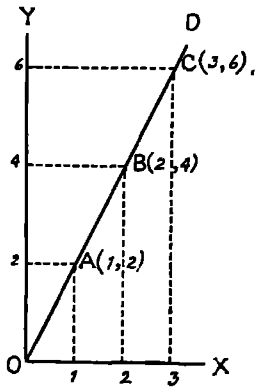


FIG 5

Values of x	$y=2x$	$y=\frac{x^2}{2}$
-4	-8	8
-3	-6	$4\frac{1}{2}$
-2	-4	2
-1	-2	$\frac{1}{2}$
0	0	0
$+1$	$+1$	$\frac{1}{2}$
$+2$	$+4$	2
$+3$	$+6$	$4\frac{1}{2}$
$+4$	$+8$	8
$+5$	$+10$	$12\frac{1}{2}$
$+6$, .	$+12$, .	18, .

The equation $y=2x$ involves the variables in the first degree and we see that the ratio of changes in the ordinates to corresponding changes in the abscissas remains constant (proportional) The triangles in Fig 5, are either equal or similar, which necessitates the equality of angles and so the line $OABCD$ is of necessity a straight line In this case as $x=0$ gave us $y=0$ the line passes through the origin of co-ordinates

The picture is entirely different in the case of the higher degree equation, $y=\frac{x^2}{2}$, illustrated in Fig 4 From the table of values of the function we see that the value of the function increases increasingly more rapidly than the values of the independent variable and so the ordinates are *not proportional* to the abscissas If in Fig 4 we connect O with A, O with B, O with C, O with D, respectively, we see that the lines OA, OB, OC, and OD have *different* angles with the axis $X'X$, the respective triangles are not similar, and so there is no proportionality The lines OA, OB, OC, OD , do *not* represent a straight line as they have all different angles with the axis XX' and so the points A, B, C, D . , cannot lie on a straight line but represent a *broken* line which, in the limit, when the points plotted become sufficiently near together, becomes a smooth and continuous curve

The fact that equations in which the variables are only of the first degree, represent straight lines, and that equations of higher degrees represent curved lines is very important, as will appear later on We must notice also that the problem of *linearity* is connected with *proportionality*

These few simple notions concerning the use of co-ordinates will allow us to explain the geometrical meaning of the derivative and the differential.

Consider P_1 and P_2, (Fig 6) two points on the curve, $y=f(x)$, referred to the axes OX and OY Drop perpendiculars P_1M_1 and P_2M_2 from P_1 and P_2 to OX. These are the ordinates $y_1=f(x_1)$ and $y_2=f(x_2)$ of the points P_1 and P_2, and OM_1 and OM_2 are the abscissas x_1 and x_2 of the points P_1 and P_2 Through P_1 draw the secant P_1P_2, the tangent to the curve P_1T, and the line P_1Q parallel to OX Then P_1Q represents $\Delta x=x_2-x_1$ the change in the variable x, and P_2Q represents $\Delta y=y_2-y_1=f(x_2)-f(x_1)$ the change in the function y

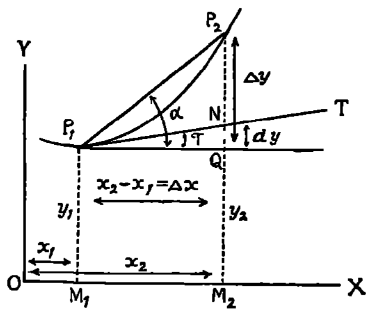


FIG 6

In the right triangle P_1QP_2 the ratio P_2Q/P_1Q is a measure (the tangent) of the angle P_2P_1Q $(=\alpha)$ that is, $\tan\alpha = P_2Q/P_1Q=\Delta y/\Delta x=\dfrac{f(x_2)-f(x_1)}{\Delta x}$ or, since $x_2=x_1+\Delta x$ we may write

$$\tan\alpha=\frac{f(x_1+\Delta x)-f(x_1)}{\Delta x}$$

As P_2 approaches P_1 along the curve, the secant P_1P_2 rotates about P_1 approaching P_1T as its limit, and the tangent of α approaches the tangent of τ, τ being the angle which P_1T, the tangent to the curve at P_1, makes with P_1Q But as P_2 approaches P_1, $\Delta x=x_2-x_1=M_1M_2$ approaches zero or symbolically as $\Delta x\rightarrow 0$, $(\Delta y/\Delta x)\rightarrow \tan\tau$, that is $\tan\tau=\lim\limits_{\Delta x\to 0}(\Delta y/\Delta x)$ We see that the $\lim\limits_{x_2\to x_1}\dfrac{y_2-y_1}{x_2-x_1}=\lim\limits_{\Delta x\to 0}\dfrac{\Delta y}{\Delta x}$ represents nothing more or less than the derivative of the function representing the curve In other words, the geometrical interpretation of the analytical process of differentiation is the finding of the slope of the graph of the function The increment Δy of the function is represented by P_2Q, the differential dy is equal to NQ and $\Delta x = dx=P_1Q$, $\tan\angle TP_1Q=\dfrac{dy}{dx}$

From the above considerations we see that the differential calculus gives, by the application of some extremely simple structural principles, a method of analysis by which we can discover a tendency at a particular stage rather than the final outcome after a definite interval From such fundamental yet simple beginnings the whole calculus is developed Most of these developments are not needed for our purpose, but we will explain one specially important theorem The theorem in question is that the derivative of the sum of two functions is equal to the sum of their derivatives In symbols

$$D_x(u+v)=D_xu+D_xv.$$

Let us symbolize $u+v=y$ and select a special value

$$y_0=u_0+v_0 \qquad (4)$$

then $\quad y_0+\Delta y=u_0+\Delta u+v_0+\Delta v$ By subtracting (4),

we have $\quad \Delta y=\Delta u+\Delta v$ Dividing by Δx,

we have $\quad \dfrac{\Delta y}{\Delta x}=\dfrac{\Delta u}{\Delta x}+\dfrac{\Delta v}{\Delta x}$. When Δx approaches zero the left-hand side approaches $D_xy=D_x(u+v)$, and the first term of the right-hand approaches D_xu, while the second term approaches D_xv and so,

$$D_x(u+v)=D_xu+D_xv$$

The symbol D_x means also that certain operations are to be performed upon our function, namely, to find its derivative When used in this sense it is called an operator The operator D_x can be also written in its differential form as $\dfrac{d}{dx}$, and similarly for higher derivatives

2 *MAXIMA AND MINIMA*

It will be useful to have some applications of the differential calculus explained.

If a function $y=f(x)$ is continuous in an interval $a<x<b$ and has larger (or smaller) values at some intermediate points than it has at or near the ends, then it has a maximum (or minimum) at some point $x=x_0$, inside this interval If Fig 7 represents the graph of the function, it is obvious that at the maximum (or minimum) the tangent to the curve is parallel to the axis and therefore the slope of this tangent is zero As this slope is given by the derivative and the slope is zero we have a simple method of finding the maximum (or the minimum) of a function by equating the first derivative to zero, namely, $D_xy=0$ when $x=x_0$

It is useful to be able to discriminate between the maximum and the minimum of a function. Fig 7 shows that this can be done by finding means to discriminate between the two cases when our curve is concave upwards or concave downwards The slope of a curve for a particular value of x is given by the value of D_xy, corresponding to that value of x If the value D_xy is positive, y increases as x increases, and the curve slopes up as we move to the right, if the value of D_xy is negative, y decreases as x increases, and the curve slopes down as we move to the right

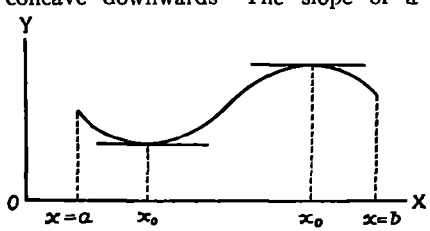


FIG 7

If we consider the curve $y=f(x)$ which has its concave side turned upward (Fig 8), the slope of the curve itself is a *function of* x, $\tan\alpha=f'(x)$ If we consider a variable point P on a curve $y=f(x)$, together with the tangent to the curve P, as following the curve in the direction of increasing values of x, the curve is concave upward whenever the slope is increasing algebraically, that is when $D_x \tan \alpha = 0$ In other words, the curve is concave upwards for those values of x for which $D_x \tan \alpha$ is positive, or since $\tan \alpha = D_x y$ for those values of x for which $D_x \tan \alpha = D_x(D_x y) = D_x^2 y$ is positive. Similarly a curve is concave downwards for those values of x for which $D_x \tan \alpha = D_x(D_x y) = D_x^2 y$ is negative. These results can be expressed thus

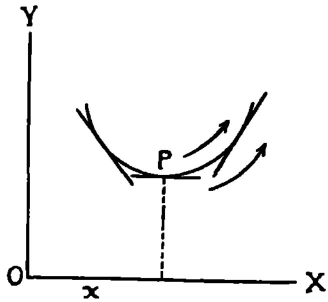


FIG 8

A curve $y = f(x)$ is concave upward when $D_x^2 y > 0$, or, in words, when the second derivative is positive, and the curve is concave downward when the second derivative is negative, or, in symbols, when $D_x^2 y < 0$.

From Fig. 7, we see that for a maximum we must have our tangent parallel to the $X'X$ axis and our curve concave downwards, hence for these conditions the first derivative $[D_x y]_{x=x_0} = 0$, and the second derivative $[D_x^2 y]_{x=x_0} < 0$ For a minimum the first derivative must again be zero and the second derivative positive, whence the concave side of the curve is turned upwards It should be noticed that the problems of maxima and minima play an extremely important structural psycho-logical and semantic role in our lives All theories, somehow, are built on some minimum or maximum principle involving evaluations which are fundamental factors of all semantic reactions. In daily life we apply these structural and semantic notions continually In science this tendency made its appearance quite early The problem of maxima and minima was treated seriously as far back as the second century B C In the eighteenth century Maupertuis formulated a 'supreme law of nature', that in all natural processes the 'action' (energy multiplied by 'time') must be a minimum Euler and Lagrange gave an exact basis and form to this principle, and finally Hamilton, in 1834, established this principle structurally as a *variational* principle, known as the hamiltonian principle, which appears to be of extreme generality and usefulness It facilitates the derivation of the fundamental equations of mechanics, electrodynamics and electron theory It has also survived, in a generalized form, the einsteinian revolution, for it contains nothing whatever which would connect it with a definite co-ordinate system, it involves only pure numbers and so is invariant to all transformations It is structurally one of the most important invariants ascribed to nature, being independent of the systems of reference of the observers

It is very desirable that this problem should be investigated further from the structural psycho-logical semantic and neurological point of view, as the very foundations of human psycho-logics are fundamentally connected with such a principle, which itself is an *invariant* in human psycho-logics

Its importance is still increasing, and the hamiltonian principle plays a most remarkable role in all the newest advances of science Any reader need only look attentively at his daily life to realize that there too this principle plays a predominant role.

3. *CURVATURE*

In modern scientific literature we hear often the fundamental term 'curvature' mentioned, and a few words about it will not be amiss. If we take two perpendicular lines $X'OX$ and OY and select on OY a number of points A, B, C, D , further and further away from O and describe arcs of circles with these points as centres with radii AO, BO, CO, DO . . respectively Fig. 9, we find each successive arc flatter and closer to the line $X'X$ than its predecessor. In other words, the larger the radius of our circle, the flatter its arc is In the limit as the radius of the circle becomes indefinitely large, the arc approaches a straight line by intuition and by definition We notice also that the curvature of each circle is uniform, that is, one-valued at every point, but that when we pass from one circle to another of different radius, the curvature changes

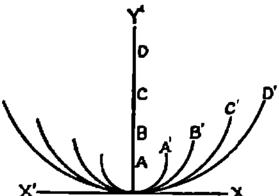


FIG. 9

If we consider a curve and two points on it, M_1 and M_2, (Fig 10) and draw two tangents at these points, then the angle between these two tangents will depend on two factors, the sharpness of the curve and the distance between the points M_1 and M_2 If we take the points near enough and designate the length of the arc between them by Δs, the angle between the two tangents by $\Delta\theta$, then the limiting value of the ratio $\Delta\theta/\Delta s$, as M_2 approaches M_1, becomes $d\theta/ds$, and is a measure of the rate of change of the direction of the tangent at M, as M moves along the curve Let us designate the rate at which the tangent turns where the point describes the curve with unit velocity as the curvature, or $k=\pm d\theta/ds$, but as k is essentially a positive number or zero we accept only the absolute value of this ratio To find $d\theta/ds$ we notice that $\tan\theta=dy/dx$

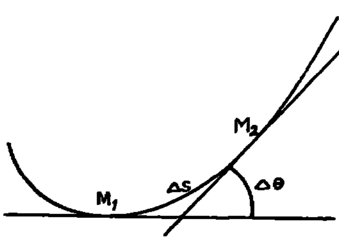


FIG 10

or $\theta=\tan^{-1}\dfrac{dy}{dx}=\tan^{-1}y'$, whence $d\theta=\dfrac{dy'}{1+y'^2}=\dfrac{y''dx}{1+y'^2}$

But $k=\dfrac{d\theta}{ds}$ where $ds=\sqrt{(dx)^2+(dy)^2}=\sqrt{1+y'^2}dx$

whence $k=\dfrac{y''}{(1+y'^2)^{3/2}}$

The reciprocal of the curvature is called the radius of curvature The radius of curvature of a circle is its radius. The curvature of a curve is measured by the radius of the osculating circle, that circle which fits the curve the most closely in the neighbourhood of our point.

4 *VELOCITY*

Until now we have treated our independent variables as *any* quantity but there are many problems where the independent variable represents 'time' For instance, if we travel by railroad the distance increases as 'time' increases, plants and animals grow with 'time', By the average velocity with which a given point moves for a given length of 'time' we mean the distance s traversed divided by the 'time' elapsed If, for instance, a train makes 5 miles in 10 minutes we say that its average velocity is 30 miles per hour, or, in symbols Velocity, $v=\frac{s}{t}$ In this case we were considering uniform velocity, but very often we have to deal with velocities which are not uniform and which might be increasing or decreasing In such a case we can describe the velocity approximately at any given moment if we take a short interval of 'time' immediately after the moment in question and take the average velocity for this short interval

For instance, the distance a stone falls is according to the law, $s=16\,t^2$ We want to find how fast it is going after t_1 seconds when $s_1=16\,t_1^2$, and a short interval after we have, let us say, $s_2=16\,t_2^2$ Obviously the average velocity for the interval t_2-t_1 is $\frac{s_2-s_1}{t_2-t_1}$ feet per second If we take $t_1=1$, $s_1=16$, and the difference $t_2-t_1=0\,1$ of a second then $s_2=16\,t_2^2=16\times1\,21=19\,36$ and $\frac{s_2-s_1}{t_2-t_1}=\frac{19\,36-16}{0\,1}=\frac{3\,36}{0\,1}=33\,6$ ft per second

If we take the interval of 'time' smaller, for instance, $1/100$ of a second we would have $\frac{s_2-s_1}{t_2-t_1}=32\,2$ feet per second, and if we take the intervals as $1/1000$ of a second the average velocity would be $32\,0$ feet per second We see that we could determine the speed of the stone at any instant with any degree of accuracy by direct calculation, but this is not necessary If we regard the interval t_2-t_1 as an increment of the variable t, that is as Δt, and $s_2-s_1=\Delta s$ which represents the increment of the distance considered as a function of the 'time' we would have the average velocity $=\Delta s/\Delta t$ As Δt approaches zero in the limit, the average velocity approaches a limit and this limit is the velocity v at the instant t_1, or in symbols

$v=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$. In words, the velocity of a point is the 'time' derivative of the space traveled

If the velocity is not uniform, the rate at which the velocity is increasing is called the acceleration and may be written as $a=\frac{dv}{dt}$, but as we have already seen dv is itself $d\left(\frac{ds}{dt}\right)$, hence $a=\frac{d^2s}{dt^2}$ In words, the acceleration is the second derivative of the distance with respect to 'time'.

In the above notes we have not attempted to give the reader more than some structural and methodological notions, and what amount really to short structural explanations of definitions which will be useful later on The reader can find many excellent books which give all the additional information he may want

Section C On the integral calculus

So far, we have been studying a method by which to find the variation of a given function corresponding to an indefinitely small variation of our variable We saw that the *rate of change* of our function was given by the first derivative, which in turn was also a function (usually different) of our independent variable and so could itself vary and have a rate of change, and so give us a second derivative , .

And now we must explain briefly the inverse problem, namely, given the derivative to find the function In symbols, given $u = D_x U$, find U

The function U is called the integral of u with respect to x, or, in symbols, $U = \int u\,dx$

To integrate a function $f(x)$ is to find a function $F(x)$ which when differentiated gives again the function $f(x)$ with which we started As in this work we are not interested in computations, but only in the structural, methodological, and semantic aspects, the inverse problem of differentiation, namely, integration, is less important for us here, and I will explain only a single example We have already differentiated the function $y = 2x^3 - x + 5$ and found its derivative $dy/dx = 6x^2 - 1$ Just as the derivative of the sum of a number of functions is equal to the sum of their derivatives, a similar rule holds for the integrals, namely, that the integral of the sum of a number of functions is equal to the sum of their integrals Hence we can take in our example only the first term of our equation In symbols $D_x(2x^3) = 6x^2$, in words, the derivative of $2x^3$ is $6x^2$.

In a problem in integration we would have $6x^2$ given and we would have to find the original function from which $6x^2$ was obtained by differentiation In our case the solution is already given, namely, $\int 6x^2 dx = 2x^3$ * In general the solution of problems of integration is largely dependent on the ingenuity of the solver, although we have a number of standard formulae and methods The geometrical meaning of integration is much more interesting for us and we will give a short explanation of it

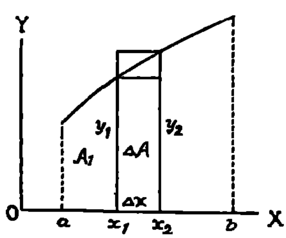


FIG 11

If we consider the curve given by an equation $y = f(x)$ and the area bounded by the X axis, the two ordinates whose abscissas are $x = a$ and $x = b$ and the curve, we may find the area as follows

*The constant of integration is omitted so as not to confuse the reader

If we select an arbitrary value $x = x_1$ for which $y = y_1 = f(x_1)$, denoting the corresponding value of the area A by A_1 (Fig 11) and give to x_1 an increment Δx, then the area A_1 would receive the increment ΔA We can approximate ΔA by the help of two rectangles, one of height $y_1 = f(x_1)$, the other of height $y_2 = y_1 + \Delta y = f(x_2) = f(x_1 + \Delta x)$

We see that ΔA is larger than the smaller rectangle In symbols

$$y_1 \Delta x < \Delta A < (y_1 + \Delta y)\Delta x,$$

hence

$$y_1 < \frac{\Delta A}{\Delta x} < (y_1 + \Delta y)$$

As we pass to the limit and let Δx approach zero, we have

$\lim\limits_{\Delta x \to 0} \dfrac{\Delta A}{\Delta x} = y_1$ That is, $D_x A = y_1 = f(x_1)$ when $x = x_1$, which means that the ordinate of the curve at any point is equal to the x derivative of the area at that point In general, $D_x A = y$, and hence, $A = \int y\,dx$

The consideration of what is called the definite integral is still more instructive Let us take the curve in Fig 12 represented by an equation $y = f(x)$ and a pair of ordinates which intersect the X axis at the points $x = x_0$ and $x = x_n$ Let us divide the interval $x_0 x_n$ into n equal parts and erect ordinates at each point of division Let us construct a set of pairs of rectangles with these ordinates as we constructed the single pair of rectangles in Fig 11 By inspection of the figure we see that the area under the curve is slightly greater than the sum of the areas of the included rectangles and slightly less than the sum of the areas of the including rectangles When n is allowed to increase without limit the sum of the areas of either set of these rectangles approaches the area bounded by the curve, the X axis, and the end ordinates In symbols, the area of the first rectangle beneath the curve is $f(x_0)\Delta x$, where Δx denotes $x_1 - x_0 = \dfrac{x_n - x_0}{n}$ The area of the second rectangle is $f(x_1)\Delta x$, The sum of these areas is $f(x_0)\Delta x + f(x_1)\Delta x + \quad + f(x_{n-1})\Delta x = \sum\limits_{i=0}^{n-1} f(x_i)\Delta x$

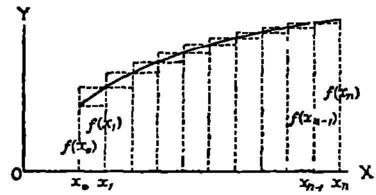


FIG 12

If we allow n to increase without limit we have the area under the curve

$$A = \lim_{n \to \infty} [f(x_0)\Delta x + f(x_1)\Delta x + \quad + f(x_{n-1})\Delta x] = \lim_{\Delta x \to 0} \sum f(x)\Delta x$$

$$= \int_{x=x_0}^{x=x_n} f(x)\,dx = \Big[F(x)\Big]_{x=x_0}^{x=x_n} = F(x_n) - F(x_0)$$

In words, the above formula indicates the fundamental process of the integral calculus, namely Let $f(x)$ be a continuous function of x throughout the interval $x_0 \leqq x \leqq x_n$. If we divide this interval into n equal parts by the points $x = x_0, x_1, \quad . \quad , x_n$, and form the sum $f(x_0)\Delta x + f(x_1)\Delta x + \quad + f(x_{n-1})\Delta x$, as we let n increase without limit, this sum will approach a limit, which can be found by integrating the function $f(x)$, that is, by finding the function $F(x)$ of which $f(x)$ is the derivative, and by taking the integral between the limits $x=x_0$ and $x=x_n$, that is, by taking the difference between $F(x_n)$ and $F(x_0)$

It must be noticed that in our first example, the case of the indefinite integral, we considered integration as the inverse of differentiation, in the second example, we considered the definite integral as the limit of a sum

The symbol of the integral, $\int$, had its origin in the letter S from the latin word 'summa', the integral being historically understood as the definite integral, or the limit of a sum

Section D Further applications

1 *PARTIAL DIFFERENTIATION*

When we have more than one independent variable, for example, two, we have to become acquainted with what is called partial differentiation This process is important, as in practice we usually deal with several independent variables It presents very little that is new from a structural and methodological point of view, but we give it here, simply to explain the meaning of the term, as the reader may find it used in other works

If we have a function z of two independent variables x and y, $z=f(x,y)$ which geometrically represents a surface, we may differentiate with respect to one of the variables, let us say x, and hold the other variable y fast, that is, treat it as a constant In this way we should then have a partial derivative of z with respect to x Similarly, if we treat x as a constant and differentiate in respect to y, we should have the partial derivative of z with respect to y The above definitions give us the rules for partial differentiation—that is, following the ordinary rules, considering each variable individually and treating all the other variables as constant

The notation for partial derivatives is similar to the ones explained before, except that the lower case letter d is replaced by the script form ∂ or a subscript is used to indicate the variable with respect to which the differentiation is performed, for instance, $\frac{\partial f}{\partial x}=\frac{\partial z}{\partial x}=f_x'=z_x'=D_xf=D_xz$, Higher derivatives are obtained without difficulty in like manner If $z=f(x,y)$ and $\frac{\partial z}{\partial x}=f_x'(x,y)$ and $\frac{\partial z}{\partial y}=f_y'(x,y)$, the partial derivatives themselves are in general also functions of x and y and can in turn be differentiated Thus, $\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x^2}=f_{xx}''(x,y)$, or $\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial^2 z}{\partial x\partial y}=f_{xy}''(x,y)$ The order in which we differentiate is immaterial provided that the derivatives concerned are continuous. The *total differential* of a function of two variables, for example, $f(x,y)$, $df=d_xf+d_yf=\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy$ is equal to the sum of the partial differentials of the first order if we neglect terms of higher orders, whose values are indefinitely small quantities relative to the first order differentials In symbols, $df = d_xf + d_yf = \left(\frac{\partial f}{\partial x}\right)dx + \left(\frac{\partial f}{\partial y}\right)dy$ In words, the total differential of $f(x,y)$ is found by finding the partial derivatives with respect to x and y, multiplying them respectively by dx and dy, and adding.

2 *DIFFERENTIAL EQUATIONS*

A natural development of the invention of the calculus was the introduction of differential equations Differential equations differ from the ordinary equations of mathematics in that in addition to variables and constants they contain also derivatives of one or more of the variables involved Differential equations are of extreme importance, and arise in many problems Newton solved his first differential equation in 1676 by the use of an infinite series, eleven years after his discovery of the calculus in 1665 Leibnitz solved his first differential equation in 1693, the year in which Newton first published his results From this date on, progress in the development and application of differential equations was very rapid, and today the subject of differential equations occupies in the general field of mathematics a central position from which important and useful lines of development flow in many different directions

To integrate or solve a differential equation means, analytically, to find all the functions which satisfy the equation In geometry, it means to find all the curves which have the property expressed by the equation In mechanics it means to find all the motions that may possibly result from a given set of forces , The *degree* of the differential equation is defined as the degree of the derivative of the highest order which enters the equation The *order* of the differential equations is the order of the highest derivative it contains.

Equations in x and y, of the first degree in y and its derivatives with respect to x, y', y'' , are called *linear equations* The main equations of physics are *linear* differential equations of the second order, since y, the primitive function, y', the first derivative, and y'', the second derivative, appear only in the first degree For instance the equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = X$, or $y'' + a_1y' + a_2y = X$, when X represents a function of x alone is such an equation It is linear, or of the first degree, because the second derivative, y'', appears only to the first degree It is of the second order because that is the highest order derivative in the equation As we may recall, the derivative of a function gives us the *rate of change* of the function when we give successive values to the independent variable When we study the rate of change of the rate of change of our function, we study the rate of change of the first derivative which expresses the rate of change of the function, whence we obtain the derivative of the second order, and so on If we equate our derivatives to zero, or choose a value of the variable for which our derivative becomes zero, the rate of change of our function becomes zero In other words, the value of our function is momentarily constant, it has a stationary value.

Quite naturally, differential equations which involve derivatives involve implicitly and explicitly the whole fundamental structural framework of the calculus as explained in this chapter by expressing the 'rate of change' of some natural process If the rate of change is zero, it might express some 'natural law', or some uniformity as found in nature In other words, differential equations express differential laws, which in turn express the momentary tendencies of processes whose outcomes are given by the process of integration

From what has already been said here, it is obvious that differential equations and the differential laws which they express are of extreme structural importance. They formulate not only the uniformities and tendencies found in nature, but also of necessity somehow involve causality Besides which, they are also in accord with the physical structure and function of the nervous system. We shall return to this most important subject in the next chapter, in which we shall analyse the physical significance and aspects of what has been explained here

3 *METHODS OF APPROXIMATION*

In discussing the above fundamental notions of the calculus we considered a portion AB, of the curve given by the equation $y=f(x)$, (Fig 13) and two points on this curve P_1 with co-ordinates (x_1, y_1) and P_2 with co-ordinates (x_2, y_2) moving along the curve, the secant, or chord, P_1P_2 rotates about P_1, its length steadily diminishing, and in the limit as the length of the chord P_1P_2 tends toward zero, the slope of the secant approaches the slope of the tangent P_1T We saw that the slope of this tangent was given by the value of the first derivative of the function which represented the curve We were trying to get some knowledge of the direction of our curve at a given point by considering the slope of a *straight line* of smaller and smaller length When we studied the curvature of our curve we considered the rate of change of the slope of our tangent and so, by the aid of a second derivative, we found the curvature In this case we approximated our curve to a circle of radius equal to the radius of curvature of the curve at a given point

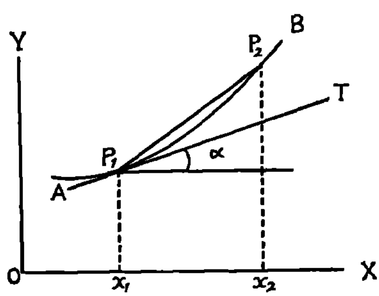


FIG 13

In attempting to determine the length of a portion of our curve a point cannot be regarded as a piece of the curve but only as marking a position on it For the purpose of determining the length of an arc it is convenient to replace each small element of the arc by its chord, a *lineal element* By definition the length of an arc of a curve is the limit, if such limit exists, toward which the sum of the lengths of the chords of its small subdivisions tends as the number of chords increases indefinitely and their individual lengths all approach zero uniformly. For example, the circumference of a circle is the limit approached by the perimeter of an inscribed polygon as the number of its sides increases indefinitely, the lengths of the individual sides all approaching zero.

Similarly the length of a curve may be approximated by the sum of the lengths of segments of tangents at successive arbitrarily chosen points, merely by choosing the points nearer and nearer together. For example, the circumference of a circle is the limit approached by the perimeter of a circumscribed polygon as the number of its sides increases indefinitely, the lengths of the individual sides all approaching zero. In either case, a point on a curve taken with a vanishingly small portion of the tangent to the curve at that point may be called the *lineal element* of the curve.

The above definitions apply equally well in either two or three dimensions. The lineal element in two dimensions may be defined by three co-ordinates x, y, p, of which x and y are the co-ordinates of the point through which the lineal element passes and p is the slope of the element. This slope, as we already know, is to be found by differentiation, and is given by the formula $p=dy/dx$. In geometrical problems which relate the slope of a tangent to that of other lines, it is not the tangent that is of real importance but the *lineal element*. From this point of view a curve is made up of infinite numbers of vanishingly small lineal elements which are tangent to it, which is the point of view of the differential calculus. Or the curve is composed of infinite numbers of vanishingly small chords which are the sides of an inscribed polygon, which is the point of view of the integral calculus.

Obviously, in the limit, both points of view are equivalent, although as a matter of convenience they may be different. In any case, it must be obvious to the reader that using *straight lines* instead of pieces of a curve, or using as closer approximations arcs of circles, facilitates our study of the curves, indeed renders such study possible at all, and in practice we can carry our work to any degree of approximation we choose. But in theoretical work we require precision, hence we think in terms of infinite numbers of vanishingly small steps. The differential and integral calculus supply the only perfect technique for these processes of analysis and synthesis.

4. *PERIODIC FUNCTIONS AND WAVES*

We have already said that the most important relations of physics are represented by linear differential equations of the second order. It is important to know the connection of these equations with the general theory of waves or oscillations.

If on a circle of unit radius, as shown in Fig. 14, we take several points P_1, P_2, P_3, P_4, and connect these points by straight lines with the centre O, we get angles XOP_1, XOP_2, . . . In trigonometry we define certain functions of these angles and a unit of measurement. For our purpose we will only define the so-called sine and cosine, as we have already met the definition of tangent

$\tan\theta = \dfrac{M_1P_1}{OM_1}$. The angles XOP_1, XOP_2 , may be specified by the ratios M_1P_1/OP_1, M_2P_2/OP_2 , respectively each of which ratios has a definite value This ratio in any case is called the *sine* of the angle, and is written in abbreviated form $\sin\theta$. If the radius of our circle is taken as unity then simply $M_1P_1 = \sin\angle P_1OM_1 = \sin\theta$, since $OP_1 = 1$. The ratio OM_1/OP_1 is called the *cosine* of the angle XOP_1, and is written $\cos\theta$

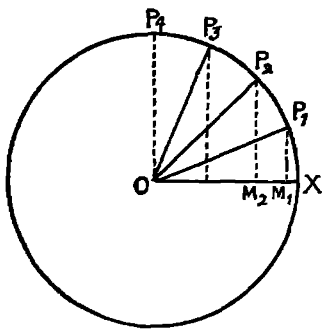


FIG 14

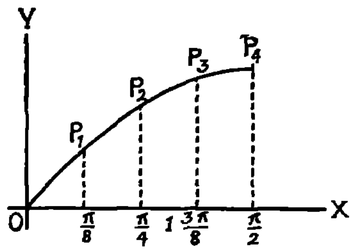


FIG 15

There are two units of measurement of angles In ordinary, or sexagesimal, measure, the unit angle is the degree, 1/360 of the entire angle about a point, 1/180 of a straight angle, or 1/90 of a right angle The degree is divided into 60 equal parts called minutes The minute is divided into 60 parts called seconds. In circular measure the unit angle is the radian, the angle at the centre of a circle whose arc is equal to the radius of the circle This angle is a constant whether the circle be large or small, due to the fact that the circumferences of circles vary as their radii, and, in one circle, angles at the centre are proportional to their arcs The constant ratio of the circumference of the circle to its radius is given by the number $\pi = 3\ 14159$ . . , this number being 'incommensurable' with unity As the length of the circumference of a circle with radius R, is $2\,\pi\,R$ we see that the entire angle about the centre, which in degrees is 360, is in radians $2\,\pi$, that a straight angle equals 180 degrees or π radians, and that a right angle equals 90 degrees or $\dfrac{\pi}{2}$ radians

Thus 1 radian $= \dfrac{180°}{\pi} = 57°17'44''\ 806$ which, as it depends on the value of π is itself an 'irrational' number The 'incommensurability' of the radian with right and straight angles makes its practical use inconvenient One of the main uses of the radian is in theory as it introduces a marked simplification in that the ratio of the sine of an indefinitely small angle to the angle itself is 1, when the angle is measured in radians In other words, the equivalence of an indefinitely small arc and chord becomes apparent numerically when the angle and sine are expressed in one unit

The following table gives the ordinary and radian measures, the sine, cosine and tangent of angles of 0, 1, 2, 3, and 4 right angles.

Angle in Right Angles	Angle in Degrees	Angle in Radians	Sine	Cosine	Tangent
0	0	0	0	1	0
1	90	$\pi/2$	1	0	$\pm\infty$
2	180	π	0	-1	0
3	270	$3\pi/2$	-1	0	$\mp\infty$
4	360	2π	0	1	0

From Fig 14 and from this table, it follows that the values of the trigonometric functions are equal for the angles 0 and 2π, or in the language of degrees, for the angles $0°$ and $360°$ We see also from Fig 14 that the angle XOP_1, or any other angle, has one measure as expressed by its trigonometric functions if we add to it $360°$ or 2π radians

The structural importance of the trigonometric functions in analysis lies in the fact that they are the *simplest* singly *periodic* functions and are therefore adapted for the representation of undulations As we have already seen the sine and cosine have the single real period 2π, which means that they are not altered in value by the addition of 2π to the variable The tangent has the period π

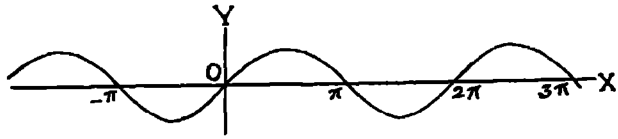
Besides the three functions defined above, we usually define three others, the secant, the cosecant and the cotangent as reciprocals respectively of the cosine, the sine, and the tangent These last three we may disregard in our present discussion

Let us consider the function $y=\sin x$, and construct the curve which this equation represents If we draw a circle of *unit* radius, Fig 14, the ordinates corresponding to the different angles XOP_1, XOP_2 , give the values of y, while the angles measured in radians, give the corresponding values of the abscissa x.

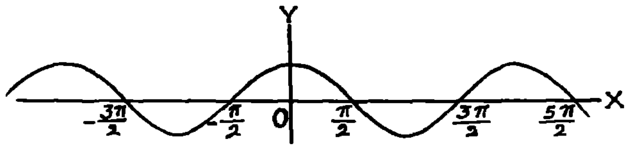
Plotting corresponding values of x and y as thus obtained in Fig 14 we get in Fig 15 the partial graph of the function $y=\sin x$ Proceeding again around our circle in Fig 14, that is, adding $360°$ or 2π, to each of our angles, hence to their abscissas of the curve in Fig 15, we add to the graph a second complete wave We may thus proceed either forward or backward obtaining as many complete waves, or undulations, as we please, as in Fig 16

The curve represented by $y=\cos x$ is obtained in like manner and is quite similar to the sine curve (See Fig 17)

To differentiate $\sin x$ we give to x the arbitrary values x_1, and $x_1+\Delta x$ and compute for y the corresponding values $y_1=\sin x_1$ and $y_1+\Delta y=\sin (x_1+\Delta x)$



$y = \sin x$
FIG 16



$y = \cos x$
FIG 17

Subtracting y_1 from $y_1 + \Delta y$, we have $\Delta y = \sin (x_1 + \Delta x) - \sin x_1$ Dividing by Δx we have $\Delta y/\Delta x = \dfrac{\sin (x_1 + \Delta x) - \sin x_1}{\Delta x}$

To express the meaning of the above geometrically we may take a circle of unit radius and construct the angles x_1 and $(x_1 + \Delta x)$, (Fig 18). Then $M_1P_1 = \sin x_1$, $M_2P_2 = \sin (x_1 + \Delta x)$, $QP_2 = \sin (x_1 + \Delta x) - \sin x_1 = \Delta y$, and arc $P_1P_2 = \angle P_1P_2 = \Delta x$.

The limit approached by the ratio $\Delta y/\Delta x = QP_2/P_1P_2$ as $P_2 \rightarrow P_1$, or as $\Delta x \rightarrow 0$, is by previous definitions the sine of the angle $M_2P_2P_1$, since *in the limit* the arc P_1P_2 becomes a straight line, the hypotenuse of the right triangle P_1QP_2, which is similar to the right triangle P_1M_1O, whence angle $\angle M_2P_2P_1 = \angle x_1$. In other words, $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{QP_2}{P_1P_2} = \dfrac{OM_1}{OP_1} = \cos x$, or $D_x \sin x = \cos x$

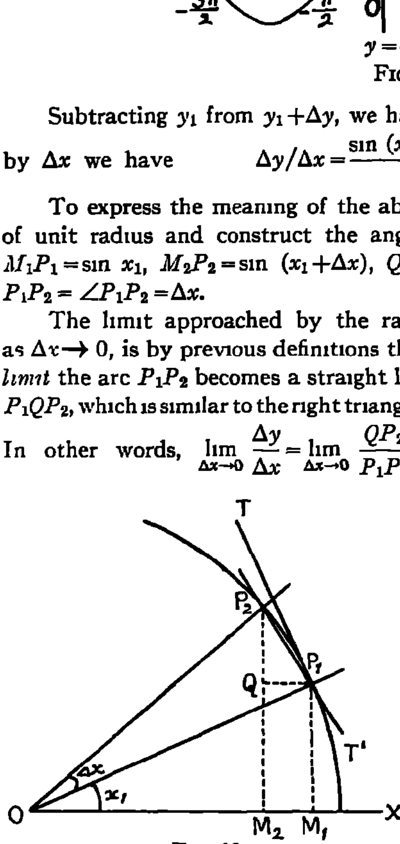


FIG 18

It may be easily and similarly shown that $D_x \cos x = -\sin x$ Our main interest is in the second derivative We see that the derivative of $\sin x$ is $\cos x$, and that the derivative of $\cos x$ is $-\sin x$

Differentiating again we obtain the second derivative of $\sin x$ as $-\sin x$, and the second derivative of $\cos x$ as $-\cos x$

The sine and cosine and their linear combinations are the only functions which when differentiated twice give us the second differential coefficient equal and of opposite sign to the original function. In symbols

$$\frac{d^2(\sin x)}{dx^2} = -\sin x, \text{ and } \frac{d^2(\cos x)}{dx^2} = -\cos x.$$

In physics we deal with many processes which are structurally periodic, which means that a definite physical condition constantly recurs after equal intervals of 'time'. The number of seconds or fractions of seconds within which the process runs its course is called the period We know already that the simplest periodic functions are sine and cosine functions of the type

$\sin(x+2n\pi)=\sin x$, and

$\cos(x+2n\pi)=\cos x$, where n may have any integral value Furthermore we have already seen that the first derivatives, therefore all derivatives of such functions are likewise simple sine and cosine functions In particular, the second derivatives of the sine and cosine functions are likewise sine and cosine functions taken with the opposite algebraic sign.

If we express the variability of a process as a function of 'time', that is, by an equation of the form $S=F(t)$, then in a periodic process, $F(t_1+nT)=F(t_1)$, where T is the period and n any integer. If the process repeats itself, as in a periodic process, we must have

$$\left|\frac{dF}{dt}\right|_{t=t_1+nT}=\left|\frac{dF}{dt}\right|_{t=t_1} \quad \text{and} \quad \left|\frac{d^2F}{dt^2}\right|_{t=t_1+nT}=\left|\frac{d^2F}{dt^2}\right|_{t=t_1}\cdot\cdot,$$

but, as we have already seen, the sine and cosine functions satisfy these conditions.

A process which can be described by an equation of the type $S=A\sin\frac{2\pi t}{T}$ is called a *harmonic vibration* or, a 'pure sine vibration', or simply a 'vibration' or 'oscillation'. The constant A, which represents the maximum value of the displacement on either side, is called the amplitude The period T is called the 'time of vibration', its reciprocal value which gives the number of vibrations in a unit of 'time' is called the vibration number or *frequency*.

As the second derivatives of sine and cosine functions are equal to the original functions taken with the opposite signs, we can describe harmonic vibrations by differential equations of the first degree (linear) and of the second order of the special type $\frac{d^2S}{dt^2}=-a^2S$, where $S=A\sin\left(\frac{2\pi t}{T}+\epsilon\right)$, A representing the amplitude, T the period, ϵ the phase of the vibration The factor of proportionality a is taken as the square of any arbitrary real quantity to indicate that the right-hand side must always have the opposite sign to that of S

The propagation of a vibration is called an advancing plane wave which has both velocity and direction.

Fourier has shown that any given form of wave may be represented by the superposition of a series of sine-waves, which gives sine-waves great theoretical and practical importance.

In writing this chapter I had two main aims One was to briefly indicate the essential semantic factors involved in the differential methods The other, to make the general reader and even specialists who are not mathematicians acquainted with some terms and rudiments of method which will be necessary for further discussion.

The differential methods involve semantic factors essential for a $\bar{A}$-system, the ∞-valued semantics of probability and for sanity and cannot be longer disregarded.

The main pressing issues are twofold One, to formulate methods which would impart the $\bar{A}$ semantic reactions of the calculus, which need not involve any technicalities, and can be imparted in the most elementary home or school education The other is to draw the attention of specialists to these semantic problems so that they will work them out

An attempt to solve the first issue has been undertaken in the present volume The second task will probably be accomplished in the not too distant future

It is earnestly suggested to all scientists, professional men and teachers, who are not mathematicians, to become familiar with differential methods and so acquire the appropriate semantic reactions Experience, in many cases, has shown that this will assist them in acquiring semantic balance and 'mental' efficiency Teachers and physicians in particular, would be greatly helped in their efforts to train children and patients in the $\bar{A}$ reactions The benefit is not in any 'calculations' whatsoever, but in the method and the related psycho-logical reactions

There is an excellent, short, most elementary and amusing account of the calculus by Sylvanus P Thompson *Calculus Made Easy* (Macmillan) which, for the present, is all that is needed for this purpose.

CHAPTER XXXIII

ON LINEARITY

The conception of linear transformation thus plays the same part in affine geometry as congruence plays in general geometry, hence its fundamental importance (547) HERMANN WEYL

It is instructive to compare the mathematical apparatus of quantum theory with that of the theory of relativity In both cases there is an application of the theory of linear algebras (215) W HEISENBERG

This "perturbation theory" is the complete counterpart of that of classical mechanics, except that it is simpler because in undulatory mechanics we are always in the domain of *linear* relations (466) E SCHRODINGER

As a result of experimental research on association, in 1904, I was led to show the complexity of the factors governing evocation . . And I have often insisted since then on this essential idea, in opposition to the simple schema of linear associative connection. (411) HENRI PIÉRON

We have already had several occasions to mention the 'plus' or additive issues as connected with linearity This problem is of structural and linguistic as well as empirical and psycho-logical semantic importance. It is sufficient for our purpose at present that we should notice two facts, namely, (1) That in one dimension, linearity expresses the *relation* of proportionality, (2) That the problems of linearity are dependent on the *relation* of additivity

The structural notion of additivity is of great antiquity. Being the simplest of such notions, it naturally originated very early in our history The earliest records show that the Babylonians and the Egyptians used the additive principle in their notation Our primitive ancestors, long before any records were written, had similar structural conditions present, open for investigation and reflection, that we have today That this was the case is not a mere guess Otherwise we would still be at their stage of development Some beginning had to be made somewhere There is little doubt that the men of remote antiquity presented many types of make-up, as we do today Some, for instance, were more curious than others, some more inventive, some more reflective . , which, as we know today, is found even among animals These more gifted individuals were, as usual, the inventors, discoverers, and builders of systems and language of their period They could not long fail to recognize the fact that a stone *and* a stone, or a fruit *and* a fruit are *different* from *one* stone or *one* fruit. For instance, the two stones might have saved the early observer's life in defence, or the two fruits might have satisfied his hunger or thirst, where one would not have done so. An accumulation of objects was obviously somehow different from a single object As these problems were often of vital importance to their lives, names for such accumulations of objects began to be invented, and one and one was called two, two and one was called three , . Number and mathematics were born as a structural semantic life-necessity for a time-binding class of life. They were an expression of the neurological structure and function and of the tendency toward induction.

In the beginning, names and generalizations were made from the simplest brute facts of life, and our primitive ancestors did not realize, that these crude generalizations might not have a structural validity, which they seldom doubted that they possessed, even as we today, seldom doubt. Those primitive scientists, (and we today differ very little from them), having produced *terms*, objectified them, and began to speculate about them. Let us examine some examples of such primitive mathematical speculations. Addition, of course, by which we *generate numbers*,—one and one make two, two and one make three . . , was all-important. They could not miss the simple fact that three which is equal to two and one, by definition, is more than two or one. A primitive generalization, namely, that the sum is always more than the summands taken separately, was still further generalized to a postulate that a part is smaller than the whole. This generalization has hampered mathematics almost up to our own day, and for many thousands of years it prevented the discovery of the notion of mathematical infinity, which we have already discussed in Chapter XIV.

It must be noticed that such generalizations involve *s.r.*, which are objective and un-speakable. If verbally formulated they should have a structure similar to that of the facts, otherwise they are fanciful and vicious, because not properly formulated. When formulated they become public structural facts (*s.r.* are personal, individual, non-transmittable, and un-speakable) and so they may be criticized, improved, revised, rejected, . All human history shows that the correct structural formulation of a problem is usually as good as the solution of it, because sooner or later a solution always follows a formulation.

After many thousands of years—in fact, practically only the other day—it was found that these primitive generalizations were in general not valid. Negative numbers were invented, and two plus minus-one was no more three but one, $2+(-1)=1$. The sum was *no* longer greater than its summands. The usual tragedy takes place here also. A few people know the facts, but the old primitive structural *s.r. remain* in some of these few, as well as in the great majority of us who did not even know the facts. That such structural *s.r.* do not vanish quickly, or generally, is proven again and again throughout history. We see it very clearly in the problems of 'infinity', or $\bar{E}$ geometries, or $\bar{N}$ physics. But the most pathetic sight is to see scientists who have *rationalized* the technique without a deeper re-education of their *s.r.* This is most clearly seen in the case of many writers on the foundations of mathematics, the Einstein theory or on the newer quantum mechanics. They *feel* in the old structural way, they *rationalize* in the new, hence their works are full of self-contradictions. Readers and students alike feel how 'difficult' and messy the whole subject is. As a matter of fact, the new theories are neither messy nor difficult. They are really much simpler and easier than the old theories, *provided* our structural *s.r.* are purged of the primitive structural tendencies to which every one of us is heir. When this semantic re-education of our structural feelings is accomplished it is the old that becomes 'unthinkable' and incomprehensible, because it gives such a structural mess.

Something similar might be said about a feeling deep-rooted in all of us namely, the 'plus' feeling In all the advances of science we struggle against it. For instance, the example of the green man-made leaf given previously shows clearly that man-made affairs may with some plausibility be considered as 'plus' affairs, but not so with non-man-made natural leaves, which appear not as 'plus', but functional affairs, where the greenness was structurally *not added* but happened, or became As a structural fact, the world around us is *not* a 'plus' affair, and requires a functional representation In chemistry, for instance, does hydrogen 'plus' oxygen produce water, H_2O? If we mix the two gases, two parts of hydrogen with one of oxygen we do not get water We must first pass a spark through the mixture, when an explosion occurs and the result becomes *water*, a *new* compound quite *different* from its elements or from a mere mixture of them Does one gallon of water and one gallon of alcohol make two gallons of a mixture? No, it makes less than two gallons Does light added to light make more light? Not always The phenomena of interference show clearly that light 'added' to light sometimes makes darkness Four atoms of hydrogen, of atomic weight 1 008, produce, under proper conditions, one atom of helium, not of atomic weight 4 032, but of atomic weight 4 The 0 032 has somehow mysteriously vanished Such examples could be quoted endlessly. They show unmistakably that structurally this world is not a 'plus' affair, but that *other* than additive principles must be looked for

The struggle against this 'plus' feeling is quite evident, but often unsuccessful, in scientific literature Man 'is' an animal 'plus' something Life 'is' 'dead matter', 'plus' some 'vitalizing principle', . In scientific literature we find curious expressions as for instance, 'It is impossible to express the conduct of a whole animal as the algebraic sum of the reflexes of its isolated segments', or, 'The individual represents heredity *plus* environment', or, 'That the abstraction does not merely take away from a number of engram groups some components and combine the rest into one sum, but forms thereby a new psychic structure is self evident and is in no way peculiar to the psyche Thus a clock work is as little the mere *sum* of its little wheels as a human being is the *sum* of his cells and molecules', and later on, 'to be exact the ego consists of the engrams of all our experiences *plus* the actual psychism' There is endless material that might be quoted, but for our purpose these few samples will suffice We do not give them with the purpose of citing authoritative examples of the need of non-plus considerations Far from it We do it to emphasize the astounding fact that, although the best men in their fields have vaguely felt this necessity, yet even they become a prey to this very old structural linguistic semantic tendency In all three cases quoted the authors were of the best we have. They have fought all their lives against the 'plus' tendency and methods, and yet, if they succeed in eliminating this tendency from one part of their subject, they plant it quite obviously somewhere else We see that we are dealing here with an ingrained psycho-logical tendency which can be remedied only by a fundamental, $\bar{A}$, structural, semantic investigation

Let us analyse these quotations In the second case, we hear, after a successful attack on *plus* tendencies, a statement that the 'individual represents heredity *plus* environment' Is this statement true? Let us take examples. There are certain fishes which are heliotropic and swim toward the light, but if we change the temperature of the water they become negatively heliotropic and swim *away* from the light Is this most complex activity of the organism-as-a-whole a 'plus environment' fact, or does the change of temperature produce some fundamental functional changes? When, for instance, a good mother rat, having been put on a different though still abundant diet, which is deprived of some minute amount of special vitamins, begins to eat her litters, is this again a 'plus' reaction, or is it a most complex functional change of the organism-as-a-whole? Or when a human being, because he received in childhood an 'emotional' shock through outside events (action or language of parents, for instance) develops a functional disorder, or even a physical ailment, is this again a 'plus environment' problem? Or, when chickens fed on eggs laid by hens kept without sunlight or violet rays, or which have only received sunlight through a glass window, develop rickets and soon die, though they do not do so when the glass windows are removed and the sunlight is allowed to operate directly upon the hens Is this again a 'plus environment' example?

One 'Smith' and one 'Smith' make two 'Smiths', as far as theatre or railway tickets are concerned, but in life, under proper conditions, they form a family and very often many more than two 'Smiths' come out of such 'addition' How about their work? Is it a mere sum? In the case of inventors who may have been influenced by one or many men directly or indirectly, do their inventions produce a sum of the work of as many men? Surely the steam engine or the dynamo produces more work than not only the inventors, but the series of other men who have been indirectly responsible for the inspiration of the inventors, could ever have produced So again it is not a 'plus' affair

In the third case we see the author attacking the 'plus' tendency on one page, and planting another 'plus' a few pages further on, which implies at once some objectified *additional* entity In this respect it must be noticed that this *additive* tendency represents a partial and important structural and semantic *mechanism of identification*, and to deal successfully with it, we must clear up the problem connected with the *additive* tendency

The numberless and endless 'philosophical' volumes, for instance, which have been written about the 'body-soul' problems, show the tremendous structural and semantic importance of the clearing up of this 'plus' versus 'non-plus' issue The reader may recall that the $\bar{A}$, the $\bar{E}$, and the $\bar{N}$ systems have one underlying structural metaphysics The $\bar{E}$ systems deal with non-linear equations and with curved lines, of which the linear equation and the straight line, (one of zero curvature), are only particular cases And the general theory of Einstein, which is the foundation of $\bar{N}$ systems, also introduces non-linear equations Ought we to be surprised to find that a $\bar{A}$-system must also solve this difficult structural and semantic problem of linearity versus non-linearity, of additivity versus non-additivity?

Indeed the problems demanding our attention are extremely baffling and difficult Even in such a perfected science as physics, we have great difficulties in using non-linear equations, and are still at the stage where we solve few equations other than linear ones To make any progress at all we must start with the *simplest* available problems in this field, namely, mathematical problems The main point at this stage is not a solution of the problem but its formulation When formulated and brought to the attention of mankind, there is no doubt that it will be eventually solved.

To better understand the additive principle, let us consider a group of elements, the individuals of which we denote by letters a, b, c, d , . Let us take two or more of these elements and produce a synthesis which results in a third or n-th entity Let this synthesis be of such a nature that the characteristics ascribed to the elements are also present in the resultant synthesis, in other words, let them have the so-called group characteristic. If our elements are, for instance, numbers, the new synthesis is also a number and belongs to the original group We must notice that the problem of *order* is important in the formulation of the additive principle If a and b are the two elements the synthesis of which we define, we must be clear that a first and b second, or b first and a second, must be recognized in the synthesis Let us assume also that only the two alternative orders a and b, or b and a, are of importance in this case The commutative law asserts that a plus b is equal to b plus a, $a+b=b+a$, which means that the two possible alternative orders give equivalent results We must notice that this does not mean that order does not enter into this synthesis, in such a case the above mentioned commutative law would make no assertion at all It is of importance that order should be involved in the synthesis It is a matter of indifference only as far as equivalence by a commutative law is concerned

We should notice for our purpose that the synthesis has the 'same' characteristics as the elements had In other words, if we know the characteristics of the elements we know the characteristics of the result For instance, if the elements were numbers, the result will be a number, and no characteristic absent in the elements will appear in the result This predictability from the characteristics of the elements to those of the result is perhaps one of the most striking characteristics of additivity On the one hand, it allows us to foretell the future, on the other hand, it limits considerably the applicability of the additive principle It is obvious that when we combine elements, and the results have *new* characteristics absent in the original elements, the new problems are structurally no more of an additive character, and the synthesis must be different

Only a few of the simplest entities in physics possess additive characteristics If we take, for instance, 'weight' or 'length' or 'time', we see that these units are additive One pound, or inch, or second, if added respectively to one pound, or inch, or second, gives us two pounds, or two inches, or two seconds

Not so, however, with temperature, or density, or many other derived magnitudes, as we call them If we have a body of temperature of one degree and combine it with another body of equal temperature the synthesis will not have a temperature of two degrees, (as in the case of weight), but of one degree This applies to density , two bodies of density one each will not give us a body of density two, but of density one

Before further analysis of the problems of linearity and additivity, it will be well to consider a few definitions

If an entity u is changed into an entity v by some process, the change may be regarded as the result of an operation performed upon u, the operand, which has converted it into v If we denote the operation by f, then the result might be written as $v=fu$ The symbol of the operation f is called the *operator* We are familiar with many such, indeed the symbols for all mathematical operations may be treated as operators So for instance the symbol $\sqrt{}$ indicates the operation of extracting the square root If we deal with a range of values for a variable x, what we have defined as the function symbol $f(x)$ may be treated as an operator whose operation on x may be indicated by the symbol fx The operation of differentiation may be symbolized by D, the result of whose operation on the variable u, Du is the derivative of u The sign of the definite integral $\int_b^a$ may be taken as indicating an operation which converts a function into a number , .

It is important to know that many of the rules of algebra and arithmetic when defined in this way, give rise to a calculus of operations The fundamental notion in such a calculus is that of a product If u is operated upon by f the result v is indicated by fu, or symbolically, $v=fu$ If v in turn is operated upon by g the result w is indicated by gv, or symbolically, $w=gv=gfu$, whence the operation gf which converts u directly into w is called the product of f and g If this operation is repeated several times in succession the usual notation of powers is used, for instance $ff=f^2$, $fff=f^3$, . Not applying the operator at all, which we would denote by f^0, leaves u unchanged, which we indicate symbolically by the equation $f^0u=u$ The operator f^0 is equivalent to multiplication by 1, $f^0=1$, whence f^0 may be called the *idem operator.* We see also that the law of indices holds; namely, the $f^m \times f^n = f^{m+n}$.

For our purpose we will analyse only one special case, namely, where we have u, v and $u+v$ as operands, and such an operator, f, that $f(u+v)=fu+fv$ Expressed in words, this means that the operator applied to the sum of the two operands gives a result equal to the sum of the results of operating upon each operand separately Such a special operator is called a *linear*, or distributive, operator.

In terms of functions we would have $f(x+y)=f(x)+f(y)$ which may be called a functional equation It has been proved that such a functional equation has one type of solutions, namely, when f is equivalent to a multiplication by a *constant*, or $fx=cx$. This fact is of great importance for us Many problems in science are stated in terms of variation. For purposes of analysis a statement that 'x varies as y' is written $y=kx$, where k is called a factor of proportionality, which enables us to convert a statement of variation into an equation If y varies inversely as x, we write $y=k(1/x)$ or $y=k/x$ A multiplication by a *constant* thus introduces a relation of proportionality, hence the importance of proportionality in a world where constants are present

It must also be noticed that the two fundamental operations of the calculus are *linear* without being equivalent to multiplication by a constant These are 'the derivative of the sum is the sum of the derivatives', that is, $D(u+v)=Du+Dv$, and 'the integral of the sum is the sum of the integrals', that is $\int(u+v)dx=\int udx+\int vdx$ But as the fundamental notion of the calculus is to substitute for a given function a *linear* function, in other words, to deal with curves as the limits of vanishingly small straight lines, this linearity underlies structurally all fundamental assumptions of the calculus, and one might say with Weyl that 'one here uses the exceedingly fruitful mathematical device of making a problem *linear* by reverting to infinitely small quantities' [1]

A vector is defined roughly as a line-segment which has a definite direction and magnitude, and any quantity which can be represented by such a segment is defined as a vector quantity

The addition of vectors is defined by the law of the parallelogram, as in the case of two forces It should be noticed that because of this definition the sum of two vectors *differs* in general from the arithmetical sum of the lengths, and only collinear, or parallel vectors obey the arithmetical summation law

The introduction by definition of mathematical entities which obey different laws from the usual arithmetical laws is an important structural and methodological innovation It gives us the useful *precedent* of defining our *operations* to suit our needs The vector calculus accepted as the definition of the sum of two vectors the law established *experimentally* in physics for the sum of two forces, and so the vector calculus from the beginning was structurally a particularly useful language in physics Only since Einstein has the value and importance of the vector calculus for physics become generally appreciated.

If we have two vectors, **a** and **b**, starting from a common origin O and complete the parallelogram as in Fig 1, then the diagonal of the parallelogram will be the required sum, $\mathbf{a}+\mathbf{b}$, by *definition*

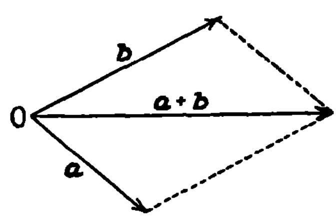


FIG 1

If we choose two co-initial vectors of unit length, one on the X axis, and the other on the Y axis, and call them **i** and **j**, we can always represent any vector **x** as the sum of two vectors, one of which is the projection of **x** on the X axis, and the other the projection of **x** on the Y axis (See Fig. 2) Let us call these vectors $\mathbf{x}'$ and $\mathbf{x}''$ respectively Then $\mathbf{x}=\mathbf{x}'+\mathbf{x}''$, by definition But $\mathbf{x}'$ differs from **i** in length only, hence it can be obtained by multiplying **i** by an appropriate number, say a Similarly, $\mathbf{x}''$ can be obtained from $\mathbf{j}$ by multiplying $\mathbf{j}$ by b, and so, in symbols, $\mathbf{x}' = a\mathbf{i}$, $\mathbf{x}'' = b\mathbf{j}$, and $\mathbf{x} = a\mathbf{i} + b\mathbf{j}$ All vectors of the plane can be obtained from $\mathbf{i}$ and $\mathbf{j}$ in this form The numbers a and b are called *components of* $\mathbf{x}$

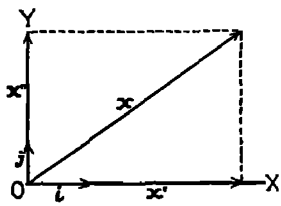


FIG 2

Now that we know how to express a vector in terms of its components, namely, $\mathbf{x} = a\mathbf{i} + b\mathbf{j}$, let us consider a vector function $f(\mathbf{x})$ which satisfies the equation $f(\mathbf{x}+\mathbf{y}) = f(\mathbf{x}) + f(\mathbf{y})$ We may take $a\mathbf{i} = \mathbf{x}$ and $b\mathbf{j} = \mathbf{y}$ and $\mathbf{x}+\mathbf{y} = \mathbf{z}$ then we have $f(\mathbf{z}) = f(\mathbf{x}+\mathbf{y}) = f(a\mathbf{i}) + f(b\mathbf{j})$ But since a and b are numbers, we have $f(a\mathbf{i}) = af(\mathbf{i})$, and likewise, $f(b\mathbf{j}) = bf(\mathbf{j})$, so that $f(\mathbf{z}) = af(\mathbf{i}) + bf(\mathbf{j})$ But $f(\mathbf{i})$ is itself a vector and therefore expressible in the form $a'\mathbf{i} + b'\mathbf{j}$, and, $f(\mathbf{j}) = c\mathbf{i} + d\mathbf{j}$ Hence, $f(\mathbf{z}) = a(a'\mathbf{i} + b'\mathbf{j}) + b(c\mathbf{i} + d\mathbf{j}) = (aa' + bc)\mathbf{i} + (ab' + bd)\mathbf{j}$ In general, the components are the coefficients accompanying $\mathbf{i}$ and $\mathbf{j}$, and so we have the components of $f(\mathbf{z}) = f(\mathbf{x}+\mathbf{y})$ in terms of the components of $\mathbf{z}$, and we see how the components of a vector are changed into the components of the linear vector function of the vector

In general terms, a continuous vector function of a vector is said to be a *linear* vector function when the function of the sum of any two vectors is the sum of the functions of those vectors, that is, the function f is linear if $f(\mathbf{r}_1 + \mathbf{r}_2) = f(\mathbf{r}_1) + f(\mathbf{r}_2)$, whence, if a be any positive or negative *number* and if f be a *linear* function then the function of a times $\mathbf{r}$ is a times the function of $\mathbf{r}$, $f(a\mathbf{r}) = af(\mathbf{r})$

Linear vector operators are also defined by a similar equation, namely, $L(\mathbf{a}+\mathbf{b}) = L\mathbf{a} + L\mathbf{b}$

Let us recapitulate If we take the functional equation $f(x+y) = f(x) + f(y)$, which might be used as a definition of *linearity*, and which is based on *additivity*, and take $x = y = 1$, then we have $f(1+1) = f(2)$ and also $f(1) + f(1) = 2f(1)$, and so our original equation becomes by substitution $f(2) = 2f(1)$ It is obvious that the original equation, $f(x+y) = f(x) + f(y)$, is the source of indefinitely many such relations for particular numbers For instance, $f(3) = f(2+1) = f(2) + f(1)$, but, in accordance with what we obtained before, $f(2) = 2f(1)$, so that $f(3) = 2f(1) + f(1) = 3f(1)$, and in general, $f(x) = xf(1)$ So, if we have an equation $f(x+y) = f(x) + f(y)$ for numbers, we know that we can obtain the value of this function for any x if we know it for 1 If we denote the function of 1, which is a *constant*, by $f(1) = k$, we have the general form of the function which satisfied $f(x+y) = f(x) + f(y)$ expressed by $f(x) = kx$ In words, a functional equation of the above type, namely, a function of the sum equal to the sum of the functions, has only one possible type of solution, namely, when f is equivalent to a multiplication by a constant, or, $f(x) = kx$ But this last means proportionality The values of the function are proportional to the arguments (variables) In fact, let us consider two arguments, that is, two values of the independent variables x and y We have, as shown before, $f(x) = kx$ and

$(y) = ky$. Dividing the first by the second, we obtain $f(x)/f(y) = kx/ky = x/y$ or, in another form, $f(x)/x = f(y)/y = k$

Let us consider an example We know from elementary geometry that if we take an angle α and draw parallels which cut the sides at AA', BB', CC', DD' , the corresponding intercepts are proportional In general, the lengths of the segments on the left side are not equal, neither are they equal to the segments on the right side If we designate the segment AB as x and BC as y, the corresponding segments $A'B'$ and $B'C'$ we may designate as $f(x)$ and $f(y)$, respectively, which means function of x and function of y $A'B' = f(x)$, $B'C' = f(y)$ But the above intercepts are proportional, which means that $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{AB+BC}{A'B'+B'C'}$

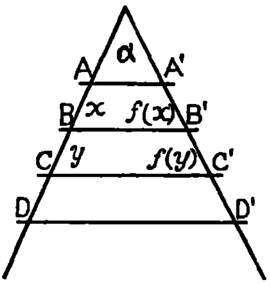


FIG 3

We easily see from Fig 3, that $AB+BC = AC = x+y$, and $A'B'+B'C' = A'C'$ and so $A'C'$ on the one hand is $f(AC) = f(x+y)$ and on the other hand it is $f(x)+f(y)$ and therefore $f(x+y) = f(x)+f(y)$ We could multiply examples by taking relations between central angles in a circle and arcs of its circumference In fact, any problem of *measure* in E geometry could be used as an example

In our development we started with definite *additive* natural tendencies, not only in our highest, yet undeveloped, mathematics, which we call our daily and scientific language, but also in our lowest, but perfected, language which we call mathematics In this perfected language the notion of *additivity* is connected with *linearity*, and the *methods* of *approximation* are also founded on additivity and linearity.

Yet the world around us in its more fundamental structural aspects is not additive, and for *adjustment* we must find means of passing from additive tendencies and formulations to non-additive tendencies and formulations Modern mathematics has developed these methods, and modern physics is beginning to apply them Let us repeat the importance of linear functions implies the importance of 'straight' lines They are important on two counts first, because they are simpler than all other curves, so that naturally we want to study them before we study other curves, such as, for instance, circles or the other conic sections in elementary geometry, and secondly, because all curves can be approximated by straight lines This point is very important, as approximation is the most powerful method we have of handling complicated situations

There are two methods of approximating a curve in the vicinity of a point If we are interested in the immediate vicinity of a point we approximate the curve by its tangent, as the tangent approximates the curve in the vicinity of a point better than any other straight line If we want to decrease the error which we make in this approximation, we have only to decrease the vicinity in which we consider it If we do not want to restrict ourselves to a small neighbourhood we have to use more complicated methods of approximation We inscribe into the curve a broken line which consists of segments of straight lines The beginnings of the study of curves consist in reducing the study of curves to (1) The study of straight lines connected with the curves' tangents, which is the point of departure of the differential calculus, and (2) The study of the inscribed broken lines, which is the point of departure of the integral calculus.

Now curves represent only the simplest dependences In other cases we have more complex kinds of functions, for instance, vector functions, but in every case we have *linear* functions, the simplest of their type, and other functions are studied by approximating them in one way or another by linear functions In using the term 'function', we mean not merely numerical functions but also *operators*, which are to the ordinary functions what ordinary functions are to numbers A general definition of linearity can be connected with that of proportionality in the following manner If two variables are proportional, one to another, then to the sum of any values of the first corresponds the sum of the corresponding values of the second

The simplest part of any field is the consideration of linear, additive questions, linear equations (equations of first degree in algebra), linear differential equations, linear integral equations, linear matrices, linear operators , But sooner or later we come to the more difficult and more interesting non-linear problems Perhaps the main importance of the General Theory of Einstein lies in the fact that the equations of physics become *non-linear* Now, although non-linear equations can be approximated by linear equations, the character of a world determined by non-linear equations must be entirely different from a world determined by linear equations In a linear world electrons would not repel each other but would travel independently of each other, and there could be no relation between the charges of different electrons But we know that electrons do repel each other, and attract protons, and that their charges are equal In physics, if a system can be described by linear differential equations, the causal trains started by different events propagate themselves *without interference*, with simple *addition* of effects

The properties of systems which can be described by linear differential equations have, as we have already seen, the property of *additivity* This means that the result of the effects of a number of elements is the sum of the effects separately, and no new effects will appear in the aggregate which were not present in the elements In such a universe there is 'continuity', fields are superposable, wave disturbances are additive, 'energy' and 'mass' are indestructible , . In such a universe we can have two-valued *causality*, as causal trains started by different events propagate themselves without *interference*, and with simple addition of effects, and the present can be analysed backwards into the sum of elementary events, that is, a two-valued causal analysis is possible

If our equations are not linear, the effects are not additive and a two-valued causal analysis is not possible.

The joint effect of *two* causes working together is *not* the *sum* of their effects separately[2], and we need ∞-valued causality

Analytically, if we have *linear* differential equations and we have one solution $y_1 = f(x)$ and another solution $y_2 = F(x)$ then their sum is also a solution, namely, $y_3 = f(x) + F(x)$ If the differential equations are non-linear and if $y_1 = f(x)$ and $y_2 = F(x)$ are two solutions, then $f(x) + F(x)$ is *not* a solution

Linear problems and linear equations play a very important structural role in science and there is little doubt that linear equations preponderate enormously, although many fundamental events cannot be described by such equations. A universe which can be described by linear differential equations of the second order has definite structural characteristics—in the main in rough accord with observation As such differential equations give us the tendency of a process, we may use them to describe large-scale phenomena by integration, or the statistical phenomena of great numbers

Unfortunately, the study of non-linear problems is structurally very difficult and largely a problem of the future

There is one very important point which we should not miss We know already that there is a fundamental difference between different orders of abstractions Physical abstractions have always characteristics left out, and our higher order abstractions are further removed from life, but they have all characteristics included The problem of sanity being a problem of adjustment, we must somehow correlate these abstractions in which characteristics are *left out* with those which include all characteristics, and so *must* proceed *by approximations* Mathematical methods, particularly those of the differential and integral calculus, have evolved the best technique of *approximation* in existence today, which, as we have seen, is strictly connected with *linearity* or *additivity*.

A similar urge which prompted us in the expression of our additive tendencies and methods in the structure of language, has led to the production of the calculus For organisms which abstract in so numerous and such different orders, the methods of the calculus are therefore fundamental psycho-logical devices, conditioning sanity

In conclusion, we should notice two quite important facts One of these is that the nervous system, being in a state of nervous tension, cannot structurally be a simple additive affair in all its functions, a fact which every one of us has experienced Too many stimulations dull, or abolish, or change reaction in an enormous variety of ways Piéron, as a result of experimenting in association, has not only shown the complexity of these processes, but also reaches the conclusion that the associative connections are non-linear [3] The other most important point is that structurally the term 'and' implies addition When we confuse orders of abstractions or levels of analysis, the 'and' additive implications falsify the issues Thus for instance two atoms of hydrogen *and* one atom of oxygen *and* a spark produce water. The second 'and', at least, is used illegitimately, as it applies to an entirely different level (the spark) from that of the atoms. Linguistically we introduced additive implications, while empirically we are dealing with most complex non-additive, non-linear higher-degree functions When we confuse orders of abstractions, as we all do, the 'and' is bound to introduce structurally false implications, which it is very difficult to avoid—the more so since these semantic problems are generally entirely neglected.

CHAPTER XXXIV

ON GEOMETRY

At the same time it will not be forgotten that the physical reality of geometry can not be put in evidence with full clarity unless there is an abstract theory also . Thus, for example, while the term electron may have more than one physical meaning, it is by no means such a protean object as a point or a triangle (259) OSWALD VEBLEN

Euclidean space is simply a group (417) HENRI POINCARÉ

It is only in Euclidean "gravitationless' geometry that integrability obtains (551) HERMANN WEYL

The fundamental fact of Euclidean geometry is that the square of the distance between two points is a quadratic form of the relative co-ordinates of the two points (*Pythagoras' Theorem*) *But if we look upon this law as being strictly valid only for the case when these two points are infinitely near, we enter the domain of Riemann's geometry* (547) HERMANN WEYL

.. *parallel displacement of a vector must leave unchanged the distance which it determines Thus, the principle of transference of distances or lengths* which is the basis of metrical geometry, carries with it *a principle of transference of direction,* in other words, **an affine relationship is inherent in metrical space** (547) HERMANN WEYL

But before dealing with the brain, it is well to distinguish a second characteristic of nervous organization which renders it an organization in levels (411) HENRY PIÉRON

Section A Introductory.

The main metrical rule in geometry is the familiar pythagorean theorem In 1933 this rule is no longer considered as generally valid outside of the euclidean system, as its proof depends on the doubtful postulate of parallels. It is considered as an empirical generalization in which the relative error decreases when the distances become smaller Indeed the small element of length, *ds*, given by the pythagorean rule is considered convenient and reliable in our exploration of the world

The pythagorean rule states that in any right triangle, ABC, the square of the side opposite the right angle (the hypotenuse) is equal to the sum of the squares of the two other sides (the legs). In symbols, $AB^2=AC^2+BC^2$ If we build squares on all three sides of the triangle ABC and denote the areas of the squares by C', A', and B' then we have $C'=A'+B'$.

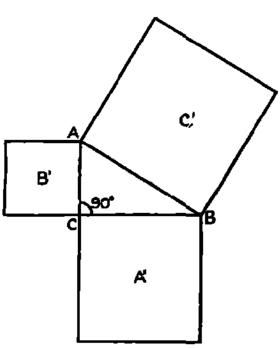


FIG. 1

The above rule is also the main metrical rule for co-ordinate geometry, which gives us the length of the line segment joining any two points. Consider, for example, two points in two dimensions, P_1 and P_2, whose co-ordinates referred to a pair of axes in the plane are (x_1, y_1) and (x_2, y_2) By drawing the lines P_1Q and P_2Q parallel to the X and Y axes respectively, a right triangle P_1QP_2 is formed whose legs P_1Q and P_2Q are equal to x_2-x_1 and y_2-y_1 respectively, whence P_1P_2, the hypotenuse of the right triangle, or the distance s between the points, is equal to $\sqrt{P_1Q^2+P_2Q^2}$, or $s=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ If we pass to indefinitely small quantities and choose to deal with differentials we have $ds^2=dx^2+dy^2$ where $dx=x_2-x_1$ and $dy=y_2-y_1$ Usually the physicists treat their differentials as very small quantities and we may do likewise, although this is not precisely what a differential represents

In three dimensions similar formulae appear, namely, $s^2=x^2+y^2+z^2$ for the distance of a point from the origin and $s=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$ for the distance between two points P_1 (x_1, y_1, z_1) and P_2 (x_2, y_2, z_2), and also $ds^2=dx^2+dy^2+dz^2$, for the indefinitesimally small distance between two points

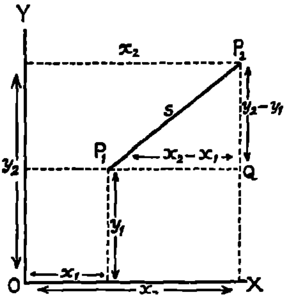


FIG 2

In referring our geometrical entities to co-ordinate axes, or frames of reference, as they are called, we are interested in the properties of our geometrical entities and not in the accidental characteristics of our frames of reference, or the accidental characteristics of the form of representation we are using. Mathematicians discovered long ago that the form of representation is not of indifference to the results they obtain Speaking roughly they have discovered that in one form of representation, they obtained characteristics $a, b, c, d,$ $m, n,$ in another form, characteristics $a, b, c, d,$ $p, q,$ and in still another form, characteristics $a, b, c, d,$ s, t , In cases where direct inspection was possible they find by checking up predicted characteristics, that such characteristics as a, b, c, d in our example actually belong to the subject of our analysis, whereas the characteristics $m, n,$ $p, q,$ $s, t, .$, do *not* belong to our subject at all, but *vary* from one form to another depending on the form of representation Such facts make mathematicians distinguish between characteristics which are *intrinsic*, which actually belong to the subject independently of the form of representation, and those which are *extrinsic*, which do not belong to the subject, but are accidental and vary with the form of representation we happen to use

If we mix intrinsic and extrinsic characteristics we have a structurally distorted knowledge of our subject Obviously we are interested in methods by which these two types of characteristics can be separated and distinguished

Such methods are found in what we call the transformation of co-ordinates, which means the passing from one form of representation to another, from one system of co-ordinates to another which corresponds to translation from one language to another Obviously those characteristics which are intrinsic to our subject are and must be *independent* of the accidental selection of our form of representation, and therefore should remain unchanged when we pass from one frame of reference to another Any characteristic which is changed by such a transformation of our systems of reference is clearly an extrinsic characteristic injected by the form of representation and not belonging to our subject, and so the transformation of co-ordinates is precisely the test we need and use.

Let us take for instance the line segment P_1P_2, as in Fig 3 We may refer P_1P_2 to a system O, or to a system O' Obviously the length of the line P_1P_2 is independent of the axes of reference used, and the formula for the length of a line is not altered, although the values for the x's and y's are different in the two systems In other words, the sum of the squares of the differences of the co-ordinates remains invariant In symbols,

$$s=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(x_2'-x_1')^2+(y_2'-y_1')^2}$$

Such expressions, however, as x_1+x_2 or y_1y_2 , are *not* characteristics of our subject but characteristics of the particular frame of reference used, and so are mostly of no interest to us

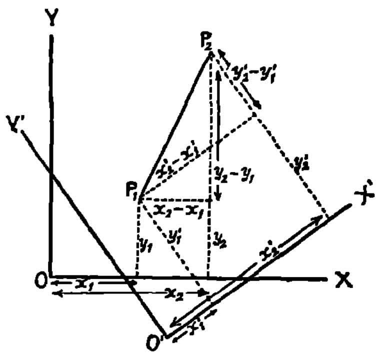


FIG 3

In such an elementary example as given here we are directly acquainted with our entities, and so we can inspect them directly and check for intrinsic and extrinsic characteristics But when we deal with geometries of more than three dimensions, such checking becomes very difficult, if at all possible, and so new methods have to be invented

If we wish to eliminate the unit by which we measure our lines, this can be done by using a relation called a *ratio* Let us, for instance, select 3 points A, B, C, and write the invariant formulae for the distance AB and AC in the form $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ and $\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}$

then the *ratio*

$$R=\frac{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{\sqrt{(x_3-x_1)^2+(y_3-y_1)^2}}$$

is independent of our unit of measurement If, for instance, this ratio $R=1$, we conclude that $AB=AC$, a characteristic which belongs to our lines and which is *independent not only of our system of reference but also of the unit which we have used*

A great step forward in the formulation of methods which lead to invariant and intrinsic formulations was made in the invention of what is called the vector calculus and its extension in the modern tensor calculus A few explanations of this principle will be of interest

A vector is roughly a directed segment of a straight line on which we distinguish the initial and the terminal points A vector has thus magnitude and direction. In practice we deal with two kinds of entities; some are purely numerical, establishing a specific, mostly asymmetrical, relation and have no direction, as for instance, mass, density, temperature, energy, electrical charge, population, mortality , These quantities which do not involve direction are called scalar quantities

Such quantities as velocity, acceleration, electric current, stresses, flow of heat or fluids , which involve not only magnitude but also a definite direction, are called vector quantities, and have given rise to a special calculus called the vector calculus

The invention of the vector calculus was a most revolutionary and beneficial structural and methodological step It was originated independently by Hamilton and Grassmann The benefits of this method are manifold, but we are interested mainly in but two of them. The first is that vector equations are simpler and fewer in number than co-ordinate equations The second, and most important, is that the language of vectors is independent of choice of axes, and of frames of reference It is naturally invariant for any transformations of axes If axes are needed we can easily and simply introduce them, but we always have means to discriminate between intrinsic and extrinsic characteristics The modern tensor calculus which made the general theory of Einstein possible is simply an extension of the vector calculus

The above methodological and structural remarks are of fundamental semantic importance to us in all our affairs Human life and affairs are never free from linguistic issues Their role is similar to that of *mathematics*, that is to say, a form of *representation* gives us not only the characteristics which are intrinsic in our subject, but also introduces extrinsic characteristics which do not belong to the subject of our analysis but are due to the particular language we use and its *structure* The analysis of these linguistic issues is much belated and extremely difficult because of the structural complexity of our language These issues were discovered first in mathematics because of its clear-cut structural *simplicity*, and it is important that we should be aware of such new and unexpected fundamental semantic problems We will not enlarge upon this phase of the problem here, except to mention that the whole of the present work, which uses a different language, of a different *structure*, already shows the usefulness of the new method Sometimes we discover new characteristics, and sometimes we are led to emphasize characteristics which are known but have not yet been sufficiently analysed

To carry our linguistic analogy further, we may take, for instance, the statement, 'knowledge is useful' We could translate this statement into any other language and it would preserve its meaning. But if we make the statement, 'knowledge is a word which has six consonants and three vowels', such a statement may be false when translated into another language Mathematics being a language, has difficulties similar to ordinary language, but in mathematics it is often much more difficult to separate from other statements those which are purely about the language used The so-called tensor calculus attempts to perform this last task.

The tensor calculus is an extension of the vector calculus, which has become famous since Einstein It gives us formulations independent of any special frame of reference In using it we are automatically prevented from ascribing to the events around us characteristics which do not belong to them The tensor equations give us absolute formulations, absolute being understood as relative, no matter to what Obviously the only language fit to express the 'laws of nature' should be independent of the particular point of view or language of some observer It should give us formulations invariant for any and all systems of reference, although we might use preferred systems of reference, as, for instance, the principal axes of an ellipse, without any danger The reader should not miss the point that such an ideal should be considered as the highest ideal in science. It is the mathematical species of a theory of 'universal agreement' The above *sounds* simple and innocent, but, when actually applied, plays havoc with most of our old 'universal laws' These laws do not survive this important and uniquely valid test, and so become mere local gossip instead of being the 'universal laws' that they claim to be We will return to the structural problem of invariant formulations later At present we must explain some other simple considerations

On any surface we need two numbers or 'co-ordinates' to specify the position of a point, and so a surface is called a two-dimensional manifold Points in three-dimensional manifolds require three numbers, points in four-dimensional manifolds four numbers, and similarly for any number of dimensions.

For our purpose, it is enough to speak in two dimensions, as our statements can easily be generalized to any number of dimensions If we want to localize a point on a surface it is enough to divide the surface into *meshes* by any two line-systems which cross each other By labeling the lines of each system with consecutive numbers, two numbers, one from each system, will specify a particular mesh. If the meshes are small enough we will be able to locate any point accurately

These specifying labels or numbers require that we know what kind of mesh we are using *Distances* between points are *independent* of mesh systems.

For the above reasons it is important to have more data about the mesh system we are using which means that we have formulae which express the *distance* between two points, which is independent of the mesh systems, in terms of the mesh system.

We have already seen, in our study of the differential calculus, that, as a rule, it is simpler to deal with very short distances, and that it is easy to pass to larger distances by the process of integration As yet we have used only plane rectangular systems of meshes in our illustrations, but this restriction is not necessary If we use oblique co-ordinates (Fig 4), the formula for the elemental distance is $ds^2=dx_1^2-2kdx_1dx_2+dx_2^2$, where k = cosine of the angle between the lines of partition

The polar co-ordinates (Fig. 5) of the point P are the distance $r=OP$, of the point from the origin O, and the angle, $\theta=\angle XOP$, between the line OP

and the axis OX The formula for the elemental distance in polar co-ordinates is $ds^2 = dr^2 + r^2 d\theta^2$

Fig 6 shows the co-ordinates frequently used by geographers, namely, geographic longitude and latitude, where the distance $ds^2 = d\beta^2 + \cos^2 \beta d\lambda^2$.

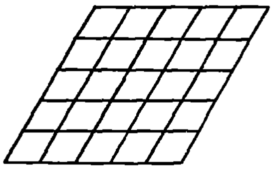


FIG 4

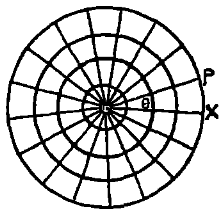


FIG 5

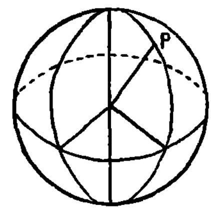


FIG. 6

It should be noticed that these formulae for different systems of co-ordinates are *different* To make it still more obvious to ocular inspection, we will tabulate them in one lettering, thus

$$\begin{aligned}
ds^2 &= dx_1^2 + dx_2^2 && \text{for rectangular systems} \\
ds^2 &= dx_1^2 + x_1^2 dx_2^2 && \text{for polar systems} \\
ds^2 &= dx_1^2 - 2k dx_1 dx_2 + dx_2^2 && \text{for oblique systems} \\
ds^2 &= dx_1^2 + \cos^2 x_1 dx_2^2 && \text{for latitude and longitude systems}^{[1]}
\end{aligned} \qquad (1)$$

It must be noticed that the values for the variables are not equal in these different equations It is not necessary for the reader to know in detail how these formulae are obtained, but it is necessary to see that they are *different*, that they have different structure The numbers of different co-ordinate systems we can use are infinite, but in practice we use only a few well-known types There are also definite and simple formulae for passing from one system of co-ordinates to another

We should not assume that in practice we always know what system of co-ordinates we are employing For instance, before we learned that our earth is 'round', we did not know whether in our measurements we were employing the flat co-ordinates of a plane or spherical co-ordinates We made some measurements and then we had to discover what kind of formulae would fit these measurements

To find out what kind of co-ordinate system we are using, we select two points, let us say (x_1, x_2) and (x_1+dx, x_2+dx) very close together, make our measurements of ds, and then test our ds to find which formula it fits If we find for instance that our ds^2 is always equal to $dx_1^2 + dx_2^2$ we may assume for simplicity and our purpose that our co-ordinate system is plane and rectangular

If our measurements fit any of the first three formulae (1), we may assume for simplicity and our purpose, that we are dealing with a plane surface, as each of these systems belongs to the plane But if we find that the actual measurements of ds^2 are such that they never fit these first three formulae, but only the fourth one, we know, that our surface is not plane but curved like a sphere. Try as we may, we shall be unable to build on a plane any co-ordinate system which will fit the last formula. Thus we arrive at an important conclusion, namely, that from *measurements* we have a *structural* hint as to the *kind of world* we are in.

Section B. On the notion of the 'Internal Theory of Surfaces'

Let us imagine some two-dimensional beings confined to their surface and unable to have a look at that surface from our third dimension. For them our third dimension would be 'unthinkable', and therefore the surface of a sphere like our earth which is curved in the third dimension would also be 'unthinkable' or 'beyond them'. Should they conduct some measurements in their 'world' and find that these measurements did not fit any of the first three formulae but only the fourth, they would have to reconstruct radically their 'world conception' and conclude that their world was a spherical surface. Our own situation does not differ radically from the situation of the inhabitants of this hypothetical two-dimensional world.

If we find ourselves so restricted as not to know whether we are finally dealing with a flat or spherical surface, we can select a point O and with a definite radius R describe from this point a circle ABC. Then we can measure the circumference of this circle. Now we know from geometry that **in *the plane*** the circumference of the circle $L = 2\pi R$ where R is the radius of the circle and $\pi = 3.1415$ . . . If our surface is flat ($ABCD$), our measurement of L and R will satisfy the relation expressed in the formula. But if the surface is curved, our $R = OA$ will be larger than $R' = AO'$, and we shall find that our π is not 3.1415 , but smaller. We see once more that the *metrical properties of our world throw some light on its structural character.*

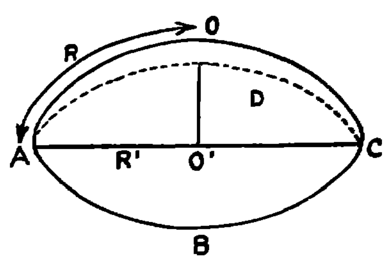


FIG 7

We should notice also that the curvature of a two-dimensional surface is in the third dimension and that it is the means of giving us data about the surface without our leaving the surface and going into a third dimension. It is easy to convince oneself about these facts by taking 12 wires or strings of equal length and constructing the figure shown in Fig 8. If we build it on a flat surface the 12 equal wires will fit exactly. But if we try this experiment on a curved surface, for instance on a pillow-, or saddle-shaped surface, the last closing wire will not fit, and will be too short or too long depending on the kind of surfaces we have.

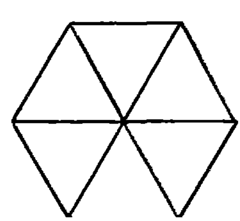


FIG. 8

The formulae (1) have been generalized to

$$ds^2 = g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + g_{22}dx_2^2 \tag{2}$$

for two dimensions and to

$$\begin{aligned} ds^2 = g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 + g_{44}dx_4^2 + 2g_{12}dx_1dx_2 \\ + 2g_{13}dx_1dx_3 + 2g_{14}dx_1dx_4 + 2g_{23}dx_2dx_3 + 2g_{24}dx_2dx_4 + 2g_{34}dx_3dx_4 \end{aligned} \tag{3}$$

for four dimensions It is easy to see that $ds^2 = dx_1^2 + dx_2^2$ is obtained from (2) by taking $g_{11} = 1$, $g_{12} = 0$ and $g_{22} = 1$ This applies to formula (3) out of which we can have any of the other formulae by equating some of the g's to zeros, or to one or to other values Formula (3) is called 'the generalized pythagorean rule', of which the ordinary form as given previously is only a particular case We see, by comparing the formulae (1) with (2) and (3), that these g's are not equal for different systems of co-ordinates, and that they are factors in measure-determination which represent the geometry of the surface considered It is customary to write the above formulae in an abbreviated form thus, $ds^2 = \sum\sum g_{mn}dx_m dx_n$, where we give to m and n the values 1, 2, 3, 4, or, (m, $n = 1$, 2, 3, 4) and where the symbol $\sum$ means summation

We will now explain briefly the above generalizations and the meaning of the g's given in the expressions

In the beginning of the nineteenth century the mathematician Gauss formulated the *internal* theory of surfaces without reference to the plenum in which they are embedded This theory perhaps is and will remain a model on which all theories should be built He introduced also a new kind of co-ordinates which have become of paramount importance, and which since Einstein are called gaussian co-ordinates Gauss investigated the theory of surfaces, which are in general curved, embedded in three-dimensional 'space' In 1854 the great mathematician Riemann generalized the two-dimensional gaussian theory to a continuous manifold of any number of dimensions Historically, both Gauss and Riemann can be considered as the precursors of Einstein.

Let us imagine a surveyor to have the task of mapping a thickly wooded hilly region Because of the conditions of his work, he can not use optical instruments, and he has no 'straight lines' to deal with So euclidean geometry will, in general, not be applicable to the region as a whole. It can be assumed, however, that euclidean geometry may be applied to *very small* regions which can be considered flat What we know already about the differential and integral calculus shows us that such approximations, when taken on a very small scale, are perfectly reliable and justifiable

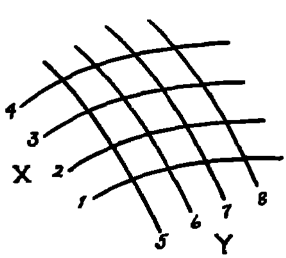


FIG. 9

The surveyor would lay out on his ground a network of smoothly curving lines, in two families, an X family and a Y family. (Fig. 9) All the curves of the X family would intersect all the curves of the Y family but no X curve would intersect another X curve, nor a Y curve another Y curve

Let us take the surveyor's network and label the curves by consecutive numbers in each family The essential point is that these numbers, (let us call them the X and Y numbers) do *not* represent either lengths, or angles or other measurable quantities, but are simply labels for the curves, much as when we label streets by numbers

But such numbering does not lead us far We must introduce some ***measure relations*** We have at our disposal a measuring chain and the *arbitrary* meshes of the network which we have introduced. The next step is to measure the small meshes one after another and plot them on our map. When this is done we have a complete map similar in structure to our region. Because of the smallness of the meshes we can consider them as small parallelograms, and such parallelograms can be defined by the lengths of two adjacent sides and one angle.

We may however, proceed differently, as shown on Fig. 10

Let us select one mesh, for instance the one bounded by the curves, 3 and 4, and by the curves 7 and 8 Let us consider a point P within the mesh, and let us denote its distance from the point O ($x=3$, $y=7$) by s This distance could be directly measured Let us draw from the point P parallels to our mesh lines and label the intersections with mesh lines by A and B, respectively. Let us also draw PC perpendicular to the x axis

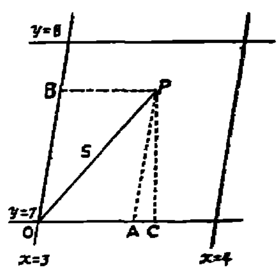


FIG. 10

The points A and B then also have numbers or labels or gaussian co-ordinates in our network. The co-ordinate of A may be determined by measuring the side of the parallelogram on which A lies and the distance of A from O We can regard the relation called the *ratio* of these two lengths as the *increase* of the x co-ordinate of A towards O. We shall denote this increase itself by x, choosing O as the origin of the gaussian co-ordinates. Similarly we determine the gaussian co-ordinate of y of B as the ratio in which B cuts the corresponding side We see that these two *ratios* which for brevity we call x and y are the co-ordinates of our point P.

As x and y are *ratios* they of course do not give us the lengths of OA and OB but the lengths are given, for example by ax, and by, where a and b are definite numbers, to be found by further measurements. If we move the point P about, its gaussian co-ordinates change but the *numbers* a and b which give the ratio of the gaussian co-ordinates to the true lengths *remain unchanged.*

We find the length s which is the distance of the point P from O, from the right triangle OPC by the pythagorean rule $s^2=OP^2=OC^2+CP^2$. But $OC=OA+AC$ and therefore by substituting and squaring we have $s^2=AO^2+2OA\ AC+AC^2+CP^2$. From the right triangle APC we have $AC^2+CP^2=AP^2$, whence substituting again we have $s^2=OA^2+2OA\ AC+AP^2$. But $OA=ax$, $AP=OB=by$, and as AC is the projection of $AP=by$, it also has a *fixed ratio* to it, whence we may put $AC = cy$, and so we obtain the important formula $s^2 = a^2x^2 + 2acxy + b^2y^2$, in which a, b, c are *ratios* given by *fixed numbers* Usually this formula is represented differently, a^2 is designated by g_{11}, ac by g_{12}, and b^2 by g_{22}; whence our formula becomes $s^2 = g_{11}x^2 + 2g_{12}xy + g_{22}y^2$ in which the numbers 11, 12 and 22 are simply ordering labels without quantitative values, mere subscripts, labels, indices , which indicate that the different g's have different values We see that the above formula is the one which was given previously by (2)

The g's with different labels serve just as sides or angles for the determination of the actual sizes of the parallelograms and we call them the *factors* of the measure *determination* They may have different values from mesh to mesh, but if they are known for every mesh, then, by the last formula, the true distance of an arbitrary point P, within an arbitrary mesh from the origin can be calculated [2]

The procedure by which we can locate any point on the surface is simple If our point P is between the two curves $x=3$ and $x=4$ we can draw nine curves between these two curves and label them 3,1, 3,2, ; 3,9 If P now lies between curves 3,1 and 3,2 we can draw nine curves between these two curves and label them 3,11, 3,12, , 3,19 , We could do similarly with the y curves and in this way we would succeed in assigning to any point as accurate a pair of numerical labels as we pleased, and so finally have the gaussian co-ordinates of any point We used nine curves simply to get the very convenient decimal method of labeling The cartesian co-ordinate systems which we use in plane geometry obviously represent only *special cases* of gaussian systems

As we have already seen, our g's are *ratios*, and so represent numbers Such numbers may be regarded as tensors of zero rank for convenience of the mathematical treatment, and the quantities g_{xx}, g_{xy}, g_{yy}, may be treated as components of a tensor Since this tensor determines the measure relations in any particular region, it is called the *metric fundamental tensor* Its value must be given for the region in which we want to make our calculations It determines the full geometry of the surface in a given region, and, conversely, we can also determine the fundamental tensor in a given region from measurements made in that region, without any previous knowledge of how our curved surface is embedded in 'space' at the place in question The fundamental tensor in general varies continuously from place to place, and so every geometric manifold may be regarded as the field of its metric fundamental tensor

Purely mathematical investigations show that the fundamental tensor defines a number called the 'Riemann scalar', which is completely independent of the co-ordinate system and leads to the definition of the *curvature tensor*, which can be connected with the 'matter tensor' [3]

The main importance of the introduction of such arbitrary curves is to produce formulae for the surfaces which remain unaltered for a change of the gaussian co-ordinates—in other words, which remain invariant This was achieved by the introduction of the relations called *ratios* which are pure numbers, and so the geometry of surfaces becomes a theory of invariants of a very general type.

On curved surfaces there are in general no straight lines—there are *shortest lines*, which are called 'geodetic lines.' To find them, we divide any arbitrary lines joining two points into small elements, which we measure, and select the line for which the sum of these elements is less than for any other line between the two points.* Analytically we can calculate them, when the g's are given, by the aid of the generalized pythagorean theorem. The geodetic lines, and also the curvature, are given by invariant formulae, which represent intrinsic characteristics of the surface independent of any co-ordinates. All higher invariants are obtained from these invariants.[4]

We shall not attempt to give an explanation of the tensor calculus, as at present there is no elementary means of presenting a brief explanation; short of a small volume—at least the writer does not know of any.[5]

The name 'tensor' originally came from the Latin word *tendere* = to stretch, whence *tensio* = tension. Nowadays, however, it is used in a more general way; namely, to express the *relation* of one vector to another, and not necessarily to imply stress or tension. As an example, we can give the representation for stresses occurring in elastic bodies, which originally led to the name.[6]

As we have already seen, when we deal with *relations* of vectors our expressions become additionally independent of units. Such equations, independent of the measure code, are called tensor equations.[7]

As we are interested in equations which are invariant under arbitrary transformations, certain functions, called tensors, are defined, with respect to any system of co-ordinates by a number of functions of these co-ordinates, called the components of the tensor, from which we can calculate them for any new system of co-ordinates. If two tensors of one kind are equal in one system, they will be equal in any other system. If the components vanish in one system, they vanish in all systems. Such equations express conditions which are independent of the choice of co-ordinates. By the study of structural laws of the formation of tensors we acquire means of formulating structural laws of nature in generally invariant forms. Obviously, such methods and language are uniquely appropriate for physics and the formulations of the laws of nature. If a law cannot be formulated in *some such form*, there must be something wrong with the formulation and it needs revision.

The tensor calculus is also peculiarly fitted to describe processes in a *plenum*. We do not use it to describe the metrical conditions but to describe the *field* which expresses the physical states in a metrical plenum.

Eddington gives an excellent example of the fact that it is definitely necessary to look into the way we build up our formulae (structure) and the method of handling them.

*More generally, the geodetic represents a track of minimum or interval-length between two distant events, either of them being unique (one-valued) in a given case.

The problem is to determine whether a particular kind of space-time is possible We must investigate the different g's which give us different kinds of space-time, and not those which distinguish different kinds of mesh systems in one space-time This means that our formulae must not be altered in any way if we change the mesh system

The above condition makes an extraordinarily simple test of laws that have been or may be suggested Among others, Newton's law is swept away How this happens can be shown in two dimensions

If in one mesh system (x,y) we have $ds^2=g_{11}dx^2+2g_{12}dxdy+g_{22}dy^2$, and in another system (x',y') $ds^2=g_{11}'dx'^2+2g_{12}'dx'dy'+g_{22}'dy'^2$, one law must be satisfied if the unaccented letters are replaced by accented letters Let us suppose that the law $g_{11}=g_{22}$ is assumed We change the mesh system, for instance, by spacing the y lines twice as far apart, that is, we take $y'=y/2$ and keep $x'=x$ Then $ds^2=g_{11}dx^2+2g_{12}dxdy+g_{22}dy^2=g_{11}dx'^2+4g_{12}dx'dy'+4g_{22}dy'^2$. We see that $g_{11}'=g_{11}$ and $g_{22}'=4g_{22}$ Whence if g_{11} is taken equal to g_{22}, g_{11}' *cannot* be equal to g_{22}'

A few examples would convince us that it is extremely easy to change a formula entirely by the mere change of mesh systems It seems unnecessary to emphasize the fact that 'universal laws', to be 'universal', should not depend structurally to such an extent on the accidental and, after all, unimportant, choice of reference systems [8]

To remedy such a state of affairs, impossible in mature science, the tensor calculus was invented The whole general theory of Einstein seems to demand that the equations of physics should ultimately be expressed in tensor forms, in other words, that 'universal laws' should cease to be 'local gossip', a demand which must be granted, and *on this point* the Einstein theory is beyond criticism and is an epochal methodological advance of an irreversible structural linguistic character

Section C Space-time

FIG.-11

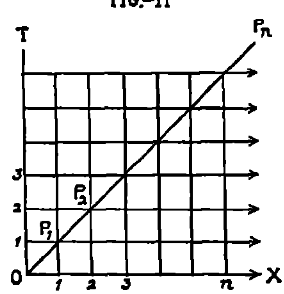


FIG 12

In dealing with co-ordinate systems we have heretofore used them to represent only 'spatial' entities, spreads of different dimensions It is desirable to become acquainted with a different use of co-ordinates, in which one of them will represent 'time'. The last use is just as simple as the former, but the graphs which we obtain are different

Let us take the simplest example, of a point P moving uniformly along a straight line OX with the velocity of one inch per second We could represent its movement in one dimension, as in Fig 11, and *say* that our point P is at P_1 after one second ($t=1$), at P_2 after two seconds ($t=2$) , at the point P_n after n seconds ($t=n$)

But we could also represent this movement in a different way We could choose two mutually perpendicular axes OX and OT as in Fig 12, OX representing the 'spatial' actual direction of the movement and OT, which we have heretofore used to represent a second 'spatial' co-ordinate, now representing the 'time' co-ordinate

We would lay off on the X axis our inches, 1, 2, 3, . . n, and on the T axis our seconds 1, 2, 3, n In our two-dimensional *space-time* our point P would be at the point O ($x=0$, $t=0$) After one second it would be at the point P_1 ($x=1$, $t=1$), after two seconds at the point P_2 ($x=2$, $t=2$) , after n seconds at the point P_n ($x=n$, $t=n$). We see that the position of our *point* P in two-dimensional *space-time* would be represented by a series of points each given by two data· one 'spatial', the other a corresponding 'time' If the intervals are taken indefinitely small, in the limit our *'moving' point would be represented by a static line inclined to the X axis.* We could then speak either of our 'moving' point, or else *not use* the term 'moving' but speak of infinitely many *static points*, each given by two numbers, one representing a distance, the other 'time' Our 'moving' *point* would become a *static world-line.* The reader should notice that in this case we have structurally changed our *language* from dynamic to static, and raised the dimension Our mathematical 'moving' 'point', which had no dimension in our one-dimensional 'space', is in our two-dimensional *space-time* represented by a *static* one-dimensional *line*

In this example we had uniform translation. We did not introduce acceleration The distances were proportional to the 'times', hence our line was 'straight' and inclined to the X axis at a constant angle

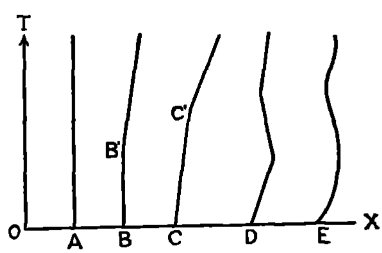


FIG 13

Using such space-time representation we see that a point when it is not 'moving', but is stationary, is represented by a line parallel to the 'time' axis T, as shown at A on Fig 13 Our point A is getting older, so to speak, but does not 'move' In the next case, the point B does not 'move' until it is some seconds old, when at B' it begins to 'move' with constant velocity. Point C 'moves' in the beginning at one constant velocity until C' where it acquires a certain different velocity and the direction changes

In Fig 13, D represents a point experiencing a series of sudden changes of velocities The graph is a succession of short straight lines forming a broken line or open polygon As the changes of velocity occur more and more frequently the sides of our polygon become smaller and smaller, and in the limit, as the changes of velocity become continuous, our broken line becomes a smooth curve E.

Motion with continuously changing velocity is called accelerated or retarded motion The rate of change of velocity is called acceleration and is represented by the second derivative of the distance with respect to the 'time', symbolically, $A = dv/dt = d^2s/dt^2$

It is important to notice that in space-time an *accelerated* motion is represented by a *curved* line In uniform (constant velocity) motion the distances are proportional to the 'times', and the line is straight and its equation is of the first degree In accelerated motion the distances are *not* proportional to the 'times', the lines are curved and the 'time' element dt enters in the second degree at least, namely, as dt^2

For example, let us study the graph of the motion represented by the equation $x = At^2/2$ which means that the distance x is proportional to the square of the 'time'.

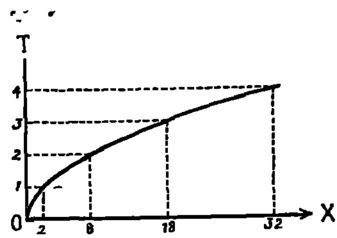


FIG 14

Let OX (Fig 14) be the 'spatial' axis, and OT the 'temporal' axis We lay off on our T axis equally-spaced points, representing the seconds 1, 2, 3, 4 , and calculate the distances x for each of these values from the equation $x = At^2/2$ where A represents a constant acceleration

Let us assume that the constant acceleration A is given as 4 metres per second per second The equation $x = 4t^2/2$ becomes $x = 2t^2$. Corresponding to the values $t = 0, 1, 2, 3, 4$, we have the values $x = 0, 2, 8, 18, 32$, If we plot these points, and assume that the change is continuous, we may join the points by a continuous *curve*, which represents the motion of the point as a *curved* world-line

Similarly, in three-dimensional space-time, a point moving uniformly in the *plane* XY would be represented in the *plane* XY by the *line* AB, and in three-dimensional space-time by the static line AB', where the 'times' are proportional to the distance

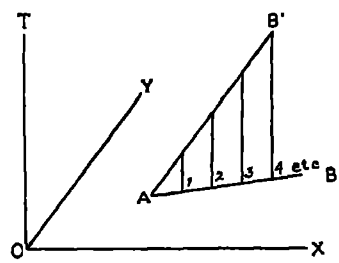


FIG 15

As we have already seen, non-rectilinear motion may be considered as accelerated motion We will generalize the above to the case where any *curved* path is traversed with *constant* velocity In this case the direction of the velocity is changed If we take the motion of a point which describes a circular orbit with *constant* velocity, it is easy to find its acceleration, which is called in this case centripetal acceleration

Let us consider a point P, moving in a circular orbit with a *constant* velocity v, as given in Fig 16 If at a certain 'time' it is at A, after a short interval t, it will be at B The direction of the velocity will be changed from AA' to BB'.

If we construct the triangle DCE by drawing CD parallel and equal to AA', and CE parallel and equal to BB', we see that the angle $\angle DCE$ is equal to the angle $\angle A'A''B'$ because the sides are parallel, and it is also equal to the angle $\angle AOB$ whose sides are perpendicular to AA' and BB'. The triangles ABO and CDE are similar because they are isosceles and the angles between the equal sides are equal. Clearly the side $DE = w$, represents the supplementary velocity which transforms AA' into BB' We know that in similar triangles the sides are proportional so we can write $DE/CD = AB/OA$. By inspection of our figures we see that $DE = w$, $CD = v$, $OA = r$, the radius of the circle. The chord AB may be taken as the arc AB of the circle, provided the 'time'-interval is taken sufficiently small Let us write chord $AB = s$. We have $w/v = s/r$ or $w = sv/r$. If we divide both sides of our equation by t we have $w/t = sv/tr$ But $w/t = A$, the acceleration, and $s/t = v$ hence $A = v^2/r$ In words, the centripetal acceleration is equal to the square of the velocity in the circle divided by the radius

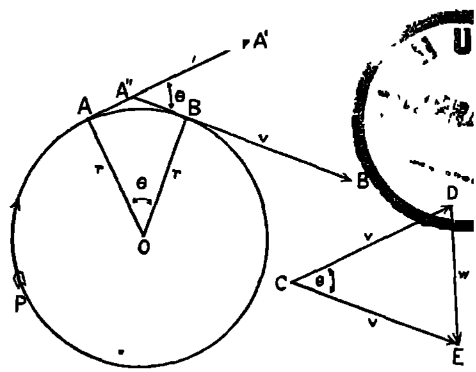


FIG 16

The above formula is of structural importance because it is the foundation for the empirical proof of Newton's law of gravitation For our purpose it is important for other reasons, to be stated later

There are two more diagrams which should be considered, in this connection Fig 17 represents the plane circular motion of a point P whose orbit in the plane XY is the circle PAB In three-dimensional space-time the plane circular orbit of motion would be represented by the static cylindrical helix (or screw-line) with axis parallel to the 'time' axis T (Fig 17) We should note that the motion is *dynamically circular* in the XY plane, yet a three-dimensional space-time representation gives us a stationary helix

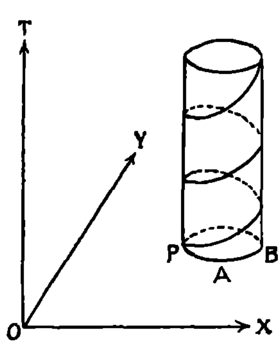


FIG 17

Similarly for vibrational movements which could be represented in one dimension by to-and-fro movements on the X axis from A to B and from B to A (Fig 18) If we introduce our space-time form of representation by introducing the T axis, our vibrational world-line would be represented structurally by a wave-line along the T axis In particular, if the vibrational motion is simply harmonic, a proper choice of the 'time' unit makes the wave-line a sine curve [9]

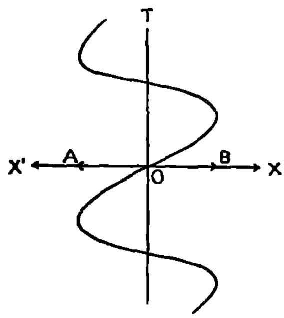


FIG 18

Becoming thoroughly familiar with these few simple examples takes away a great deal of mystery from the Minkowski-Einstein and the new quantum world. We see that after all there is nothing extraordinary in the fact that in languages of different structures we get different forms of representations and pictures, and that in a world where accelerations abound we may very profitably use the term 'curved'

When we come to speak about the Einstein theory, the four-dimensional space-time world of Minkowski, and the new quantum mechanics, we shall have considerable use for these few notions and illustrations

Section D The application of geometrical notions to cerebral localization

In the present work we are dealing in the main with structure and the adjustment of the structure of our languages to empirical structures, and at this point it will be of use to suggest some of the consequences which follow from what has been said

The question of cerebral localization is a difficult and vital problem In former days it was supposed that the brain had individualized centres with strictly defined functions Attempts were made to ascribe to definite cerebral parts definite functions such as 'memory', 'intelligence', 'morality', 'talents' , In the meantime, experimental facts disproved such structural views, and as a reaction another tendency appeared, namely, to deny any localization

Modern researches show unmistakably that both of these extreme tendencies are at variance with experimental structural facts It appears that the lower centres play a more important role according as the terminal, or higher centres, are less developed and that there is considerable variability, at least in man, not only from the morphological and histological aspects but also from the functional aspect It was found impossible to generalize from the particular development of centralization and functional distribution in one species to the distribution in another species Localization may vary even in one individual under different circumstances [10] Metabolism, and slight disturbances in the functioning of a neuron, were also found to have a most far-reaching influence, shown in its relations to other groups of neurons. The problems of localization are far too complex to attempt even an account of them, the more so since the reader will find excellent accounts of them in the large literature on the subject The general conclusion reached by practically all investigators is that some localization of nervous function does exist, yet it has a certain variability which depends on an enormous number of factors

The methods explained in this chapter will enable us to suggest a method by which we can orient ourselves in the bewildering complexity of the functioning of the nervous system

One of the main difficulties is that the structure of this world is such, that it is made up of absolute individuals, each with unique relationship toward environment (in the broadest sense), and we have to speak about it in terms of generalities 'Laws' , formulated in the *old two-valued ways*, can never account adequately for the facts at hand, being only approximations The mathematical methods which have already been explained give us at once a great advantage We have seen that if we have a function, $y=f(x)$, let us say, and take the graph of this function, to every point of the graph there corresponds a pair of values x and y. We have seen also that each of the four quadrants I, II, III, IV has a characteristic pair of signs In quadrant I, both x and y are positive, in II, x is negative and y positive, in III, both x and y are negative, and finally, in IV, x is positive and y negative We can easily see that the value of the variables may be thought of as variable conditions different for each individual, and that definite *localizations* correspond to them In our example we had to do with a function of one independent variable, and we had a one-dimensional line, curved in two dimensions When we had a function of two independent variables we had a surface, which in general was curved in a third dimension By analogy we may pass to any number of dimensions, where by dimension we do not mean anything mysterious, but roughly the number of variables involved in the problem

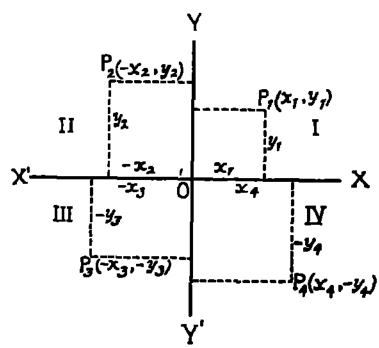


FIG 19

We see that if we think of the activity of the nervous system in terms of a mathematical function with an enormous number of variables, we shall not only have place for the uniqueness of each individual, determined by the value of the variables and the character of the function, but that this would also imply a *localization*, which is *permanent* in a given individual at a given 'time', which again implies the totality of 'circumstances' , Our function would be $N=f(x_1, x_2, x_3, \quad x_n)$

In fact it is hard to see how it is possible to analyse the activities of the nervous system in any other way The facts are, that every organism is an individual, distinct and different from others, and so we must have means to take this individuality into account Different values for different variables take care of this point Similarities are accounted for by the general structural character of the functions For instance, any quadratic equation with two unknowns gives us a conic section An equation of the type $y^2=ax$ represents a parabola, the graph of any equation of the form $xy=a$ represents a hyperbola , For every definite set of values of our variables the implied localization is also definite, which corresponds to the fact that in a given individual at a given 'time' , the localization is definite One value for the whole function can be reached by giving different sets of values to the different variables. For instance, in the function $z=5x-2y-1$, if $x=1$ and $y=1$, then $z=2$, but we can have $z=2$ by taking $x=2$ and $y=3.5$. Or if one of the conditions be non-existent, which means that the value of one of our variables is zero, for instance, $x=0$, we still could have the value $z=2$ by taking $y=-1.5$. This fact accounts for the *many-to-one* correspondences of causal factors, typified, for instance, by sunshine *or* cod liver oil producing a similar effect.

It should be understood that in what is said here, the numerical values do not matter. In most of the cases we are not advanced enough to be able to deal with such numerical values. What is to be emphasized is the structure of the language we use. The method should enable us, instead of dealing with generalizations in the old language, which somewhere have to be contradicted, to use a language of mathematical structure which shall account for the facts and leave room for the great individual varieties of organisms in structure and function.

After all, we should not be surprised that the theory of functions and language of functions is structurally appropriate in expressing, and so in understanding, the *functioning* of the nervous system, or any other system. Personally I have benefitted greatly through this method, and many baffling structural complexities have been much simplified.

Structurally, when we use the language of functions, variables . . . , we automatically introduce *extensional* structure, as already explained, and we have at our disposal methods of translation of different orders of abstractions—dynamic into static, and vice versa—which is a neurological structural necessity for being rational and sane. And surely science should try to be rational. It should be stressed again that in our problem numerical values matter very little, but structure and method, for the many reasons already explained, are of paramount importance. Perhaps even the value of numbers is due mainly to the structural fact that it has forced upon us extensional and relational methods. It is the only language which is in accordance with the structure and functioning of the nervous system, and so helps to co-ordinate these activities instead of disorganizing them.

That these simple structural dependences have been discovered so late is really astonishing. The only explanation I can give of this is that we have been so engrossed in generalizing and generalizations that we lost sight of the fact that in life we deal structurally with ***absolute individuals***, and that the only *language* which preserves the ***extensional structural individuality*** for its elements is found in mathematics—specifically, in numbers.

It may be that a study of mathematical structure and the psycho-logics of mathematics will give results of unparalleled human values, particularly for our sanity. The problems of sanity are problems of adjustment, and no means of adjustment should be disregarded. It may also be that the main importance of mathematics will be found some day to be more in the mathematical *methods* and structure which it has originated, methods forced upon the mathematician by the relational character of the entities he has to deal with, than in the possible combinations of these entities themselves

At any rate, we must sadly admit that the problems of mathematical methods and structure and the psycho-logical values of mathematics have so far received very little attention, since we have failed to realize their human importance. In the future this problem will be further, and thoroughly, investigated.

PART IX

ON THE SIMILARITY OF EMPIRICAL AND VERBAL STRUCTURES

The theory of relativity has resulted from a combination of the three elements which were called for in a reconstruction of physics first, delicate experiment, secondly, logical analysis, and thirdly, epistemological considerations (457) BERTRAND RUSSELL

The essence of Einstein's generalization is its final disentanglement of that part of any physical event which is contributed by the observer from that which is inherent in the nature of things and independent of all observers. (21) E. T. BELL

Even Leibniz formulated the postulate of continuity, of infinitely near action, as a general principle, and could not, for this reason, become reconciled to Newton's Law of Gravitation, which entails action at a distance and which corresponds fully to that of Coulomb (547) HERMANN WEYL

This limitation to what is directly observable is ultimately based on Mach's philosophy and, directly inspired by Mach, led three decades ago to the propagation of the so-called theory of "Energetics," which sought to recognise only quantities of energy as physically given and observable quantities. (481) A SOMMERFELD

CHAPTER XXXV

ACTION BY CONTACT

> The difficulty involved is that the proper and adequate means of describing changes in continuous deformable bodies is the method of *differential equations*. . . . They express mathematically the physical conception of contiguous action. (45) MAX BORN

The analysis of 'matter', 'space', and 'time' from the point of view of structure and of orders of abstractions has led us to far-reaching conclusions. Let us summarize the semantic results, and consider some of the immediate consequences

We may begin by recalling the difference between the lower order and higher order abstractions The lower order abstractions are given to us by the lower nerve centres They are 'dynamic', 'continuous', non-permanent, shifting, unreliable, and above all *un-speakable*

They have a character of immediacy, because, structurally in terms of order, they are closest to outside events They come first in order in the functioning of the nervous system We always associate with them some 'objectivity' as, by necessity, the eventual definition of an 'object' starts at this level

It should be emphasized over and over again that, speaking correctly, *on this level* we cannot define anything, since abstractions on this level are fundamentally *un-speakable* We may look, listen, handle, feel , but *cannot speak* and therefore cannot define. The moment we *define* our objects, we are no longer on the level of lower order abstractions By neurological structural necessity we have passed to the higher nerve centres (speech), and higher order abstractions This is what is meant when we say that this lower level is *un-speakable*

Because these lower order abstractions are closer to the outside events, and because they come *first in order*, they have a special character of immediacy, with which we *must start* The struggle begins when, through some primitive-made doctrines or structural assumptions (metaphysics), we try to avoid going any further than these lower order abstractions As a matter of fact, this is an impossibility, because of the very structure of our nervous system However intensely we believe that it is possible to do so, and however 'emotionally' we attempt to do it, we are cherishing delusions, which easily become *morbid* identifications, delusions, illusions, and often hallucinations

This level being *un-speakable*, the only way to function on this level is to look, listen , but to be *silent* outwardly as well as *inwardly* This last condition represents a most beneficial semantic state, really difficult, perhaps impossible, to acquire without training

The higher order abstractions appear to be products of the activity of the higher nerve centres, further removed from the external events and lacking, therefore, in immediacy But these higher abstractions are static and so may be analysed They have a separable unit 'quantum' structure, which can be treated individually It should be carefully noticed that the static character of these higher order abstractions is the origin of their separable quantum character, conditioned by the human nervous structure They are, if properly treated, reliable and are uniquely responsible for our being time-binders

Again, by the structural necessity of our nervous system, we deal first with lower order abstractions, and next with higher order abstractions It must be noticed that *no one*, unless he is (pathologically) entirely deprived of the higher nerve centres, is, or can be, an exception We all deal with lower abstractions first, and with the higher next, no matter how perfect or imperfect these abstractions may be

The general confusion of orders of abstractions, the lack of theories, and therefore of structural understanding of the entirely different characters of these distinct orders of abstractions, leads to, and must result in, identification or confusion of orders of abstractions As the different processes are going on, whether we will it or not, in every single one of us, they may result in the *delusional* ascribing of the characteristics of the higher order abstractions to the lower order abstractions, as for instance, permanence, immutability , somewhere involving 'infinities'. When objectified, we have such semantic disturbances as fanaticism, absolutism, dogmatism, finalism , which often become morbid semantic states

A similar confusion may lead to the delusional ascribing of the characteristics of the lower order abstractions to the higher ones Under such delusions we ascribe to the higher abstractions fluidity, shiftiness, non-permanence, 'non-knowability' , which results in pessimism, cynicism, disregard for science, bitterness, fright, hopelessness and other equally vicious semantic disturbances These in turn affect by structural necessity the proper working of the entire organism, which always works as-a-whole

The *A*-system and other older systems were not only built before these facts became structurally known but were actually based on such confusion Hence their viciousness By building a language and a method of this nature they perpetuated and made effective *mechanically* through the structure of language, a harmful confusion This language being not in accordance with the structure and functioning of the nervous system and the world, must produce pathological results somewhere

We have already seen that the use of the 'is' of identity is unconditionally delusional Naturally, attitudes (affective, lower order abstractions) which can assert, (higher order abstractions) that so and so on objective levels 'is' so and so, must lead to pathological results In science this is a profoundly unsatisfactory state of affairs and needs structural revision

Mathematicians, though in the main unconscious or innocent of the structural, semantic, and neurological issues involved, nevertheless have solved this problem by producing methods of passing from one order of abstractions to another, from dynamic to static, and vice versa The influence of these discoveries has also affected the other sciences *unconsciously*. Without consciously recognizing it, the modern trend of science is to banish from its habits and methods the application of the 'is' of identity

So in science we have to use an actional, 'behaviouristic', 'functional', 'operational' language, in which we do *not* say that this and this 'is' so and so, but where we describe *extensionally* what happens in certain *order* We describe how something *behaves*, what something *does*, what we *do* in our research work , If one asks, for instance, what *is* 'length', what *is* 'space', what *is* 'time', what *is* 'matter' , the only correct answer would be, 'As you asked the question verbally, and I answer it *verbally*, the above *terms remain terms*, which beside structure, have no connection whatsoever with the external world' Yet undoubtedly we are interested in this external world and we should like to use a language which would help us in understanding this world better What shall we do? It seems that if we produce a language which is *similar in structure*, to the external world, somehow, as a map or picture is similar in structure to the region it pictures, we should have a uniquely appropriate language How can we do it? It is quite simple the moment we discover the principle First of all, abandon completely the A 'is' of identity, and, instead, describe ordered happenings in an actional and functional language Such a language shares with the external world at least the multi-dimensional order of happenings, and it gives us a solution

It is easy to see that arguments (verbal) about 'matter', 'space', 'time' , will never become anything else than verbal All uses of the 'is' of identity, must lead to delusional evaluation The situation is radically changed when we use an actional or functional language, when we describe what a physicist does when he finds his 'length' or 'second' or any other entity he is interested in

We should notice here that the above procedure involves extremely far-reaching structural and semantic consequences First of all, we abandon the vicious use of the 'is' of identity, and eliminate the semantic disturbance called identification We introduce automatically the full psycho-logical working mechanism of *order*, *extensional* methods and discrimination between the orders of abstractions We introduce the four-dimensional and differential methods, we build up static units, 'quanta', and so introduce *measurement* and its language called mathematics, which leads to structure and so to knowledge at each date

It will be useful to recall why mathematics and measurements are somehow so important in our lives Our nervous system, as we have seen, exhibits different activities on different levels On one level the abstractions are shifting, non-permanent, on the other static and permanent in principle This is expressed in our lives in a longing for some permanency, some security, some 'absolutes' Mathematics formulated this tendency first and with *full success* Mathematics has not only formulated full and successful theories of 'change', as, for instance, the theory of functions and the different calculi, but also full-fledged and remarkable theories of *invariance* under transformations These new theories of invariance are actually *absolute* formulations in the only sense in which the term 'absolute' has a meaning, namely, relative, no matter to what; all of which leads to the only content of knowledge—structure.

The whole Einstein theory should, in this sense, be called the 'theory of the absolute', and can be expressed as the simple demand that 'universal laws should be formulated in an invariant form, a most revolutionary demand and yet so *structurally natural* that no one can deny it.

When we mathematize or speak about potential or actual measurements we are dealing with *ordered, extensional*, actional, behaviouristic, functional and operational entities, and so we build up a language which at least has a similar *structure* to the external events Numbers imply units, quanta but also order. It seems that number is the only abstraction upon which we all must agree. We never doubt that a statement, such as that 'I have in my pocket five pennies', may be perfectly definite and ascertainable for all The specific and unique relations called numbers seems to have absolute significance. It must be added that the existence of non-quantitative branches of mathematics does not alter what is said here In these branches, the asymmetrical relation of order remains paramount and we may treat numbers from either of their two aspects, the cardinal or the ordinal

The epoch-making significance of the Einstein-Minkowski work consists precisely in the fact that they were the first to *apply* the above, though without it is true, formulating the general principle The lack of such a general, $\bar{A}$, epistemological formulation retards considerably the understanding of their work, and so laymen miss the enormous structural, and semantic beneficial effect upon the proper working of our nervous system and our sanity

Before giving a short methodological account of the Einstein theory it will be well to recall some structural and semantic conclusions which the differential calculus suggests

When we were dealing with the notion of a variable, we saw that the variable might be *any* element selected out of an ordered aggregate of elements We can select elements relatively widely separated from each other, as, for instance, the numbers 1 and 2, or points, let us say, an inch apart. It is obvious that if we choose, we can make the gaps smaller, and postulate an infinity of intermediate steps When we make our gaps smaller, the elements are ordered more densely and closer together In the limit, if we choose indefinitely many elements between any two elements, our series become compact, if we still have a possibility of gaps, or they eventually become what we call *continuous*, when there are no more gaps

Without legislating as to whether the entities we use in physics are 'continuous', 'compact', or 'discontinuous', we may grant that the maximum elucidation of the above terms in mathematics is very useful We can easily see that in terms of *action* a continuous series gives us *action by contact*, since consecutive elements are indefinitely near each other. As the differential and integral calculus were built on the structural assumption of *continuity*, the use of the calculus brings us in touch not only with our x but also with its indefinitely close neighbour $x+dx$ We see that the calculus introduces a most important structural and semantic innovation, namely, that it is a language for describing *action by contact*, in sharp contradistinction to the structural assumption of action at a distance.

Let us illustrate the above by a structural example. Consider a series of equal small material spheres connected with each other by small spiral springs as shown on Fig. 1.

Fig. 1

These little spheres all have inertia, because of which, and because of the little springs, they resist displacement. If we displace the first of our spheres either in the transverse or longitudinal direction, it acts upon the second sphere, which in turn acts upon the third , . We see that the disturbance of equilibrium of the first little sphere is transmitted like a wave to the next sphere and so along the whole series The most significant point in the analysis of such a wave of excitation is that it is not transmitted with some 'infinite velocity', or 'infinitely quickly' or in 'no time' The action of each sphere is slightly delayed owing to its inertia, that is, it does not respond 'instantaneously' to an impulse It must be noticed that the displacement is not due to a velocity, but to an acceleration, which is a change of velocity and requires a short interval of 'time' The change in velocity again requires an interval of 'time' to overcome inertia and produce displacement Similar reasoning applies to a long train just being started by the engine The cars being coupled together by more or less elastic means, the engine may be moving uniformly and some of the last cars still be stationary The pull of the engine is *not* transmitted instantaneously but with a *finite velocity*, due again to the inertia of the cars

We see that the only structurally adequate means of describing changes in continuous, deformable materials is to be found in differential equations which express a method of dealing with *action by contact*

We have already seen that this action by contact involves also the *finite velocity* of propagation, a fact of crucial structural and semantic importance In the history of science we can distinguish three periods The first was naturally the period of action at a distance, the best exemplified by the work of two great men, Euclid and Newton In it we find of course, a superabundance of 'infinities'. With the advent of the differential calculus, and the introduction of differential equations in the study of nature, the notion of action at a distance became more and more untenable. We had a period of pseudo-contiguous action, which indeed involved differential equations, but the *velocity of propagation* was not introduced explicitly, and so there remained an implicit structural assumption of 'infinite velocity' of propagation As an example of such pseudo-contiguous action we can cite the older theories of potential, which give differential equations for the change in the intensity of the field from place to place, but which do not contain members that express a change in 'time', and hence do not take into account the transmission of electricity with finite velocity.[1]

The modern theories, as for instance, the Maxwell theory of electromagnetism, and the Einstein theory, are based on *action by contact*. These theories not only use the differential method, but they also introduce explicitly the *finite velocity* of propagation.

The invention of differential geometry with the recent contribution of Weyl, which we have already mentioned, transforms the geometry of Euclid from a language of action at a distance into contact geometry, or a language of indefinitely near action.

It should be mentioned perhaps that the riemannian differential geometry is more general than all the $\bar{E}$ geometries which preceded it, and includes them, as well as the E geometry, as special cases. Perhaps, as Weyl points out,[2] the investigation of the famous fifth postulate, which was the beginning of $\bar{E}$ geometry, was accidental in importance and the main structural value of the $\bar{E}$ geometries lies precisely in the application of the differential methods to geometry which was originated by the great work of Riemann. This work, we see, has carried us from metaphysical action at a distance to a physical action by contact. In passing from the older mechanics to electromagnetic events a very striking analogy appears, which explains the finite velocity of propagation.

In mechanics, when we have waves in an elastic medium, the finite velocity of propagation is due to the delay which occurs due to the inertia of materials. Now inertia is determined by acceleration (d^2s/dt^2), which represents the rate of change of the velocity $(v=ds/dt)$, velocity itself being a rate of change of displacement. We see that this retardation, or negative acceleration, is represented by a double differentiation.

Something analogous occurs in electromagnetic events. The rate of change of the electric field (de/dt) determines the magnetic field, and then the rate of change (dh/dt) of the latter determines the electric field at a neighbouring point. The advance of the electric field from point to point is thus conditioned by two differentiations with respect to 'time', which is quite analogous to acceleration.

It is due to this double differentiation with respect to 'time' that the formulation of electromagnetic waves are structurally possible. If the partial effects were to occur without loss of 'time', no propagation of the electric waves would occur. The maxwellian 'field equations' not only express the above-mentioned relations, but introduce structurally the finite velocity of propagation which makes the Maxwell's electromagnetic theory structurally a contact theory.

The Einstein theory is also structurally a contact theory, and it may be said that it was originated by this contact tendency, and has carried it to the limit, as we shall see later. The gaussian theory of surfaces, whose extension to any number of dimensions was made by Riemann, also represents action by contact. This theory does not state the laws of surfaces on a large scale, but only their differential properties, the coefficients of the measure determination, the invariants which we can form, and the curvature and its measure. The form of a surface and its characteristics can then be calculated by a process similar to the solution of differential equations in physics.

We are now in a position to understand why the newer physics and the $\bar{N}$ systems, which are built entirely on the foundations of action by contact, found the E-system unsatisfactory. The E-system was built on the structural assumption of action at a distance, and we had to select the $\bar{E}$ geometries as originated by Gauss, Lobatchevski, Riemann, and others, which gave to physics the necessary geometry of action by contact.

But the question of action at a distance versus action by contact has also an experimental aspect which makes the latter theory more satisfactory.

Faraday (1791-1867) was not a learned academician, and he was much freer from scientific prejudices than any of his contemporaries. From a book-binder's apprentice he became through his genius one of the founders of modern physics. His method of experimenting was to try every possible experiment and note what happened.

In 1838 Faraday made an important structural discovery, namely, that the mutual action between two electrically charged bodies depends upon the character of the intervening medium. Faraday established by this experiment that the capacity of a spherical condenser changes when another material is used as the separating medium, rather than air. He found that the capacity became twice as large when the medium was paraffin, three times as large for shellac, six times as large for glass, and about eighty times as large for water.

This experiment became the foundation of the new theory. The old 'action at a distance' theory postulated that the electrostatic field was merely a geometrical structure without physical significance, while this new experiment showed that the field had physical significance. Every charge acts first upon its immediate surroundings, and it is only through the medium of these that the action is propagated. The discovery of displacement currents necessitated an extension of his point of view to all distances.[3]

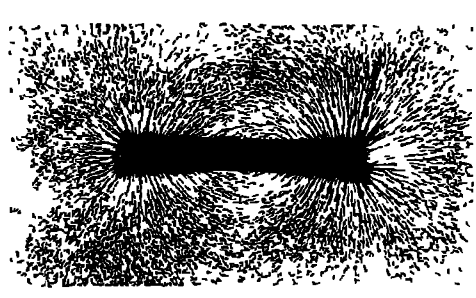


Fig. 2

Faraday was so impressed by this discovery that he abandoned the older theories of action at a distance and formulated a structurally new theory of contiguous action for electric and magnetic events. Any one can convince himself of the fact that the fields represent very actual physical conditions by taking a sheet of paper, sprinkling some iron dust upon it and putting a magnet under the paper He will find that the particles of iron dust arrange themselves in a very definite structure as shown in Fig 2

Faraday also discovered that the forces between two magnetic poles likewise depend on the medium that happens to be between them He concluded correctly that the electric as well as the magnetic forces are produced by a state of tension in the intervening medium.

These two examples will suffice as illustrations, but it can be said in general that all modern physics gives ample proofs of the correctness of Faraday's structural point of view Some physicists, for instance Helmholtz, built special devices to test the correctness of this theory As a matter of fact the success of the whole electromagnetic theory of Maxwell, which is structurally built as a contact theory, in which the velocity of propagation is considered finite, is in itself one of the best proofs of the correctness of the theory.

The finite velocity of light was discovered by Olaf Romer in 1676 and has since been repeatedly verified This velocity is usually denoted by c and is known to be approximately 300,000 kilometres per second, that is, $c=3\times10^{10}$cm /sec, or 186,000 miles per second

In 1856 Weber and Kohlrausch calculated a certain constant which appears in the electromagnetic theory, and discovered that the constant had the dimension of velocity, $[c]=\left[\frac{L}{T}\right]$, and that its numerical value was 3×10^{10}cm /sec, which is the exact value for the velocity of light This fact led Maxwell to associate light with electromagnetic waves, a view justified by experiments In 1888 Hertz not only established once more the interrelation of optics and electrodynamics but found that the velocity of propagation of electromagnetic waves is finite and exactly equal to the velocity of light [4]

Outside the exact sciences the principle of action by contact is making but slow progress, perhaps because of A s r and the lack of structural formulations of the general issues at hand We are happy to find a notable exception in the biological work of Professor C M Child, who has laid down a foundation for $\bar{A}$ biology and his system is structurally based on action by contact This $\bar{A}$ biology has been applied to neurology by Professor C J Herrick This present work, being a $\bar{A}$-system, must follow the methodological and structural advances explained here, and the $\bar{A}$ biology and neurology founded by Child and Herrick.

It is interesting to follow up the structural merging of geometry and physics There are certain Smiths and Browns who call themselves physicists There are some rooms with various instruments, which are called physical laboratories The activities of the physicists which interest us are twofold First, these scientists come to their laboratories, manipulate their instruments, note the positions of some pointers, manipulate the instruments again, note positions again, . This represents the *un-speakable* level of activity Whatever happens happens, but there is no speaking to be done on that level

Later the scientist describes his experiments in words Obviously there are two entirely different stages in building physics, which usually we do not distinguish.

Quite obviously the *un-speakable* level cannot be called 'physics', and so we must apply the term to the higher order abstractions on the *verbal level,* namely, to the reasoned verbal account of what the experimenter saw, or felt, or experienced, in general abstracted on the lower levels, summarized, generalized . , in higher orders

Physics represents then a verbal discipline Being verbal, it needs a language What language shall we select? As we want to have a science called physics, we shall naturally try to use the most structurally correct language in existence, so by necessity we must look in the direction of mathematics

In mathematics we find originally two entirely different disciplines One we may call arithmetic, the other, geometry. Becoming acquainted with these two originally separated languages, we find that the actual experiments and the stimuli for many experiences of importance to us, are outside our skins, so we try to choose the one of these two languages which is the more closely related in structure to the lower abstractions—that is, to what we see, feel , Naturally we have an inclination toward the geometrical languages, dealing with 'lines', 'surfaces', 'volumes' . , terms for which we find immediate and quite obvious applications. By further investigation we find that of late both languages have become so developed in structure, that either can be translated perfectly into the other This fact makes geometry the link between the higher order abstractions and the lower order abstractions We have seen that *physics*, as well as geometry, must be considered *verbal* disciplines and their fusion becomes a very natural fact.

It is true that, as yet, 'time' appears as the bothersome factor, but 'time' may very well be represented geometrically, except that our diagrams and figures look a little different. For instance, a *flat* circular orbit in two-dimensional 'space' becomes a helix in three-dimensional space-time, a vibrational motion in one-dimensional 'space' becomes a wave-line in two-dimensional space-time , . 'Time', when properly represented, becomes simply another geometrical dimension

It should not be forgotten that mathematicians obtain most of their structural inspirations from physics and build up mathematical theories to supply the structural needs of the physicist We see an excellent example of this in the $\bar{E}$ geometries In the days of Euclid, when physics hardly existed, we had 'emptiness', 'action at a distance', and such notions as were quite satisfactory for the needs of surveyors and builders With the development of astronomy and physics, curved lines became more and more structurally important, and the haziness of the definition of 'straight line' also became apparent. The notion of 'emptiness' also became slowly structurally untenable Such geometers as Gauss, Lobatchevski, and others, began to demand that the axioms of geometry be tested by experiment With the introduction of 'curvature', the 'straight line' became only a special case of a curve with zero curvature.

The invention of the differential calculus also had a tremendous structural influence. It introduced continuity as a basic assumption in the vast structure of science, and cleared the way for psycho-logically trained scientific workers in structural continuity, and therefore in *action by contact*

The discovery that light appears as electromagnetic waves and the finite velocity of both, made the notion of 'absolute emptiness' structurally untenable, and so E geometry with its action at a distance, 'emptiness' and neglect of gravitation and electricity, became very unsatisfactory Indeed if our universe were E, light could not reach us

Leibnitz, who invented the differential calculus independently of Newton, formulated a postulate of action by contact, and therefore could not become reconciled to Newton's Law of Gravitation which was structurally a law of action at a distance, corresponding fully to Coulomb's law in electricity The latter law states that the force exerted by two electrically charged bodies upon each other is inversely proportional to the square of the distance between them, and acts in the direction of the line joining them [5]

The introduction by Faraday of the structural notion of a 'field', instead of the notion of electrical charges acting at a distance, introduced the notion of a strain of the electrical field, which appears structurally as 'lines of forces' Here we already have a 'fulness' of 'lines' and a big step toward the structural fusion of physics with geometry has been taken

The transition from E to riemannian geometry corresponds structurally to the transition from physics based on action at a distance to physics based on action by contact The fundamental metrical theorem of E geometry is the pythagorean rule, which expresses the fact that the square of the distance between two points is a quadratic form of the co-ordinates of the points If we regard this theorem as strictly valid only in the case of points which are very near each other, we pass at once from E geometry to differential geometry By doing so we gain a notable structural advantage, as we dispense with the necessity of defining our co-ordinates more precisely, because the pythagorean law, when expressed in differential form, is invariant for arbitrary transformations [6]

Semantically, Riemann was the immediate predecessor of Einstein, although Einstein was not directly influenced by him In differential geometry we ought to start with indefinitesimally near points, and depend for the analysis of greater distances, areas and volumes, on integration The difficult notion of 'straight line' has to be replaced by the notion of the shortest line (geodetic), which is easily defined by differential methods and found empirically In the older method, the length of a curve was to be found, in general, by the process of integration. The length of a 'straight line' between two points was supposed to be defined as a whole, and not as the limit of a sum of indefinitely little bits Riemann considered that a 'straight line' does not differ in this respect from a curve. Measurements which are always performed by means of some instrument are *physical* operations, and their results depend for their interpretation upon the theories of physics. Dealing with geodetics is therefore preferable to dealing with 'straight lines'.[7]

We see that the problem was ripe for a final stroke of genius. Einstein's structural discovery of the dependence of 'space' and 'time', and Minkowski's success in giving a geometrical interpretation to the Einstein theory accomplished the probably irreversible fusion.

Three dimensional kinematics becomes four dimensional geometry, *three dimensional dynamics* can be considered as *four dimensional statics*.

We see immediately the human, psycho-logical, semantic and neurological importance of this fact. Our nervous system by its structure produces abstractions of different orders, dynamic on some levels, static on others. The problems of sanity and adjustment become problems of translation from one level to another, for which the structural advances in science supply us with methods of solution.

It should be noticed that the semantic gain due to the above facts is considerable, and that being structural, it is practical as well as theoretical. The fact that geometry has lost its old restricted status, which formerly applied principally to what could be 'intuitively visualized' and has been further abstracted to apply to what can be 'conceived', has merged geometry with the rest of mathematics. This merging represents a great structural and semantic step forward, and makes possible the treatment of geometrical problems by purely analytical means. It liberates geometry from the restrictions of lower order abstractions. By using 'geometrical intuition' (lower order abstractions) we find again a great help in analysis.

In the cyclic nerve currents, our so-called 'intuitions' (lower order abstractions) are not structurally isolated from our 'conceptions' (higher order abstractions), but both are intimately connected and influence each other. Modern advances are not only in perfect accord with the 'organism-as-a-whole' principle, but indeed give us excellent proofs that this principle is sound. 'Psychologists' miss a great deal by disregarding this important and unique form of human behaviour which we call mathematizing.

CHAPTER XXXVI

ON THE SEMANTICS OF THE EINSTEIN THEORY

> **It is precisely here, in an improved understanding of our mental relations to nature, that the permanent contribution of relativity is to be found. We should now make it our business to understand so thoroughly the character of our permanent mental relations to nature that another change in our attitude such as that due to Einstein, shall be forever impossible.**
> **(55)** P. W. BRIDGMAN

It is not my aim to expound the Einstein Theory as such. There are many excellent and competent books written on this subject. I have already explained and stressed several structural points which in the last analysis are the foundation of Einstein's work. Many 'thinkers' through the ages have felt vaguely the dangers of the structure of language and the viciousness of objectification, that is, of the delusional ascribing of objective values to verbal forms. This vague feeling, of course, is useful in individuals, but it is a private benefit, which cannot be made public without some sort of formulation. The stroke of genius of Einstein was that he produced a *non-elementalistic*, linguistic system of new structure. Einstein, being a physicist, decided rightly, as we understand now, to be entirely actional, behaviouristic, functional, and operational, and to stop gambling on words. The older *el* linguistic problems of 'matter', 'space', and 'time' were in such a mess, due to the objectification of verbal structures, that it was useless to talk any more in the old way. He decided to describe what a physicist *does* when he measures 'space' and 'time', and to abandon, perhaps unconsciously, the 'is' of identity.

It seems unnecessary to stress the simple fact that when we measure a piece of wood, for example, we mark it off with another piece of material which we have accepted arbitrarily as our 'unit of length'. The coincidence of our 'unit' with the intervals between the marks is again judged by an extremely complex electromagnetic-neural process, which was quite disregarded until Einstein. Our judgement is conditioned by the light rays travelling with finite velocity which excite our nervous system through the retina, this excitation in turn also travelling with finite velocity. We see that the apparently simple measurement of a 'length' is really an extremely complex process, in which the *finite* velocity of light and of the nerve currents plays a very important role. Naturally, if we were to assume an 'infinite' velocity of the propagation of light, our verbal speculations about 'space' and 'time' might be perhaps entertaining, but they would nevertheless be fundamentally and structurally wrong.

Similar remarks apply to the measurement of 'time'. What do we mean when we say that a train has arrived at the station at 9 o'clock? We mean no more and no less than that the arrival of the train coincided with the arrival of the pointer of a clock at a point marked 9 on the clock face. In other words, we saw 'simultaneously' the arrival of the train and the pointer of the clock reaching the mark 9

Our judgement about the results of measurements of 'time' depends on the *seen* coincidence of events—in this case, of the arrival of the train with the arrival of the pointer of the clock at the mark 9 Similar considerations, which applied to the measurements of 'lengths', apply also to the measurements of 'time'.

We see with Einstein that if we want to make any headway we shall have to investigate the two key terms, namely, 'velocity' and 'simultaneity'

The newtonians take a particular delight in accusing Einstein of being a 'psychologist' and not a physicist We have already stressed the physical subjectivity of physical instruments What is said there applies, not only to the retina of the eye, but also to a photographic camera, or to a microscope or telescope, or any other instrument Before an energetic packet, be it a light-impulse or a bullet, is able to accomplish any result it must first reach its mark, and so the finite velocity of propagation must be taken into consideration, which is a hard, established, empirical structural fact So the criticisms of the newtonians are simply shallow and unscientific (1933) They disregard most important empirical physical facts, and so simply defend a semantic disturbance without aiding science (1933)

With the einsteinians, we treat the eye on the same footing as we would treat the camera or any other physical instrument Even the newtonians must admit that when they photograph some happening on the sun, for example, the happening actually occurred (approximately) eight minutes *before* the photographic plate was affected The eight minutes is the 'time' taken by the light to reach the earth from the sun

Let us analyse the term 'velocity' first We find ourselves here, as in any other human problem, on two distinct levels of abstraction, and we must discriminate between them

Let us take up the verbal level first We see that before we can talk about our terms 'space' or 'time', 'length' or 'seconds', we have to know a great deal about the term 'velocity' How do we define the term 'velocity'? We define it as 'space divided by time', $v = s/t$ We see that on the *verbal* level the situation is perfectly hopeless and no result can be expected from verbal gambling It may be added that older notions were based on objectification, or confusion between the two levels of abstraction, and the affective belief in the magic of words, identification playing most of the structural havoc

How about the instrumental level, the silent level of the lower order abstraction? On this level, we find that physicists in their actions, behaviour, operations , have elaborated a fairly definite technique for finding the data they require So we see that there is no choice, we must *start* on this level

But starting on this level is not all, and not enough We must somehow *talk* about these doings and operations Hence we must select a language which in its *structure* will reflect the structure of these actions and operations Therefore we must abandon the 'is' of identity and *describe* in the asymmetrical language of order the happenings recorded by an instrument or by our lower nerve centres

Without going into details we may summarize the results as achieved by the physicists Experiments by the physicists, as indicated by the coincidences of pointers on different instruments, have seemingly established the fact that the 'velocity' of light, as defined by *behaviouristic, operational* instrumental means, is a *constant*, $c = 3 \cdot 10^{10}$ cm /sec , independent of the relative velocity of the observers By the 'observers' we mean again the readings on the instruments which the observer carries with him Now this result contradicts flatly the established *verbal expectations* which we reached on verbal levels through the elementalistic structure of language and the semantic disturbance, of ascribing 'objective' existence, to the *terms* 'space' and 'time'

The situation is acute Shall we follow our semantic disturbances and reject hard empirical structural facts, or shall we accept the experimental facts and eliminate semantic disturbances?

As usual, the answer is implied by the method of putting the question We accept the experimental facts and revise our semantic disturbances In this case a psychiatrist might be a useful co-worker with the physicist.

The einsteinian revolution is structurally and semantically so fundamental, that every intelligent person should be acquainted with it It will therefore be as well to consider some of its details

In classical mechanics we had the classical mechanical principle of relativity, namely, that all mechanical equations have one form for two co-ordinate systems moving uniformly with respect to each other The above has a very simple empirical meaning If we travel in a train, let us say at a velocity of 50 miles an hour, all our activities in the train have one familiar relative velocity as if the train were at rest. If we throw a ball with a velocity of 20 miles an hour to another passenger on the train in the direction of the movement of the train, the ball will not reach the other passenger with the velocity of 20 miles an hour *plus* the additional 50 miles an hour velocity of the train but will reach him with the velocity as if the train were standing still Not so, however, if the ball were thrown to an observer, standing on the tracks The ball might hurt him, because it would have, relative to him, the velocity of 20 miles an hour of the ball, plus the velocity of 50 miles an hour of the train, or in all, a velocity of 70 miles an hour.

Quite probably, even our remote ancestors who used artificial means of transportation on land or water did not overlook the structural fact that mechanical events happen in just one way, whether the system is at rest or in relative motion With the advent of *verbal* formulations of physics and mechanics, such happenings were formulated verbally, and so, slowly, the language of old structure with its consequent objectifications was built

Now on *verbal grounds*, which seemed to be justified by experimental, macro-mechanical facts, we concluded that one law should prove valid in the case of electrodynamic and optical events.

To reformulate the above in simple symbols, let us imagine two parallel co-ordinate systems, O' and O'', of which the second moves with a velocity u relative to the first in the common x direction. If we denote the co-ordinates of the first system by single primes, and the co-ordinates of the second system by double primes, then, as usual, the co-ordinates of a point, P, in the second system would be connected with its co-ordinates in the first system by the equation $x''=x'-ut$, which means that the x'' co-ordinate is less than the x' co-ordinate by the amount that our second co-ordinate system has moved, namely, by $a=ut$. We gave the diagram in two dimensions because it is simpler,

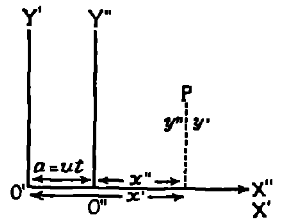


FIG. 1

and, as we have assumed that the displacement is parallel to the x axis, the other co-ordinates remain unaltered, $y''=y'$, $z''=z'$. 'Time' by the older assumptions, being 'objective' and 'absolute' would be 'the same', namely, $t''=t'$ ('absolute time'). The classical law of relative motion states that if the equation of motion in the first system is $f'(x', y', z', t)=0$, this function must also be zero when x' is replaced by its new value, namely, $(x'-ut)=x''$ so that $f'(x'', y'', z'', t)=0$.

Let us see if the above conditions hold true when we deal with the *propagation of light in spherical waves.*

$$OB^2=OA^2+AB^2$$
$$OB^2=x^2+y^2$$
$$OP^2=OB^2+BP^2$$
$$OP^2=x^2+y^2+z^2$$
$$c^2t^2=x^2+y^2+z^2$$

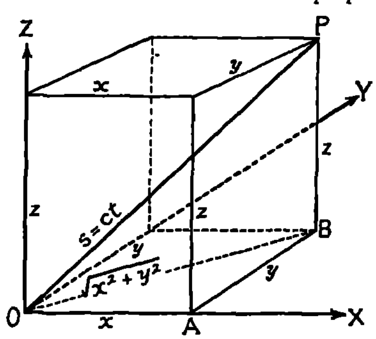


FIG. 2

If we select a three dimensional co-ordinate system O, the distance s of the point P from O is equal, by the pythagorean rule, to $s^2=x^2+y^2+z^2$. If we assume that a light ray is travelling from O to P, the distance s could then be represented by the product of the velocity of light c by the 'time' or ct. The square of this distance would then be $c^2t^2=s^2$. We have $x^2+y^2+z^2=c^2t^2$, or $x^2+y^2+z^2-c^2t^2=0$.

We can easily convince ourselves that if the last equation for light-waves holds good in the first co-ordinate system it cannot hold in the second.

Writing the last equation in our primed letters, we have $x'^2+y'^2+z'^2-c^2t^2=0$. If we pass to our second system of co-ordinates moving uniformly in the X direction with the velocity u relative to the first system, our y', z', t, do not alter by assumption, but only $x''=x'-ut$. We would have by substituting $x'-ut$ for x'', and retaining the primed values for y'', z'', t

$$\begin{aligned}(x'-ut)^2+y'^2+z'^2-c^2t^2&=x'^2-2x'ut+u^2t^2+y'^2+z'^2-c^2t^2 \qquad (1)\\ &=x'^2+y'^2+z'^2-c^2t^2+(u^2t^2-2x'ut)\end{aligned}$$

But, by assumption, $x'^2+y'^2+z'^2-c^2t^2=0$, and therefore equation (1) cannot be zero unless $(u^2t^2-2x'ut)=0$ This last condition would mean that our second system of co-ordinates is also at rest We see that for *light-waves* the older mechanical principle of relativity does *not* hold, as the equations *are altered* when we pass from one system of co-ordinates to another which moves with uniform velocity relative to the first

To indicate this more obviously, we will express it in formulae Consider two co-ordinate systems O' and O'', in which the second moves with a uniform velocity in the X direction relative to the first If for the *light-waves* the equation $x'^2+y'^2+z'^2-c^2t^2=0$ holds in the first system, a similar equation for the second moving co-ordinate system, $x''^2+y''^2+z''^2-c^2t^2=0$, cannot be true In other words, $x'^2+y'^2+z'^2-c^2t^2\neq x''^2+y''^2+z''^2-c^2t^2$, whence we have an *inequality*, fundamentally contradicting the classical principle of relativity

This extraordinary and unexpected inequality, because it contradicted structurally the classical mechanical principles of relativity, which apparently had been well established experimentally, created a baffling semantic situation which was profoundly unsatisfactory

What could we do about it? Should we abandon the older principle of mechanical relativity, or should we have two different laws, one for the older gross macroscopic mechanical relativity, and another for optical and electrodynamic events, or should we investigate the fundamental structural assumptions which underlie our formulae, and see if the discrepancy is not due to some prejudice or some structural dogma which we have overlooked for centuries?

If a solution of the last kind should be found it would naturally be most satisfactory The admission of two relativities, one for the mechanical events, the other for the optical events, would be against the whole trend of science, which requires the unification of theories

Such a structural revision, namely, the rooting out of the old unjustified dogma which made all the trouble, was the work of Einstein's genius In this epoch-making discovery he was *assisted* in the beginning by the famous Michelson-Morley experiment, since performed repeatedly with similar results, *seemingly* proving that the velocity of light is a constant no matter what the relative motion of the observer If we take the equation for the spherical propagation of light-waves $x^2+y^2+z^2-c^2t^2=0$ or $x^2+y^2+z^2=c^2t^2$,

then $$c=\frac{\sqrt{x^2+y^2+z^2}}{t}$$ in one system of co-ordinates,

and $$c=\frac{\sqrt{x'^2+y'^2+z'^2}}{t}$$ in another system of co-ordinates,

are equal, which stated in another form would mean that

$$x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t^2$$

The experiment says this relation is true, the arguments already advanced show it cannot be true So we have to hunt for some error or compensation

With Einstein's explanation, the finding of the error is simplicity itself In the older mechanical relativity our 'space' and 'time' were *objectified*, we endowed them with objective values of definiteness and rigidity, we dealt with 'absolute space' and with 'absolute time', which was 'unchanging' and 'the same for all' In the older assumptions our *velocities varied* If A had a velocity of 5 centimetres per second, for instance, and B was overtaking A with a velocity of 7 centimetres per second, the relative velocity between A and B would be $7-5=2$ cm per second The *units of 'space' and 'time' were definite, immutable and did not and could not vary*, which followed directly from the assumptions of an 'objective' 'absolute space' and 'absolute time'.

In the case of light, we came in contact with a velocity which did *not vary* for any observer no matter what his relative motion The velocity c was found to be constant, so the natural assumption to make is that our *'space' and 'time' vary* for different observers.

In the above equations as they stand, 'absolute time', $t=t$, the 'same' for all observers is assumed, which made such equality impossible Assuming different 'times' for different observers, t for the first, and t' for the second, such a compensation transforms our inequality into an equality, as demanded both by the experiment and by the theory Instead of writing

$$x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t^2,\ (t=t),$$

which *cannot be true*, we write,

$$x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2=0,\ (t\neq t')$$

which can be true We should notice that in the first equation we have on both sides t, which makes the equation impossible, whereas in the second equation we have on the left-hand side t and on the right-hand side a different t, namely, t'

The above considerations mean that there is a definite structural discrepancy between the old language and the empirical world, requiring a fundamental structural linguistic revision This revision has been accomplished, and is known as the Einstein theory It is not implied that Einstein's work is final, but that it shows clearly the structural errors of the old elementalism to which we can never return

In other words, in the older mechanics we had definite and permanent 'time' (absolute) and varying relative velocities. Dealing with *light-waves* we find experimentally that the velocity, c, of light does *not vary* with the relative motions of the observers and we must assume a *variable time* to preserve our equations.

An obvious objection can be raised to this why alter our habitual notions of 'time'? Can we not keep the old s r and find some other method of compensation, less bothersome and less revolutionary? The older physicists and Einstein give a long and convincing list of perfectly sufficient reasons for such a change, yet their arguments always leave us somehow in doubt, with the feeling of a lurking possibility that the old can be preserved

What has already been said in this work about structure and semantic disturbances and the fact that the *terms* 'matter', 'space', and 'time' *are not objects*, which they cannot be, removes perhaps for good and all, the last doubt as to the revolutionary and epoch-making significance and value of the structural linguistic discoveries of Einstein. On these grounds *alone* the return to the old is impossible The old is due to objectification of the structural peculiarities of the old *el* language, and to semantic disturbances, which at the present low level of our development is inevitably the result of *copying lower animals in our 'thinking'*, a *pathological* process for 'man'

It will be well to explain at this point why I said that the Michelson-Morley experiment only *assisted* Einstein, and only *seemingly* proved the constant velocity of light Historically, there is no doubt that the beginning of the theory of Einstein was suggested by, and had its physical basis in this experiment In reality, as the whole of this present work about structure shows, the two issues are quite independent The fact of the *finite* velocity of light has never been challenged, on the contrary it is becoming more and more solidly established, both empirically and theoretically, simply because an 'infinite velocity' has no meaning

With the structural results of this present work, and the establishment of the fact of the finite velocity of light, the whole Einstein *theory* has a perfectly solid structural, linguistic foundation (1933) Nevertheless it is extremely gratifying that the latest, very important, and painstaking work of Doctor Roy J Kennedy seems once more to add fundamental experimental support to the correctness of the Einstein theory * From the point of view of structure, Einstein merely eliminated some primitive, perhaps even animalistic, remains of objectification which still lingered in the structure of our language of 'matter , 'space', and 'time' These, being animalistic, were unfit for humans, vitiating not only our daily lives but science as well (Eddington in *The Mathematical Theory of Relativity*, p 196, uses the term 'pre-human' in a similar connection.)

It must be recalled that the definition of velocity is connected in a *circular* way with 'space' and 'time' That is, in the definition of the relation of velocity, ($v=s/t$), 'space' and 'time', the definition of any one of our three terms depends upon our definition of the other two, whence there are many possible ways of verbal adjustment

As we saw, the mechanical verbal principle of relativity with which we are all familiar was not structurally able to account satisfactorily for a similar relativity of optical and electrodynamic events The older formulae of transformations were, as already given, $x'=x-ut$, $y'=y$, $z'=z$, $t=t$ These formulae are called the Galileo transformations in honour of the founder of mechanics, and, as we have seen, structurally they are not general enough

If we consider the equation $x^2+y^2+z^2-c^2t^2=0$ and $x'^2+y'^2+z'^2-c^2t'^2=0$, we find that the galilean transformations do not satisfy them Lorentz and Einstein have found another set of transformations which satisfies uniquely the above equations These formulae of new structure are called the Lorentz-Einstein transformation, and are given by the following equations $x'=\beta\,(x-vt)$, $y'=y$, $z'=z$, $t'=\beta\,(t-vx/c^2)$, where v is the relative velocity of one system with

*See 'The Velocity of Light', in *Nature*, Aug 20, 1932, by R J Kennedy, and his latest paper (No 261) in the Bibliography respect to the other, c, as usual, represents the velocity of light, and the factor $\beta = 1/\sqrt{(1-v^2/c^2)}$

The most striking characteristic of these formulae is that if we assume that c, the velocity of light, is '*infinite*', all the expressions containing c^2 would become zero, c^2 entering only in the denominators of fractions In such a limiting case $\beta = 1/\sqrt{(1-0)} = 1/1 = 1$ and $x' = (x - ut)$, $y' = y$, $z' = z$, $t' = t$ which are the older galilean transformations

Thus there appears the astonishing fact that all the pre-einsteinian physics and mechanics which involved the structural assumption of the galilean transformation, had a *tacit structural assumption* of the infinite velocity of light This assumption, *known since 1676* to be false as to facts, remained unnoticed before Einstein

As $c = 3 \times 10^{10}$ cm /sec , $c^2 = 9 \times 10^{20}$ is a very large number, whence the fractions vx/c^2 and v^2/c^2 are very small, and β differs very little from unity

If we apply the Lorentz-Einstein transformation instead of the older galilean transformation to mechanical problems, the changes are so small that they can hardly be detected by experiments, the terrestrial velocities v^2 or vx being so small in comparison with the square of the velocity of light.

The galilean transformations are *experimentally* shown to be structurally invalid for optical and electrodynamic events The Lorentz-Einstein transformations satisfy structurally the optical and electrodynamic events, and also apply to the older mechanical problems We see that the Lorentz-Einstein transformations are *more general*, as they include the galilean transformations as a particular case when we assume $c = \infty$

In a few instances, where we deal with large velocities, the values of the fractions containing the square of the velocity of light become appreciable and allow experimental testing As yet all such experiments have verified the Einstein theory

We should repeat again that the achievement of Einstein was the building of a linguistic system similar in structure to the world, which eliminated a pathological pre-human factor of objectification of terms Such structural elimination was bound to bring some sanity to our theories, and this fact is independent of experiments in physical laboratories However, it is gratifying to find that experiments support (1933) the Einstein theory It was particularly gratifying in the beginning, when physicists and Einstein himself believed that his theory would stand or fall by experiment Today we see that this theory represents such an enormous general, structural, epistemological, psycho-logical, and methodological *non-elementalistic* advance, that no matter what the experiments show or may show in the future we cannot return to a language of the old, *el*, obviously wrong, structure of the pre-einsteinian days As usual, the negative results are the important ones No matter what experiments may show we shall never again accept the silent structural assumption of 'infinite' velocity for light, when we know positively that the velocity is finite We shall never again treat *terms* of 'matter', 'space', and 'time' as objects—lower order abstractions, when we know that they represent *terms*—higher order abstractions When once this is realized, we cannot ascribe 'finiteness' or 'infiniteness', 'definiteness', 'rigidity' . . ., to *terms*, verbal forms, forms of representation. From this point of view we may consider the Einstein theory as an irreversible gain. If it had achieved only the elimination of various structural prejudices and dogmas, it has done well, and at least this much Einstein has already achieved.

The structural, verbal, cortical quest for invariance in our formulations also becomes apparent. The older mechanics were invariant under the galilean transformation, equations preserved their form in different systems of coördinates. In the special theory of relativity the new laws are invariant under the Lorentz-Einstein transformation. In this special, or restricted, theory of relativity only uniform relative motion was taken into account. If we generalize the principle of relativity to *any* kind of relative motion we pass from the restricted to the *general* theory, which demands that the laws of physics should be formulated in a generally invariant form for any arbitrary transformations.

For this structural, cortical reason it is necessary to express all the laws of physics in tensor equations, which satisfy such conditions of general invariance. If this cannot be done, there must be something wrong with our language, as such, and with our verbal laws. We require structural revision of those laws so as to be able to express them in tensor equations. The newtonian law of gravitation and the older form of the law of conservation of energy are perhaps the most remarkable examples. They do not survive such minimal, and yet entirely justified, structural requirements as those of the general theory of Einstein, and therefore they cannot be structurally satisfactory.

We have already seen that the equation $x^2+y^2+z^2=c^2t^2$ or $x^2+y^2+z^2-c^2t^2=0$ represents the equation of the spherical *propagation of light* with the finite velocity c. The discovery that the velocity of light is a universal constant for all observers, and the above equation, led historically to the re-discovery by Einstein of the Lorentz transformation which, as we have seen, has assumed such overwhelming structural importance. The meaning of these facts is worth considering.

In Chapter XVII we analysed briefly the elementalistic language of 'matter', 'space', and 'time', and came to the conclusion that to eliminate objectification we must abandon the semantic disturbance and the use of the term 'is' of identity. Instead, we must use an actional functional language to describe ordered functioning, behaviour, or operations. By necessity we were led to a 'contact' method. We also discovered that in accepting the above structural methods we were compelled to discriminate between different orders of abstraction, since what we see, feel, and experience is *not* what we say about it. We found that on the 'objective' level of our actual activities, (manipulating instruments . . .,) which represent the silent *un-speakable* level, we could never find a situation in which the old language of 'matter', 'space', and 'time' could be used without coming violently into conflict with the properly analysed facts. We came to the conclusion that this language was not structurally satisfactory, for *verbally*, 'space', 'time', and 'matter' were supposed to be quite clear-cut and *separate* entities, while in actual experience we *never could find such separated* objective entities. It became obvious that the *structure* of the old language of 'matter', 'space', and 'time' was *different* from the structure of the outside world as we now know it We found ourselves in a situation where we had to choose either to keep the old language which, as it differed from them in structure, could never give a coherent account of facts at hand, or else to build up a new language with *structure* similar to that of the outside world, in order to have the possibility of coherent conversation about it

The invention of such a new language is of course an extremely difficult undertaking In fact, it requires some genius to invent new, more structurally similar, forms of representation for the old facts Lorentz, Einstein, and Minkowski prepared and finally produced such a structurally new language The difficulty was that verbally we had already separated what empirically could not be separated The problem was to amalgamate somehow the old structurally *elementalistic* language of 'space' *and* 'time' into a *non-elementalistic* language. The key to such an amalgamation is found in the light-wave equation which gives us the structural information about the world, $x^2+y^2+z^2=c^2t^2$.

This equation represents an equality The left-hand side is expressed in 'spatial' terms only—the *distance* between two points O and P The right-hand side expresses the 'spatial' length, but in a 'temporal' term We see that here we have means of translation, and a possibility of amalgamation of two *elementalistic* languages, which were not supposed to be intertranslatable

The Lorentz-Einstein transformation formulae are $x'=\beta\ (x-vt)$, $y'=y$, $z'=z$, $t'=\beta\ (t-vx/c^2)$ where v is the relative velocity of the two systems of coordinates, c, the constant velocity of light, and $\beta=1/\sqrt{(1-v^2/c^2)}$

The formulae for x' and t' which typify, on the left-hand side, x' a 'spatial' *length* and t' a 'time', are of particular interest We see that on the right-hand side of the expressions the value of the 'spatial' x' is given by $\beta\ (x-vt)$ which involves '*time*' The value of 'time', t' is given by $\beta\ (t-vx/c^2)$, which involves the 'spatial' *length* x So we see that our amalgamation is complete, and separation impossible. The above formulae express structurally the *simple experimental* fact that 'space' and 'time' cannot be separated At this point we are not ready to discuss 'matter'. This will be considered further on in this work (see Chapters XL and XLI)

The above formulae have also a very important physical and experimental meaning, as they introduce the 'contact' methods into our language Our actual measurements of 'space' and 'time' are strictly connected with readings on some instruments, and involve therefore coincidences between pointers and 'simultaneity'. In all instances the finite velocity of propagation of signals must be taken into consideration When our instrument, or the eye, is affected by signals there is always a delay due to the finite velocity of the propagation of the signals These delays are part and parcel of our experiment, and so our formulae must contain terms explicitly involving this finite velocity of propagation This innovation involves not only a most profound structural epistemological and semantic revolution but supplies the very factor that enables us to formulate more structurally satisfactory languages (theories), which Lorentz, Einstein, and Minkowski have produced.

We have been contrasting finite and 'infinite' velocities Let us say frankly that 'infinite' velocity is a polite way of speaking about blunders of observation 'Infinite' velocity is meaningless Velocity is defined as $v = s/t$ and if t is taken as zero or in other words, if one of the fundamental factors in our *definition* is lacking, our *definition ceases to define the term in question*—in this case, velocity So when the term 'time' is lacking, we have *no velocity*, by definition, so, to speak or speculate about 'infinite' velocity is simply making noises, and not saying anything The *negative* of this noise, namely, saying that velocity is *not* 'infinite', or in a positive sense, that velocity is 'finite', is on a different verbal footing, although it remains a polite invitation to stop talking non-sense.

It should be noticed carefully that the *general* theory of Einstein is a high structural generalization of the special theory, and that both of them are generalizations of the classical mechanical principle of relativity It is founded, not on the introduction of any extraordinary structural assumptions, but on the elimination of some unjustified and false-as-to-facts structural assumptions, such as that of the 'infinite' velocity of light

Both the theory of Einstein, and the theory presented in this work are long overdue The Einstein theory could have been formulated as soon as we discovered the *finite* velocity of light, in 1676 It should be noticed that this last discovery was also overdue, as it did *not* require experiments to establish the finite velocity of light. It was sufficient to establish the meaningless character of 'infinite' velocity, which on symbolic grounds, could have been accomplished much earlier, and to conclude, that the velocity of light *must* be finite This example shows the hampering, blocking, semantic effect which different meaningless verbal structures have on us To express this high and satisfactory structural generalization, Einstein had to select the most general and structurally appropriate language in existence He chose at some stage of his work the language of $\bar{E}$ and four-dimensional geometries in general and that of the differential geometry and the tensor calculus in particular In the latest field theory, Einstein and Mayer introduce a new *more general* and very revolutionary mathematical language where vectors and tensors in an n-dimensional spread may have m components

At present it appears that two other very general mathematical disciplines will be used increasingly in the future. One of them is the *theory of groups*, the other is *analysis situs*. In the latter we study only these characteristics of figures that are unaffected (invariant) by continuous deformation produced without tearing Two structural points are relevant for us in this connection namely, that the analysis situs is fundamentally a *differential* and also an *ordinal* discipline, based on asymmetrical relations In the next chapter, as an illustration of the actional, behaviouristic, functional, operational, differential, contact method a short account will be given of the way Einstein structurally treated 'simultaneity'. The elimination of the old structural dogma about 'simultaneity' resulting from the semantic disturbance of objectification of 'time', is one of the outstanding achievements of Einstein and is historically the beginning of his theory.

CHAPTER XXXVII

ON THE NOTION OF 'SIMULTANEITY'

> So we see that we cannot attach any *absolute* signification to the concept of simultaneity, but that two events which, viewed from a system of co-ordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system. (155)
>
> A EINSTEIN

In the older days we accepted as self-evident the structural assumption that there is sense in such a statement as that an event A on the sun was 'simultaneous' with an event B on the earth We assumed also that the 'moments of our consciousness' had a universal 'meaning' We tacitly assumed, for instance, that when we saw or photographed an event on the sun, that it happened just the moment we saw it Such structural assumptions were rudely disturbed by the discovery of the *finite* velocity of light Today we know that when we see or photograph an event on the sun, that event happened approximately eight minutes earlier, as it takes about eight minutes for the light from the sun to reach our earth We begin to realize that the moments of our perceptions have no universal significance

We inquire first what we mean structurally by simultaneity We do not need to go into details The application of functional and contact methods, even in the rough, will assist us We can speak in terms of instruments For instance, we can build a special, very fast moving picture camera, C, with two lenses D and E, at two opposite sides, and a calibrated film, F, running rapidly through the middle of the camera as shown in Fig 1. If we focus our double camera on two flashes, A and B, occurring at 'equal distances', L, from the film, we say that the flashes occur simultaneously by definition if the pictures, a and b of the flashes A and B, appear exactly opposite each other on the film, or if we have *one* picture If, under the conditions of the experiment, where the distances between the origins of the flashes and the film are equal, and our film is moving very rapidly, the pictures of the flashes do not occur exactly opposite each other, but one picture is separated from the other, then we have two pictures, and conclude, by *definition*, that the flashes are *not* simultaneous

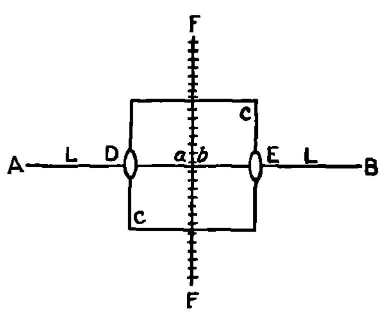


FIG 1

We introduce this hypothetical instrument to show that, in discussing physics, and the theory of Einstein in physics, we do not speak of 'psychology' or personal 'subjectivity', but that we do deal with the inherent physical subjectivity of the instruments and the finite velocity of propagation When we discuss the psycho-logical or methodological or semantic significance of science and scientific method, we deal with different subjects

When we use the term 'observer' we mean an observer so equipped that he can do whatever is demanded of him.

What was said about the definition of 'simultaneity' by the aid of the camera, applies also to ourselves

The problem of prime importance before us is to find out if 'simultaneity', as defined, has an 'absolute' and universal significance, or if it is perhaps a private and relative notion

We will carry out the analysis in two ways, the first by example, which will be instructive, though perhaps not completely conclusive, the other, by the use of the Lorentz-Einstein transformation

Let us perform our last experiment, which, with modern methods seems to be feasible, in a slightly more complicated form

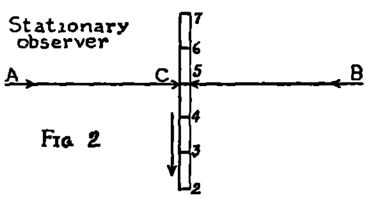


FIG 2

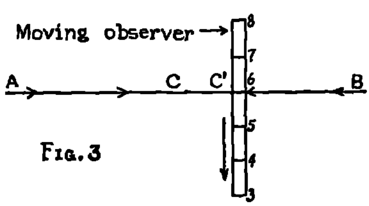


FIG. 3

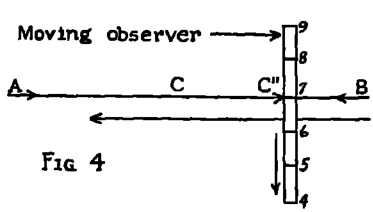


FIG 4

We can select a dark night in which flashes will photograph well even at considerable distances We can place powerful projectors at A and B, and we place our camera so that the film will come exactly at C, midway between A and B We can start the mechanism of the rapidly moving film and, by an electrical contact made at C, we can produce a short flash from each of the two projectors. Because of the assumptions, AC=CB, and equal velocity of the propagation of electrical currents and light-waves in all directions we shall have by the structural definitions which condition the experiment, *one* picture in Fig 2, say at the spot of our moving film marked by 5 The rays of light from A and B would arrive 'simultaneously'—that is, 'at the same time'—and would affect our moving film in *one spot* Our definition was for a stationary observer, and under the conditions, the experiment was fairly definite—all the underlying structural assumptions, of course, being taken for granted

Now consider an observer, as shown in Fig 3, moving uniformly in the direction from A to B.

Let us assume that he is also equipped with a similar sort of moving picture camera as the stationary observer, and that just before he passes the point C the electrical impulse to the projectors is sent Let us assume further that the mark 5 on his moving film is exactly at the focal point of the camera as C is passed The electrical impulses travelling from C to A and B would travel the distance AC = BC, produce the flashes A and B which again would travel with finite velocity in all directions During the interval these impulses and light-waves are travelling, our observer is moving from A toward B, and spot 5 on his moving film is no more at the focus of the camera Obviously he will meet the light-wave from B first, at C′, let us say, when mark 6 on his film is at the focus (Fig 3) After another short interval when he reaches C″ and mark 7 on his film is at the focus, the light-wave from A overtakes him (Fig 4)

So we see that what was 'simultaneous' (by definition) and produced *one* impression on the moving film of the stationary observer, was not 'simultaneous', (again by definition), for the moving observer, as *his* film registers *two* pictures

As both observers use similar instruments and one set of definitions, obviously both are entitled to claim that their records on the film are conclusive. So the first can claim that the flashes were 'simultaneous', the second can claim that they were not 'simultaneous'. The reverse is equally true If the moving observer had *one* picture, and claimed 'simultaneity', the stationary observer would have *two* pictures, and deny 'simultaneity'.

But when two observers are *equally justified* in making *two* opposing claims where, by their very meanings, there is only one possible, we must conclude that the claim itself is meaningless. We see that 'absolute simultaneity' is a fiction and impossible to ascertain, as it would depend on some impossible 'absolute motion', or 'infinite velocity' of propagation of signals

The analytical form of showing the impossibility of 'absolute simultaneity' is very simple, and follows directly from the Lorentz-Einstein transformation.

Let us imagine two observers, one in an S system of co-ordinates (x, y, z, t) and another in an S' system of co-ordinates (x', y', z', t') moving relatively with the velocity v

Let us assume two events happening in the unprimed system at the point (x_1, y_1, z_1) at the 'time' t_1, and the other at the point (x_2, y_2, z_2) at the 'time' t_2. According to the Lorentz-Einstein transformation the 'times' at which the two events occur relatively to the primed system are given by the formulae

$$t_1' = \beta\ (t_1 - x_1 v/c^2),\ t_2' = \beta\ (t_2 - x_2 v/c^2), \text{ where as usual } \beta = 1/\sqrt{1 - v^2/c^2}$$

If we assume that in our unprimed system S the two events were 'simultaneous', which means that they 'occurred at the same time', t_1 would be equal to t_2, that is, $t_1 = t_2$, or $t_1 - t_2 = 0$ Let us find the difference between the two primed 'times' in the moving system S', and see if this difference is zero, which would mean that the primed 'times' are equal.

Returning to our formulae which give us the values for the primed system 'times', we express their difference as

$$t_1' - t_2' = \beta\ (t_1 - x_1 v/c^2) - \beta\ (t_2 - x_2 v/c^2) = \beta\ (t_1 - t_2 + x_2 v/c^2 - x_1 v/c^2). \qquad (1)$$

But we assumed $t_1 - t_2 = 0$, therefore $t_1' - t_2' = \beta\ (x_2 v/c^2 - x_1 v/c^2)$.

This last formula shows clearly that $t_1' - t_2'$ cannot be zero, or in other words, t_1' cannot be equal to t_2' unless $x_1 = x_2$.

The two events which, for an observer in the unprimed system, happen 'simultaneously', ($t_1 = t_2$, or $t_1 - t_2 = 0$) at different places, (or x_1 not equal to x_2, $x_1 \neq x_2$), cannot be 'simultaneous' for the moving observer in the primed system S', but will happen at different 'times' (t_1' is not equal to t_2', or $t_1' - t_2' \neq 0$).

It is extremely instructive to consider further what happens in measuring 'times' and 'lengths' in systems which are moving relatively to each other.

If, in the equation (1) above, we assume $x_1 = x_2$, this means that both events occur at one place in the stationary unprimed system S

By changing the signs and cancelling the terms with x_1 and x_2, which are equal and of opposite signs, we have $t_2' - t_1' = \beta\ (t_2 - t_1)$ whence, substituting for β its value $1/\sqrt{1-v^2/c^2}$, we obtain

$$t_2' - t_1' = \frac{t_2 - t_1}{\sqrt{1-v^2/c^2}}$$

This last formula brings out a few remarkable issues In terrestrial velocities the square of the velocity of the motion v^2 of the observer in the primed system S' is very small as compared with the square of the velocity of light c^2, so the fraction v^2/c^2 is small, $\sqrt{1-v^2/c^2}$ differs very little from unity but the whole denominator is *less* than unity, and so $t_2' - t_1'$ is *not* equal to $t_2 - t_1$, but greater

In other words, the interval of 'time' between the two events appears *larger* to the observer in the primed moving system than to the observer in the stationary unprimed system In general, among all systems in a state of uniform relative motion, that one in which two events occur at *one place*, is characterized by the fact that the 'time' interval between the two events appears *shortest* to an observer in this system The shortest interval means that to an observer in the system, the events run their course most rapidly A process which, with reference to a given system, occurs in *one* place, appears to run its course most rapidly to an observer in that system, but more slowly to a moving observer in any other system

The more rapid the relative motion, the slower the process will appear, and, in the limit, if an observer could move with the velocity of light, $v^2 = c^2$, the denominator of our equation would become $1—1 = 0$ and $t_2' - t_1'$ would become 'infinite' and all events would be at a standstill

As the formulae for length, x and x', involve the 'times' and, as we see, the intervals of 'time' are dependent on the relative velocities, by a similar process of reasoning we find that the standards of length are also relative, and that the length L' in the primed system is represented by $L' = L\sqrt{1-v^2/c^2}$. In other words, to an observer who sees the rod in motion, it will appear 'shortened', and among all systems in a state of uniform relative motion, the one in which the rod is at rest is distinguished from all others by the fact that in it the rod appears longer than in any other system For instance, a metre rod lying on the earth in the direction of its motion would appear to an observer on the sun to be shortened by 5×10^{-7} cm In the limit, when $v = c$, the fraction $v^2/c^2 = 1$, $1 - 1 = 0$ and $L' = 0$, which means that to an observer moving with the velocity of light, a three-dimensional body would appear as two-dimensional, or a two-dimensional figure as one-dimensional The co-ordinates y and z, as we have seen, do not enter into consideration as they are equal in both systems moving relatively in the X direction, and the 'time' co-ordinates are independent of them.

If a body at rest appears to the observer in the unprimed system as a sphere, it will appear as an *oblate spheroid* to an observer in the primed system

We see that structurally not only 'simultaneity' and 'time' are not absolute but also that length, and therefore *shape*, is relative

We have seen that the 'shortest' and 'longest' values are important characteristics of the motion This suggests why in the general theory of Einstein we are interested in, and introduce, geodetics

It should be mentioned here that the Lorentz transformation has been reached by difficult considerations involving Maxwell's electromagnetic field equations, unrelated to the Einstein theory Einstein found the Lorentz transformation by the *simplest consideration* connected with his theory The finding of such important equations by two methods, entirely different structurally, must be considered as a convincing proof of the fundamental importance of such formulae, the more so since they follow from very simple and fundamental structural principles which in themselves cannot be denied because they are negative in character Negative statements are on a different footing in the new systems, they follow structurally from a $\bar{A}$ orientation, just as the older positive dogmas were the structural results of aristotelianism and the delusional results of identification

The facts mentioned concerning the measures of length and the behaviour of clocks do not present any paradoxes They simply say that these discrepancies are mutual and inevitable, as any measurement is only a measurement when it can be registered by an instrument, or seen, or recorded in some way If the measuring rods and clocks are moving relatively to us, what we see or what our instruments record is *not* what is happening on the moving system, which no one can see or record from outside the system What reaches us is simply what the light-waves or other signals moving with finite velocity (and therefore retarded by a motion away from us) bring to us As all existing methods of communication and all known signals have *finite velocities*, these structural differences which are conditioned by the inherent characteristics of the world should be taken into consideration in modern science

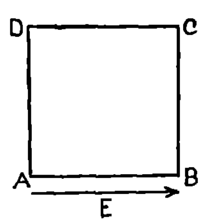


FIG 5

If we draw a square $ABCD$ (Fig 5) and an aviator E were to pass this square sign with a velocity of 161,000 miles a second* in the direction AB,

*I deliberately select such a velocity so as to make the contraction given by the formula $L'=L\sqrt{1-v^2/c^2}=1/2$ With this aim we must make the fraction represented by $v^2/c^2=3/4$, then $1-3/4=1/4$ and $\sqrt{1/4}=1/2$ We find the square of our velocity v by taking 3/4 of the square of the velocity of light $v^2=\frac{3}{4}c^2$ and find $v=\frac{c}{2}\sqrt{3}=161,000$ miles a second he would see—and any instrument carried by him would register it—the sides of *our square* ($AB = BC = CD = DA$) in the direction of his flight, namely AB and CD, as 'contracted' to half their length If he turned at right angles, the sides AB and CD would 'expand' and the other sides, which are at right angles, BC and DA, would 'contract'. For us the sides AB and BC are *equal*, for him one appears twice the other To him our square appears oblong

Under such *natural structural* conditions it is a fundamental fallacy to ascribe to 'lengths' or 'shapes' or 'times' any 'absolute' significance If we grasp the structural fact that 'length' and 'duration' are not *things* inherent in the external world, nor are 'matter', 'space', and 'time', but that they appear as relations between events and some specified observer, and forms of representations, then all paradoxes would disappear

A suggestion which concerns visualization may be helpful If we realize the structural fact that words *are not* the objects they represent, we shall always discriminate automatically between what we *see, feel* , on the level of lower order abstractions, and what we *say* on the level of higher order abstractions When we have conquered that single difficulty we could never then identify the two different orders of abstractions. We would evaluate the *terms* 'matter', 'space', and 'time' as forms of representation, and non-objects, and we would describe events in a functional, operational, behaviouristic language of order If we realize and feel the *finite* velocity of propagation of all processes, we may visualize all that has been explained here Diagrammatizing and even following with one's hand, the *visualized order* of occurrences, helps enormously Try to visualize how the aviator in the last example is flying away and how much more slowly the light impressions from the earth are reaching him or his instruments, and the difficulties will soon vanish

We shall also be greatly helped in our power of visualization when we become acquainted with the structure of the Minkowski four-dimensional world. An explanation of this appears in the next chapter.

CHAPTER XXXVIII

ON THE 'WORLD' OF MINKOWSKI

> Moreover, the really fundamental things have a way of appearing to be simple once they have been stated by a genius, who was in this case Minkowski (431)
>
> G Y RAINICH

We have already freely used the structural term 'dimension' and only hinted at its meanings Before we approach the Minkowski world we must summarize roughly what for our purpose we should know about dimensions

There is nothing mysterious about the term 'dimension' First of all, the dimensionality of a manifold is not inherent in the manifold as such It is a characteristic of *order* and so of structure A manifold can be ordered in different ways, so that it follows that one manifold may have different dimensionality, depending on how we *order* it

A manifold which has linear order and structure is called one-dimensional A two-dimensional manifold is then a linearly ordered manifold of linearly ordered manifolds ,

Usually we speak about our 'space' of daily experiences as a three-dimensional manifold, but this is true only with reference to points, and not true with reference to lines or spheres The manifold of all spheres in 'space' is, for instance, a four-dimensional manifold, so also is a manifold of lines

Let us explain the line-dimensionality of our 'space' in terms of lines. A line can be given by two points—one, let us say, in the floor of our room, the other in the ceiling Each of these points is given by two co-ordinates, it has two degrees of freedom, and so our 'space' is a four-dimensional (2×2) manifold *in lines* This means that to distinguish any line in our 'space' from any other line we would have to have *four data* Similarly, if we deal with spheres, a manifold made up of spheres requires four data, three for locating the centre and one giving the radius of the sphere. The above examples, of course, do not exhaust the structural possibilities [1]

The term 'dimension' does not apply solely to what we call 'space' The term applies to any manifold which we can order in some particular way Manifolds or aggregates abound everywhere in our lives The domain of colours, for instance, is a manifold, and so is the domain of tone, or of remembrances, No manifold in itself has any dimensionality To ascribe dimensionality to the manifold we must first order it and the number of its dimensionality, or its ascribed or discovered structure, may differ according to the principle of ordering used

In discussing dimensionality we have two purposes First, to dispel the semantic fright about this simple term, and, second, to suggest *means for visualization*, which for our purpose are of great neurological importance

When we say that the world is structurally a four-dimensional manifold, we mean only that according to our experience and the structure of our nervous system, the world of our experience is represented by a fourfold order We can order the events as to the right and to the left, forward and backward, up and down, and sooner and later In our experience this fourfold order is completely united, and cannot be separated unless we deliberately choose to *neglect* some of these orderings

Nor does it mean that all these dimensions are 'identical' We are accustomed, for instance, to consider the three dimensions of 'space' as 'identical', or at least equivalent Is this true in life? Can we disregard, for instance, the structural difference between vertical and horizontal? If we did, quite probably, as Eddington remarks, we should come to an untimely end, and break our necks

Obviously if we *visualize* our *plenum*, as made up of lines or particles, by necessity we visualize structurally a four-dimensional manifold It should be noticed that a four-dimensional 'absolute void', or 'absolute nothingness', besides being non-sense, cannot be visualized at all, because it could have *no structure*

We see that all metaphysical 'fourth dimensions' are not only non-sense, but usually indicate a pathological semantic disturbance The intensity of such disturbances is often high, because it is entirely impossible for a sane person to deal with such meaningless noises The victim is obsessed with attempts to do the impossible,—a semantically hopeless and painful task

Such objectifications of terms are very dangerous and science should try, by proper emphasis, to eliminate them Outside of science the term 'dimension' has *no meaning* and ought to be definitely abandoned in our speculations, for the sake of sanity

The notion of 'time' as a 'fourth dimension' is by no means new It appeared in a vague form centuries ago The notion however was not formulated properly, and therefore was unworkable Instead of helping science, it only hindered it

Inspired by Einstein's work, the mathematician Minkowski, whose work had been mainly in the theory of numbers, began to work at the theory of manifolds of any number of dimensions In 1908 he delivered his famous and semantically epoch-making address on *Space and Time* which fused geometry and physics structurally In this address he insisted that the connection between 'space' and 'time' as given by the Lorentz-Einstein formulae is not accidental but exhibits that inner connection or structure to which we had not paid enough attention [2]

In our *experience*, 'space' and 'time' can never be entirely separated, as already explained, and so Minkowski combined them into a higher entity which is called the 'Minkowski world' In the world of experience the datum appears to be, not a place and a point of 'time', but the *event* or the *world point*—that is, a place at a definite *date*

The graphic picture of a moving point is a *world-line* Rectilinear uniform motion corresponds then to a straight world-line, accelerated motion, to one that is curved

The *event* is the most elementary notion We shall use it from now on in this work in the sense of a four dimensional volume of space-time which is small in all four dimensions We do not posit whether events themselves have structure or not, but it is preferable to assume that they have no space-time structure, which means that the event has no parts which are external to each other in space-time The order of events is fourfold, as previously shown

The aggregate or manifold of all point-events is then called the world The point-events are given by four numbers representing the co-ordinates, three giving the 'space' co-ordinates, and the fourth the 'time' co-ordinate

The term 'space-time continuum' or 'space-time manifold' is used often and implies that the numbers x, y, z, t, are to vary continuously

In such a space-time continuum all happenings are structurally the intersections of world-lines, and if we could describe the world-lines of all points of the universe we would have a full account of the universe, 'past' and 'future' We see that all physics, with the *rest of our problems*, must then be considered as a chapter of the *general* structural and semantic study of continuous manifolds of four dimensions

But we are already acquainted with such theories For instance, the internal theory of surfaces may be considered as a part of the subject in two and three dimensions We have seen that different surfaces are characterized by the expression for the line element $ds^2 = g_{11}dx_1^2 + 2g_{12}dx_1dx_2 + g_{22}dx_2^2$, or by that group of transformations which leaves the line element invariant We know already that in the E as well as riemannian geometries we have similar expressions and characteristic transformations

If physics is to be considered a branch of the theory of four-dimensional manifolds, we should naturally look for some such transformations The manifold represents the world, the *generalized** theory of relativity gives the desired answer. Minkowski proposed a postulate, which he calls the *postulate of an absolute world*, or the world-postulate which asserts the *invariance of all the laws* of nature in relation to linear transformations, for which the function $x^2+y^2+z^2-c^2t^2$ is invariant

The reader is already familiar with the expression x^2+y^2, which gives the invariant length in E geometry in two dimensions, and $x^2+y^2+z^2$ which gives it in three dimensions It would be natural to expect that in four dimensions we should have an expression of the type $x^2+y^2+z^2+t^2$ but in this case our expression is $x^2+y^2+z^2-c^2t^2$ It should be noticed that the above different types of expressions have different origins The first two arise in pure geometry, and the last has its roots in physics The problem was to bring an experimental expression into harmony with a familiar geometrical expression Minkowski introduced the expression $ict = u_4$, where i is as usual the square root of minus one, $(i=\sqrt{-1})$ Then of course $-c^2t^2$ becomes $(ict)^2 = u_4^2$

*I use the term 'generalized' to embrace the unified field theory and eventually the quantum theory, although, for our purpose, I utilize only the special and general theory

If we change the lettering, and denote $x=u_1$, $y=u_2$, $z=u_3$ then our expression $x^2+y^2+z^2-c^2t^2$ becomes $u_1^2+u_2^2+u_3^2+u_4^2$, a simple formula for distance in four-dimensional geometry It would not be profitable for us to speculate upon this substitution, it was introduced merely for the sake of mathematical, verbal treatment and can be easily translated back into the usual terms of c and t

We have already seen that the expression $x^2+y^2+z^2-c^2t^2=x'^2+y'^2+z'^2-c^2t'^2$ is invariant under the Lorentz-Einstein transformation This fact of invariance is fundamental, and it is well to convince ourselves that it is so The Lorentz-Einstein transformation was $x'=\beta(x-vt)$, $y'=y$, $z'=z$, $t'=\beta(t-vx/c)^2$ where $\beta=1/\sqrt{1-v^2/c^2}$ As the co-ordinates y and z are equal in both systems, we can disregard them, and verify that $c^2t'^2-x'^2=c^2t^2-x^2$ Let us substitute for t' and x' the values given by the Lorentz-Einstein transformation We have then:

$$c^2t'^2-x'^2=c^2\beta^2(t-vx/c^2)^2-\beta^2(x-vt)^2=c^2t^2\beta^2+\frac{c^2\beta^2v^2x^2}{c^4}-\frac{2c^2\beta^2tvx}{c^2}-\beta^2x^2-\beta^2v^2t^2$$

$$+2\beta^2xvt=c^2t^2\beta^2(1-v^2/c^2)-x^2\beta^2(1-v^2/c^2)=c^2t^2\frac{1-v^2/c^2}{1-v^2/c^2}-x^2\frac{1-v^2/c^2}{1-v^2/c^2}=c^2t^2-x^2;$$

since $\beta^2=\dfrac{1}{1-v^2/c^2}$ Similarly it is easy to show, if we take an event-particle, as, for instance, a momentary spark, which has the co-ordinates x_1, y_1, z_1, t_1, in one system of co-ordinates, say S, and let another event-particle occur in that system at x_2, y_2, z_2, t_2, that the formulae remain invariant If we designate the distance between the two events by r, its value would be given by $r^2=(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2$

In a different system, S', moving uniformly relatively to S, r'^2 would in general not be equal to r^2, but the expression $r^2-c^2(t_2-t_1)^2$ would be equal to $r'^2-c^2(t_2'-t_1')^2$, $r^2-c^2(t_2-t_1)^2=r'^2-c^2(t_2'-t_1')^2$

The above expression is called the *interval* and expresses a most fundamental structural characteristic, namely, that the interval is invariant for all systems in uniform relative motion This result is quite general and independent of the relative orientation of the axes, or of the angle the velocity has to the axes The interval plays in the theory of Einstein a similar role which the pythagorean rule played in the E geometry

Because of the finite velocity of our measuring signals, our formulae must involve finite velocity Therefore the interval is the only actual measurement which we can ever make in practice Hence its fundamental semantic importance

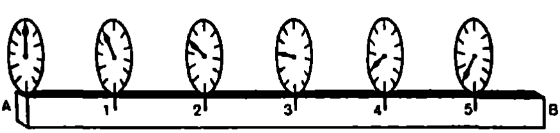


FIG 1

Eddington gives a very fine diagram in explaining how intervals are measured. I reproduce it herewith (Fig 1).

The expression ct, where c is the velocity of light, 300,000 kilometres per second, gives us the *distance* travelled by light in the 'time' t It is natural to regard the velocity of light, which is a constant and translates easily into the language of length, as a unit of 'time' In the Minkowski world it is customary, because of its convenience, to regard 1 second as the equivalent of 300,000 kilometres and measure lengths or 'times' in seconds or kilometres indiscriminately

Let us imagine a scale graduated in kilometres, and clocks whose faces are also graduated in kilometres (1/300,000 of a second) If the clocks are set correctly and we look at them from A the sum of the reading of any clock and the scale division beside it is one for all because the scale reading gives the correction for the 'time' taken by light, travelling with unit velocity, to reach A

If we lay the scale in line with the two events and note the clock and scale readings, t_1 and x_1, of the first event, and the corresponding readings, t_2 and x_2, of the second event, then $s^2 = (t_2 - t_1)^2 - (x_2 - x_1)^2$ where s represents the 'interval' mentioned above

If we set the scale moving in the direction AB then the divisions would have advanced to meet the second event and the difference $(x_2 - x_1)$ would be smaller. But this is *compensated*, because $(t_2 - t_1)$ also becomes altered When A is advancing to meet the light coming from any of the clocks on the scale the light arrives too quickly, and the reading of the clock appears smaller.

The net result is, roughly, that it does not matter what uniform motion is given to the scale, the final results for the interval s are always equal [3]

We can now understand the vital importance of the minus sign with the 'time' co-ordinate In fact, if in our equations all the signs were plus, using the 'space' and 'time' of one observer, one value of s would be obtained, but using the 'space' and 'time' of another observer, a different value would be obtained. With the minus sign for the 'time' co-ordinate, we see that we can have values of s which are equal for all observers If the distances increase, the 'time' element increases also, and so the *difference* may not be changed, but with the positive sign this would not be the case

We see that the interval s represents something which concerns only the events under consideration The corresponding entity in ordinary geometry is *distance*, which is independent of the accidental choice of co-ordinates. The minus sign makes the geometry of space-time *non-euclidean*

To familiarize ourselves with what has been already explained about simultaneity and the geometry of space-time, we will work it out once more, but now by the Minkowski method

It will be enough to use two dimensions, one represented on the X axis, the other on the T axis Let us consider three points A, B, C, at rest in our system O on the X axis (Fig 2) In our space-time they will be represented by three parallels to the T axis Let C be midway between A and B so that $AC = CB$ Let us assume that light signals are sent in both directions from C at the moment $t = 0$ We assume that the system is 'at rest', which means that the light signals propagate themselves to the right and to the left with equal velocities Hence we can represent them by straight lines equally inclined to the X axis These lines are called 'light-lines'

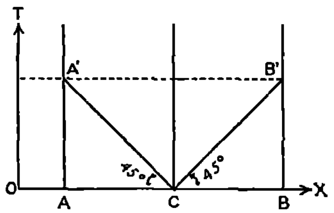


FIG. 2

The points A', B' which are the intersection of the 'world-lines' of the points A and B with the light-lines give us the 'times' at which the signals arrive It follows from the drawing that $A'B'$ is parallel to the X axis, which means that A' and B' are 'simultaneous' (equal 'times')

Let us now take another case in which our points A, B, C, move uniformly with an equal velocity (Fig. 3) Their world-lines will also be parallel to each other but *inclined* to the axis In the drawing the light-lines will be represented by similar lines but their intersections with the world-lines of A and B will not be on a parallel to the X axis, and so they will not be simultaneous

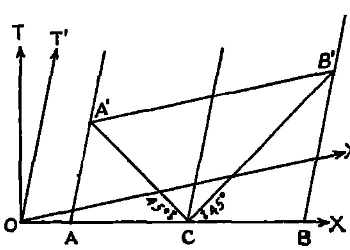


FIG 3

We should notice that an observer who moved with the system in the direction OX' would be perfectly entitled to claim that A' and B' are *simultaneous to him* His co-ordinate system would be $OX'T'$, in which the points A' and B' are on a parallel to *his X' axis* as he is *at rest* in his system $OX'T'$ The world-lines A, B, C, are parallel to the T' axis because the points are supposed to be at rest in this system and hence the x's have equal values for all t's

An important point should be noticed, namely, that we have only one space-time and that the indefinitely numerous ways different observers partition their 'space' and 'time' represent merely the indefinitely many ways in which it can be partitioned If we keep the whole of it under consideration we see that we cannot divide it into 'space' *and* 'time', as any subdivision has both aspects [4]

The Minkowski method of representation makes the change in our measurements of length, as given by the Lorentz-Einstein transformation, very obvious

A measuring rod is not purely a 'spatial' configuration, as in the actual world such a thing does not exist, but it is a space-time configuration

Every point of the rod exists at each moment of 'time' We see that in space-time we cannot represent our rod as a segment on the X axis but must represent it structurally as a *strip* in the XT plane We assume here for simplicity that the rod is one-dimensional (Fig 4)

A rod which is at rest in a system is represented by a strip parallel to the T axis. If it is moving, its strip is inclined to the T axis. The 'contraction' does not affect the strip at all but it is rather a section cut out of the X axis In actual experience, it is only the strip as a manifold of world points which has *physical reality*, and not the cross sections, which, as we see, are not equal on different axes. The 'contraction' is not a change in physical reality' but merely a consequence of our way of regarding things We see that the notorious argument as to whether the 'contraction' is real' or apparent' is based on a misunderstanding Born gives an excellent example If we slice a cucumber in different directions it is fallacious to argue that the smallest slice which is perpendicular to the axis is the real' one and the larger oblique slices only 'apparent' Similarly in the Einstein theory, a rod has various lengths according to the motion of the observer

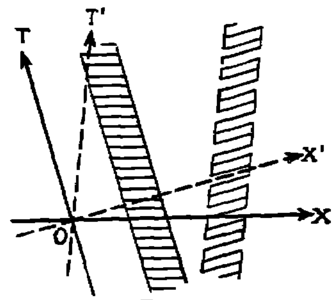


FIG 4

One of these lengths the static length, is the greatest, but it is no more 'real than any other Similar remarks can be made about 'time

Attention should be given to one extremely important semantic point concerning the Minkowski four-dimensional world We already know that for our nervous systems the passing from dynamic to static, and vice versa, is a most vital structural problem The first step of this translation has already been given in the notion of the variable The calculus carried it a step further In the Minkowski world we reach the complete solution of the problem

As Keyser points out in his *Mathematical Philosophy* we had two verbal methods of dealing with 'time' One was the method of Newton, the method of the structural importation of time' From the objective dynamic world of the lower order abstractions 'time' is imported into the static world of the higher order abstractions We import it with motion' we say things 'move'. Such language is structurally unsatisfactory, even on the earlier level of our development It hampers analysis, and is contrary to the structure and function of the *human* nervous system It breeds tremendous metaphysical impasses, and is ultimately based on semantic disturbances due to identification

If we introduce dynamic, shifting entities into static higher order abstractions, rationality is impossible and we drift toward mysticism

A very real semantic problem appears here We want to give the best possible account of the structurally *dynamic* world around us yet our higher order abstractions are structurally *static*, and for their proper working they must use static means. Here seemed to be an impasse which for milleniums had defied solution 'Philosophers' of different schools were preaching and teaching that we should never be able to be 'rational' and understand this world and ourselves Anti-intellectual schools began to flourish, to the bewilderment of all

The issue, after all, was simple, the moment some one discovered and stated it. We did not need to change either the world around us or ourselves;

we simply had to discover a structurally new method of dealing with the c squ problems without changing them

The new method is given by Einstein and Minkowski Instead of making the static world of higher order abstractions dynamic, which cannot be done at all without producing semantic disturbances, they invented structural methods for dealing with the dynamic world by static means The key was found in the handling of the troublesome factor, 'time' Minkowski decided to put 'time' in its proper place by introducing the structurally new four-dimensional world

In the case of particle P, we habitually used to say that the point P at an instant t was at a 'space' point (x, y, z) At the instant t' it was at the point (x', y', z'), . We always needed four numbers, which gave us the where and when in respect to some frame of reference

As we have seen, Minkowski decided to look structurally at this tetrad of four numbers (x, y, z, t) 'as-a-whole'. In other words, he placed himself on a higher level of abstraction He took under consideration the older results, combined them, and called the combination by one single name, the 'world point' Such a world point has also four numbers (not 3 plus 1, but just 4) A world made up of such points is a four-dimensional world in which all the points co-exist The flux of the lower order abstractions and 'time' is abolished There is no more 'motion' in a 'flow of time'. In such a world the term 'where' has completer structural meanings, it has absorbed the *when* If we ask, where in such a world the particle P is, we answer, at the point (x, y, z, t) Where is the particle P'? At the point (x', y', z', t')

We see that the particles of such a world *are never 'the same'*, they do not 'change' or 'pass', they co-exist, and all is *static* In this way the three-dimensional dynamics become four-dimensional statics

It should be noticed that we are now dealing with a language of new structure, uniquely befitting the structure and function of our nervous system Of course we have altered nothing in the world around us The example of the moving picture and the static film as given before is an excellent example of this structural innovation

The fact that in this new world nothing repeats itself because it has a different date, unless the time-lines are closed, has very far-reaching consequences, of which we have already spoken and which we will analyse in more detail later on.

CHAPTER XXXIX

GENERAL REFLECTIONS ON THE EINSTEIN THEORY

> For, beyond the bounds of science, too, objective and relative reflection is a gain, a release from prejudice, a liberation of the spirit from standards whose claim to absolute validity melts away before the critical judgment of the relativist (45) MAX BORN

It is extremely instructive to follow the elements of identification as they appear in the evolution of the Einstein theory.

We know that the results of the Michelson experiment which disclosed that light is propagated in all directions with equal velocities for all observers, irrespective of their relative velocities, could not be reconciled with the older mechanics These results disturbed the physicists profoundly and attempts were made to solve this apparent impasse In what follows we shall have to analyse incidentally the activities of some of our great scientists, men who have added enormously to our knowledge, and this fact should be appreciated What we have to say is not intended as criticism—far from it—but simply as a structural and a semantic analysis

The feeling that we objectify unduly and that we should not use a language of 'is' of identity, but that we should use an actional, behaviouristic, functional, operational language and methods, is not new in science, although the need was not formulated structurally, it is true, and therefore it never became a workable foundation The main successes in these fields were rather accidental, and were the personal prerogatives of those few men whose psycho-logical make-up urged them to achieve Objectification, which as we know, is a semantic ascribing of *objective* existence and values to terms, was bound to make its appearance somewhere.

This struggle against identification is apparent in all science, but it will suffice to point out the most striking example in the relation of the works of Lorentz and Einstein Lorentz objectified, Einstein did not We here come across a tremendous semantic fact which has to do with the *interpretation* of mathematical formulae. Lorentz on elaborate and difficult grounds, connected with Maxwell's field equations, produced what is usually called the Lorentz transformation He gave it an *objectified* interpretation Einstein introduced an entirely different fundamental interpretation of the structural *principle* involved The formulae look alike but they now have different and very simple meanings.

Hertz, whose epoch-making discoveries made wireless possible, advocated long ago what is termed the phenomenological point of view, which in our language is approximately the actional, behaviouristic, operational, functional language and method. In his writings he implicitly refused to use the vicious term 'is' of 'identity', and so to objectify his terms, which refusal in picturesque language he expressed as a refusal to legislate about 'essences'

The old E and N language of 'absolute space' and 'absolute emptiness' were for a long while structurally unsatisfactory Physicists felt that somehow they could not deal with it, but it never occurred to them that this 'absolute nothingness' is objectively meaningless, and that therefore no one can possibly deal with it Not knowing that, they politely called this non-sense a 'metaphysical question' and evaded issues by leaving the solution in the hands of 'philosophers', never to be solved

By now I hope that the reader is quite aware that meaningless problems cannot be solved by any one, and that there are no such things as *meta*-physical questions. There may, however, be a question about *enlarging* the domain of physics

Being forced to abandon this 'absolute emptiness', physicists went to the other extreme and postulated some kind of 'material' ether Let us note that such a postulate involves structurally the 'is' of identity and objectification. Lorentz in opposition to Hertz postulated an 'ether' which was 'motionless' in 'absolute space'. Note that here we have a perfect example of structural *objectification of terms* 'Absolute space' is for him semantically some kind of 'absolute emptiness', which, not being satisfactory for the physicist, is filled with some 'material', 'motionless' ether. 'Motionless' is itself an objectification of language, as such a term has here *no* physical or objective meanings at all

In pursuing the speculations on *objectified terms* (semantic disturbances) it was natural to expect, as the earth is not at rest with respect to the sun, the other planets , that some 'ether wind' or 'ether drift' should appear which would make the constant velocity of light impossible for observers moving with different relative velocities But these structural expectations were not fulfilled The velocity of light, as shown by many experiments, was a constant for all observers The 'motionless material ether' also became structurally impossible, as might be expected, if we stop objectifying terms

In 1892 FitzGerald suggested an *objectified* theory, assuming 'absolute' 'length' and 'time' *superior to measurement*, which involve identification and do not allow the use of the actional, behaviouristic, operational, functional attitudes, language, and methods FitzGerald assumed that every body 'moving' with the velocity v in the 'ether' is shortened in the direction of motion It should be noted that every mention of 'shortening' or 'contraction', *presupposes some 'absolute'* standards of 'rest' or 'motion' or 'length', which do not, and cannot, exist outside of our skin, but are only semantic disturbances, inside our skin, which occur when we identify and ascribe *objective* existence and value to *terms*

How deeply and completely these objectifications permeate our daily and scientific lives is best shown again in the case of Lorentz Even in 1917, in his Haarlem lectures, he expressed structural hopes that a 'material', 'substantial' ether can be preserved, that 'space' and 'time' can be sharply separated, and that 'simultaneity' can have an absolute meaning

In the *Theory of Relativity* of Whitehead, and in some others writers who deal with the theory of Einstein, and particularly in all critics of Einstein, we find a similar objectification of terms

They still *feel* the older E and N 'absolute emptiness', 'absolute space', 'absolute time', to which *terms* they ascribe structural objectivity In such works the term 'contraction' is used frequently

Let me recall the mechanism of objectification If we do not reject explicitly and implicitly the 'is' of identity, we automatically identify different orders of abstractions and ascribe objective characteristics to terms Thus the term 'time' which represents a label for a feeling inside our skin, is given an objective evaluation If 'objective' it must have a 'property' of 'simultaneity', a semantic process taken over from comparing two objective sticks when the two ends are made to coincide On the objective external level, we never deal with 'time' but we simply *compare processes* When we select an arbitrary unit-process on the objective level, whatever we might *say* that it 'is', well, it *is not*, and the difficulty is found exclusively in the use of the 'is' of identity

If we abandon entirely the 'is' of identity, we stop objectification, we do not ascribe objective existence and values *outside* our skin, to terms and semantic reactions *inside* our skin But then of course we have to change the *structure* of our language, as otherwise the old *s r* will continue to play tricks on us An actional, operational, functional language of *order* is the structural solution of our semantic difficulty.

If we objectify 'space' into 'absolute space', we must objectify it as 'absolute emptiness' for only such an 'absolute space' can be at 'absolute rest', that is, static in the E or N sense Similarly only objectified 'time' can have the 'property' of 'absolute simultaneity'

If we realize that these 'absolutes' are only the semantic objectifications of terms, (where the activities of the lower nerve centres are structurally ascribed to the activities of the higher nerve centres and vice versa), we begin to differentiate between different order abstractions, and to keep them differentiated In terms of our structurally new language we become 'conscious of abstracting', and then habitually *and unconsciously* use the behaviouristic language and methods of *order*

If we picture this 'absolute emptiness' or 'absolute nothingness' (which cannot be done successfully, as it has no meaning), and try to compare it with a plenum, or 'fulness' (a cloud of smoke, for instance), we see at once that only this 'absolute emptiness' can be static, homogeneous , a condition that is impossible with a dynamic fulness

Perhaps we can now appreciate the tremendous semantic significance of the Einstein theory, which introduces structurally a *non-objectified*, human, sane attitude of proper evaluation toward this world We should not be surprised to find that a $\bar{A}$-system which is an inevitable general structural concomitant of the $\bar{E}$ and $\bar{N}$ systems of geometry and physics should *formulate as a general structural and semantic issue* what the $\bar{E}$ and $\bar{N}$ systems have *done* in their special fields, without such general formulation

From our structural point of view there is no retreat, the Einstein work is irreversible. In the younger scientists of today the non-objectified attitude toward *terms* of 'space' and 'time' is already an accomplished semantic fact, entirely independent of what future experiments may show For experiments can never justify identification, and so can have no detrimental effect upon this fundamental and most beneficial structural, linguistic, and semantic revolution Our $\bar{A}$ task was to formulate these issues *in general* so as to make us conscious of them, and I assume that it is at this semantic point that the tremendous value of Einstein's work will manifest itself in life. Indeed we shall see later on in this volume that the newer quantum mechanics, which have begun to spring up rather rapidly, is made possible only by the semantic background imparted *unconsciously* (as yet) to younger physicists by the Einstein theory It is my hope that the present work may make the above issues *conscious*, and so enable us not only to impart this semantic attitude more easily and with less labour but also to benefit by them more universally in our *daily life* The problems of science and life do not differ in this respect. In both we are equally hampered by semantic disturbances, 'emotional stupors', identification, and similar difficulties, the elimination of which means better adjustment for all of us, as well as swifter progress in science

A study of the history of science shows how slow and painful scientific progress has been Now we begin to see why 'Geniuses', as history shows, are men who at least in some fields are freer from identification and false evaluation than others They are not hampered to a similar extent by 'emotional stupor'; hence they can evaluate the old *anew* Lorentz, for instance, produced the formulae, but his objectifications *prevented* him from evaluating properly the new formulae. As a fact of history the formulae of Lorentz were discovered by Voigt a number of years before, but identification made impossible the evaluation of these formulae, and so *delayed* the discovery of the Einstein theory. This factor of identification can be found all through recorded history as a retarding semantic blockage

If we could find methods of eliminating these semantic disturbances, an extremely hampering, paralysing psycho-logical factor would be eliminated, and 'geniuses' could be made the rule rather than the exception Let me say again in the old days morons were made and geniuses were born, in the new days, perhaps, this can be reversed, and *morons will be born but geniuses made.* We witness something of this kind among the younger post-einsteinian physicists, where the number of 'geniuses' is growing rapidly, in spite ot the fact that the above structural issues are not as yet consciously applied in general education The secret of creative work is freedom from structural bondage, and particularly the structural semantic bondage of words.

The reader should not assume that the few simple structural explanations given in this book exhaust the Einstein theory I have not even attempted to summarize the theory, I have only given a few semantic facts, which belong to general semantics and to the theory of knowledge The Einstein theory is indeed such a tremendous structural linguistic achievement that quite probably its full semantic significance and meanings will not be worked out for many years to come. We have given here only the minimum of explanation necessary for our special purpose.

The historical development of a theory has usually little to do with the semantic importance of the theory or its deeper meanings. The constancy of the velocity of light for all observers, which started the ball rolling, was an historical beginning and it served its purpose well, though the objectified 'contractions' and formulae of FitzGerald and Lorentz also did their share, as they helped Einstein and Minkowski to produce their epoch-making structural challenge to old prejudices such as 'absolute space' and 'absolute time', which were semantic remains of a primitive, perhaps pre-human, remote past. Once this is accomplished, no matter how, there is no return possible. Of physical structural facts, all that we need is the *finite* velocity of the propagation of events,* which as we already know involves far-reaching structural and semantic issues. Of the psycho-logical issues involved, we need only to eliminate semantic disturbances which still occur when we copy animals in our nervous processes and do not discriminate between different orders of abstractions—which animals do not recognize. This elimination can be done by training in the $\bar{A}$ methods explained before, with the net result that we become 'conscious of abstracting' on different levels and so can instinctively and by feeling discriminate habitually between orders of abstractions, which structurally and semantically could not be done by the old disciplines.

The theory of Einstein has manifold applications but we need only mention a few, which we shall utilize later on.

First, and above all, there are no possible 'absolute' meanings to 'space' and 'time', beyond the relations established by measurements. The structure of our language involving 'space' and 'time' should be similar to the structure of experimental facts, which ultimately show the impossibility of sharply dividing them.

If any one challenges this statement, he could not *a priori* be criticized. Such criticism would be entirely against the whole tendency of the present work. But such a person might be approached with no little curiosity and expectation. He could be asked: 'You claim that you can absolutely divide

*'But,' some reader may ask, 'though you assume a finite velocity of propagation, may it not happen that some day an "infinite" velocity will be discovered?'

Such a question would show that the reader has missed the point in the present work. We are confident in saying that an 'infinite' velocity has no meaning, and that no matter what we discover, this will never be discovered. This becomes still clearer if we use the differential *definition* of 'velocity'. Velocity is defined as the 'time' derivative of 'space' travelled. If 'time' is taken as zero, or if we have 'no time', there can be no 'time derivative' by our very assumption, and, therefore, no 'velocity'. There is, therefore, no danger that we shall ever discover in the actual world an 'infinite' velocity.

"space" and "time" on the objective level That would be an epoch-making structural discovery *Please demonstrate how to do it.*'

The fact is, of course, that he cannot demonstrate the process, because he refers to identifications inside his skin, yet he is claiming to be able to show it *objectively* outside his skin That ends this problem.

While speaking of Einstein's Theory, it will be well to mention a few of the many structural differences between the older newtonian and the new einsteinian mechanics

In the N-system, relative velocities were simply added $W_N = v + v'$ In the einsteinian system which we will denote by $\bar{N}$, it is not so structurally simple. We must introduce the finite velocity of propagation of our signals, which *alone* give us the data, and so

$$W_{\bar{N}} = \frac{v+v'}{1+vv'/c^2}$$

The above formula involves the remarkable constant, c, the velocity of light If we assume in the above formula that our velocity v' is equal to the velocity of light, c, we would have

$$W_{\bar{N}} = \frac{v+c}{1+vc/c^2} = \frac{v+c}{1+v/c} = c$$

This means that the addition of some velocity to the velocity of light does not alter the velocity of light, which thus appears as a *limiting* velocity

This applies to the difference of velocities where

$$W_{\bar{N}} = \frac{v-v'}{1-vv'/c^2}$$

Let us here give an example of Eddington's Let us assume two relative velocities each differing by only 1 km /sec from the velocity of light Let us say that one is 299,999 km /sec and the other 300,001 km /sec Now let us calculate the relative velocity This relative velocity will be found to be 180,000 000,000 km /sec For in our formula $v - v' = (c+1) - (c-1) = 2$, and

$$(1 - vv'/c^2) = 1 - \frac{(c+1)\ (c-1)}{c^2} = 1 - \frac{c^2-1}{c^2} = 1 - 1 + 1/c^2 = 1/c^2,$$

whence $W_{\bar{N}} = \dfrac{2}{1/c^2} = 2c^2 = 2 \times 300{,}000 \times 300{,}000 = 180{,}000{,}000{,}000$

We see that a particle which might try to overtake light by having a velocity of one km /sec greater than the velocity of light could never succeed When the velocity 299,000, for example, was reached, the particle would find itself further away from its goal than when it started [1]

Similar general considerations apply to mass If we designate the mass of a particle at rest by m_0 its mass in motion

$$m_{\bar{N}} = \frac{m_0}{1-v^2/c^2}$$

As the denominator is smaller than unity the mass in motion, $m_{\bar{N}}$ is larger than m_0, the mass at rest In the limiting case, when the velocity would become equal to c the denominator would become zero and our mass $m_{\bar{N}}$ would tend toward infinite values, which is another way of saying that it is physically impossible

In the N-system we had two kinds of energy, one was called *vis viva*, or kinetic energy, and was represented by $T = mv^2/2$, the other was called potential energy, or capacity for work, and was denoted by U The law of conservation of energy in the N-system was expressed by the statement that the sum, $T + U = E$, or the total mechanical energy of a system remains constant (zero variation) during the motion of the body.

We see that as the above formula involves the terms m and v, the older formulae for energy must be altered, especially since they do not survive a Lorentz-Einstein transformation It is found that

$$T_{\bar{N}} = c^2 (m - m_0), \text{ or } m = m_0 + T_{\bar{N}}/c^2,$$

which formula appears rigorous as a definition of kinetic energy even if members of order higher than the second are taken into account In words, the mass in motion differs from mass at rest by the kinetic energy divided by the square of the velocity of light.

This expression suggests immediately that the static mass, m, is similarly related to the energy content in the body at rest Generalizing our results we would have $m = E/c^2$, an equation which holds generally between mass and energy This fact has been called by Einstein the law of the *inertia of energy* It has been verified repeatedly by experiments, and is one of the most striking structural results of Einstein's theory The above statement means that the two fundamental notions of 'mass' and 'energy' are equivalent and thus we have a clearer vision of the structure of 'matter' The two older structural laws of 'conservation of matter' and of 'conservation of energy' become fused into one Mass becomes structurally and verbally nothing else than energy concentrated at a point, and it appears as a form of energy manifestation [2]

The above considerations have also led to a revision of our structural notions about 'energy' which we do not need to explain here Suffice it to say that the old 'potential' energy is not associated structurally any longer with any features of this world. It can be made to vanish by a proper selection of co-ordinates, hence it is no longer considered as energy of any kind [3]

With the Minkowski world we became acquainted with a new *language* which represents structurally more nearly the facts of experience (lower order abstractions) and shares the structure of our higher order of abstractions So we have the language of 'space-time' How about 'matter'? The bumping against something hard is not to be disregarded True, *we need a language of new structure, but that is all* In the Einstein theory, 'matter' of course is not treated separately as such It is an offspring of the field, and is connected with the curvature of the world The reader should not be surprised to find that the Minkowski world, which has accelerations, must be curved in this structurally new form of representation

We have already defined a most fundamental entity called 'action' Naturally in a space-time manifold, energy multiplied by 'time' should be a more fundamental entity than energy, and we call it 'action' When we speak about some continuous material present in 'space' and 'time' we speak in terms of density Density multiplied by a three-dimensional volume of 'space' gives us mass, or what appears as its equivalent—*energy* From a four-dimensional, or space-time point of view, density multiplied by a four-dimensional volume of space-time gives us action We see that the multiplication of density by the three dimensions of 'space' gives us mass or energy A fourth multiplication by the dimension of 'time' gives us mass or energy multiplied by 'time' which becomes action by definition It is obvious that, structurally, action must be more fundamental than the older quantities

In terms of curvature, action represents the curvature of the world, because where we find 'action', we also find 'matter', acceleration, gravitation , [4]

'Action' is fundamental, because structurally in a four-dimensional metrical manifold it takes the form of the simplest integral *invariant* that can exist at all On this form of action Maxwell's electromagnetic theory is built The quantity action appears as a pure number,[5] a unique, specific relation which conditions structure

We should expect that the action represented by the number 1 would be most interesting and would eventually represent the indivisible atom of action The modern quantum theory seems to favour such a point of view.

When we encounter a pure number having such crucial significance in this world we should not wonder that such a number intrigues us As yet it is impossible to state that action cannot have fractional numbers What, then, would the action represent?

Eddington suggests that the number may represent a *probability* or some function of a probability

We combine probabilities by multiplication, but we combine actions in two regions by addition We see, therefore, that the logarithm of a probability gives the function indicated and Eddington suggests the provisional equivalence of action with the negative of the logarithm of the statistical probability of the state of the world around us Such a suggestion is extremely appealing and important because the principle of *Least Action* can be stated as the principle of *greatest probability* The laws of nature appear to be such that the actual state of the world is represented by that which is statistically the most probable[16]

That such structural conclusion can be drawn at all is of tremendous semantic importance for us because, as we are *abstracting* in different orders all through, the only appropriate language in which we can eventually hope to speak correctly, is the language of probabilities, statistical averages ,

Action is one of the terms of pre-einsteinian physics which has survived unmodified, the only other one being entropy The law of gravitation, the laws of mechanics, and the laws of electromagnetism, can all be, not only summed up, but also deduced, from a single principle of least action This important structural unification was accomplished even before the advent of the einsteinian theory, and only the addition of gravitation to this list is new [7]

In this brief structural and semantic survey we have had neither the opportunity nor the necessity of analysing the general theory of Einstein, which embodies and unifies most of the laws of mechanics, that of gravitation included * In this unification lies the unrivalled grandeur of the theory As we shall see later, the newer quantum theories have been already very much influenced by the Einstein theory As all possible theories are dependent on *human ingenuity* and never can be the events themselves, we can rest assured that once freed from 'emotional stupors' and semantic disturbances, the world will not be long in producing a whole structurally unified system of science

In our discussions we deal with 'apparent', 'real', 'actual', and similar *m o* terms. We should recall that mathematics is *exclusive* in one respect, namely, that it has no content. It is entirely a product of higher abstractions created by definition from undefined terms We have seen that mathematics must be considered as a language of special structure which is, however, similar to the structure of the world around us

Our daily A language, among others, being based on the 'is' of 'identity', can never give a structurally satisfactory picture of this world or ourselves, but actually prevents such an achievement Having abandoned a language which leads to identification, we shall be able to apply a new language, with new structure, by which we achieve better means for representing the events around us. From this point of view, mathematics and our daily language do not differ Terms, being *not* the things they represent, must by necessity be creatures of definitions and undefined terms The solution of many baffling semantic problems is found in the *structure* of a language which involves different semantic and unconscious attitudes.

*In fact, a few months ago, Einstein and Mayer succeeded in reducing the laws of gravitational and electromagnetic fields to a single basis This was accomplished by the aid of a very revolutionary mathematical discovery that it is possible to introduce into a 'space' of n dimensions, vectors with m components Although at present the results of the quantum theory are not included in this theory, there is no doubt that shortly, because of this mathematical discovery, these will be included in a *generalized* theory of relativity.

PART X

ON THE STRUCTURE OF 'MATTER'

Rather against my better judgment I will try to give a rough impression of the theory. It would probably be wiser to nail up over the door of the new quantum theory a notice, "Structural alterations in progress—No admittance except on business", and particularly to warn the doorkeeper to keep out prying philosophers. (149) A S EDDINGTON

CHAPTER XL

THE OLDER 'MATTER'

And yet when I hear to-day protests against the Bolshevism of modern science and regrets for the old-established order, I am inclined to think that Rutherford not Einstein, is the real villain of the piece (149)
A S EDDINGTON

Micro-mechanics appears as a refinement of macro-mechanics, which is necessitated by the geometrical and mechanical smallness of the objects, and the transition is of the same nature as that from geometrical to physical optics. (460)
E. SCHRODINGER

From the dawn of history, man has had to deal with different bits of materials, some hard and solid like stones, some soft like fruit or flesh, some liquid In remote antiquity air and gases were not considered as 'matter'.

In those days 'matter' was structurally only what could be seen, or felt, or touched : anything else was some kind of 'spirit', and everything 'existed' in an 'absolute void' But even in remote antiquity our primitive ancestry could not miss the fact that the bits of materials they dealt with could be divided into smaller bits Naturally, if we can subdivide bits into smaller bits, an interesting question arises How far can this division be carried on? It seems that Democritus (about 460-360 B C) was the first man on record to formulate an atomistic theory He already postulated structurally a subjective world picture, to be contrasted with an 'absolute' or objective world in which 'motion' was all important This theory started us on the mechanistic road formulated for macroscopic events, and also on the road of individualization, the study of smaller and smaller bits of materials and the search for some unit bricks out of which this world appeared to be built, all of which was already a search for *m o* structure.

With the advent of chemistry some further fundamental structural light was thrown on the problem of individualization It was found that certain materials, as, for instance, iron, copper , remain one material, no matter how far we carry our subdivision These were called 'elements' At present we recognize 92 elements, a number which is supposed to represent all possible elements Out of these a remaining few were at first predicted theoretically, and just the other day discovered experimentally All other materials do not stand division so well At some stage they decompose into their elements The smallest bit of one of these last materials which still has the characteristics as the bulk, is called a molecule. The molecule is found to be built up from atoms of the elements For instance, the molecule of water still has the characteristics of water, and consists of two atoms of hydrogen and one atom of oxygen, which are no longer water but elements of entirely different characteristics *

Electrochemistry taught us in the meanwhile an important structural lesson; namely, that definite electrical charges are combined with the atoms Such electrified atoms are called 'ions' (Greek for traveller) For instance, a

*The above statements are over-simplified, but satisfactory for my purpose molecule of water is broken up into a positively charged hydrogen ion consisting of two hydrogen atoms, and a negatively charged oxygen ion consisting of one oxygen atom

But electricity had more structural surprises in store for us About 1880 new facts were discovered One of them was that a moving electrical charge has the effect of an electrical current, namely, it can deflect a magnet just as a current does Such moving electrical charges were called convection currents, and the fact that they produce effects similar to an electric current led J J Thomson to a surprising conclusion According to the Maxwell theory of electromagnetism a certain amount of energy must be associated with every electric or magnetic field If an electrical charge in motion can produce magnetic effects, hence energy, it was concluded, and verified by experiment, that energy was required to set an electrical charge in motion From which it follows structurally that an electrical charge possesses a characteristic in common with other materials, namely, inertia, which can be overcome only by the application of energy This inertial mass of the electric charge was called electromagnetic mass

Here we see two fundamental structural issues involved One is that electricity seemingly has an inertial mass similar to that of 'matter' The other is, that in convection currents, we find means to study electromechanical parallelism, and so discover the relationship between electrical and mechanical theories

In the year 1895 Lorentz proposed the electron theory He assumed structurally that moving molecules contain electrical charges and so produce convection currents These charges are further assumed to be one electrical quantum and were called electrons The electron theory proved to be enormously fruitful and all further advance in our structural knowledge is intimately connected with it

As knowledge advanced, the more convincing became the structural evidence for some electronic theory As already noted, a moving electrical charge produces magnetic effects So also should a moving electron, whence we succeeded in accounting for magnetic effects in terms of moving electrons which in this case represent molecular convection currents

Granting this, a revolving electron should represent also a small magnet and a mechanical gyroscope as well Einstein in 1915 verified this assumption by an experiment If these structural assumptions were true, then, by a quick reversal of magnetism, a soft iron rod should turn by a slight but definite amount The reverse effect has also been verified, namely, a soft iron rod when rotated rapidly around its axis becomes magnetized

The discovery of radioactive materials was also of enormous structural importance, as it gave us a means of studying directly the rays emitted from these materials The rays were found to be of three kinds and were called by the first letters of the Greek alphabet The α-rays have been found to be similar to positive rays, the β-rays similar to cathode rays, and finally the γ-rays to Rontgen-rays Further investigation revealed that the α-rays were atoms of helium charged with a double positive charge of electricity, and that the β-rays were negatively charged particles with the charge of an electron

These few remarks already make apparent the structural fact that electromagnetic phenomena exhibit characteristics quite similar to those of 'matter', some of their processes are atomic, they have inertia ,

In the older days we tried to apply macroscopic mechanical structural laws to electromagnetic phenomena, but were not very successful The laws which applied to those sub-microscopic levels were seemingly different from those which applied to gross macroscopic levels, just as the psycho-logics of the individual differ from the psycho-logics of the mob

An epoch-making semantic step was taken by Rutherford when he formulated the electromagnetic theory of the structure of 'matter' In this theory the atoms represent complex structures built up of positive and negative electrons, and their number and arrangements (structure) determine the chemical and physical characteristics of the atom in question The old structural dogma of the immutability of the elements became untenable, and today, theoretically, and in a few instances experimentally, it has been established that a transmutation of elements is not only a possibility but a rather well-established structural fact of 1933

It should be noticed that once more one of the fanciful linguistic, structural, 'infinities' has been abolished The elements appear as transitory processes with a 'life' of a *limited* span of years The experimental structural evidence which physicists and chemists have gathered is overwhelming and, though the *positive* theories (verbal structures) may not always be satisfactory, the *negative* results leading to the rejection of the older theories are conclusive This point is of supreme structural importance to us

A short description of the different atomic models comes later in this chapter, but first something must be said about the old quantum theory, which represents at present the central problem in science and out of the solution of which the most revolutionary consequences are bound to follow

The main problems of the quantum theory may be described and contrasted somehow as follows If we take a line $X'X$ and select a point O as the origin, we may fix the position of a point P on this line by the co-ordinate x In practice we find the values of x by measurement If we assume that x varies *continuously* we can expect that by refined measurements we can find values of x as close together as we please

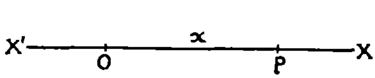


FIG 1

Experiments show that for the processes going on 'inside of the atom' the structural conditions are somehow radically different In comparing them with the above example, we should have to give our point only the freedom of occupying certain discrete points, let us say 1, 2, 3 , but all fractional values, such as 1/2 or 2/3, would be impossible If the only possible values of x are whole numbers, then the possibility of finding values of x as close together as we choose in order to make more precise measurements is excluded. If we find, for instance, that our x is neither 1 nor 2 nor 4, and cannot be larger than 4, the only solution is that our x must be 3

These conditions, which are actually found in the atomic mechanics, represent an entirely new and unexpected structural state of affairs The whole of the older quantum mechanics can be summarized in the statement, that its peculiarity consists in the fact that structurally characteristic *discrete numbers* make their appearance, and that the processes 'inside the atom' are to be described by *discrete* numbers

The usual classical quantum mechanics demands that x be allowed to take all possible continuous values, but that the *integral values* of x represent the so-called stationary states through the quantum conditions Under such conditions the intermediate fractional values had no meanings

It is safe to say that, when in 1900 Planck formulated his quantum theory along the structural lines sketched here, it meant a complete and revolutionary structural and semantic departure from all accepted standards in life and science, for studying this world

Planck has shown that it is impossible to explain the spectral distribution of the energy radiated by a black body under the older assumptions that energy can be divided indefinitely into smaller and smaller parts, but that it may be explained on the structural assumption that the energy exists in quanta of finite size $h\nu$, where ν is the frequency of the radiation and h is a constant ($h = 6\ 54 \times 10^{-27}$ erg sec)

These observations lead to the revolutionary structural conclusion that the emission of radiation occurs *discontinuously*, and so the characteristic discrete numbers make their appearance

It seems natural that because of this peculiar appearance of whole numbers, periodic processes such as rotations or oscillations should be closely related structurally with the quantum theory As a matter of fact the most important structural and semantic reconciliation of continuous differential equations of the older mechanics with the appearance of discontinuous whole numbers has been solved by the newer quantum mechanics on this basis as explained in the following chapter

The kinetic theory of heat and the atomistic theory of electricity have shown an enormous productivity It is quite natural that these theories (verbal structures) should be highly workable, considering the structure of our nervous systems, as explained in the foregoing chapters It was this principle of individualization which helped so greatly The quantum theory is a structural attempt to extend this method of individualization, or the atomistic principle, to processes themselves

As in the older days we introduced units or elementary quanta of mass, and later, an elementary quantum of electric charge, so in our newer knowledge we have need for an elementary quantum of action Action is defined as energy multiplied by 'time', or $A = Et$

Naturally such a product as energy multiplied by 'time' must play an extremely important structural and semantic role in this world of space-time, where nothing happens 'instantaneously', but all action requires 'time'. If we could discover some unit of action, we could change from the language of 'energy' and 'time' to the language of 'action' and 'times'. This language, by the way, is much more satisfactory and structurally closer to experience than the old languages. 'Action' as structurally defined (product of 'energy' by 'time') is one of the two fundamental entities of pre-relativity physics which have survived the Einstein revolution It is really a universal term which we can apply without danger of degrading science into private gossip From the neurological standpoint, as it deals with definite units and times, such a term has all the structural earmarks of an abstraction of highest order and of being really semantically important. Energy in space-time must by necessity be reformulated as 'action'. The quantum theory posits structurally that the action of physical processes is built up of a number of elementary quanta of action

From the fact that electromagnetic waves and light-waves have one velocity, Maxwell concluded that light-waves are of an electromagnetic character, a conclusion which further experiments have fully justified Einstein, in 1905, successfully applied the quantum structural principle to the theory of light, and in 1907 to the theory of heat of solid bodies

The *evolution of our theories concerning the internal structure of atoms* has, until lately, closely followed our astronomical theories, but with the newest quantum mechanics this structural analogy seems less useful The first atomic model on an electrical basis was proposed by J J Thomson He assumed the atom to consist of a uniformly dense spherical volume charge of positive electricity, within which electrons described circular orbits. But the discovery of radioactivity and the fact that the alpha-rays could pass through several centimetres of atoms (which means penetrating through many thousands of atoms), without their direction being altered made these assumptions structurally untenable

The Wilson photographs (Fig 2) show clearly that a single atom can deflect the alpha-particle by a large angle which makes it clear that the nucleus of an atom must be considered as a very small part of the volume of the atom The large deflections of the alpha-particles show also that the mass of the nucleus must be much larger than the mass of the deflected particles Observations show also that the deflections increase with the atomic weight of the deflecting materials These and similar facts led to the structural assumption that the mass of the atom is principally concentrated in the nucleus and that the mass of the electrons must be very small in comparison with that of the nucleus

FIG 2

As the atoms are in general electrically neutral, we had to assume that the positive charges of the nucleus are compensated by the negative charges of the electrons These and other structural considerations led Rutherford to propose a different atomic model, which was much more successful for a while

The Rutherford atom is supposed to be composed of a nucleus of positive electricity surrounded by negative electrons The simplest atom is that of hydrogen, and was assumed to consist of one electron revolving around the simplest positive nucleus or proton, each having a charge, $e=4\ 77\times10^{-10}$ *e s u*, and different masses, the mass of the nucleus or proton being 1845 times the mass of the electron The other atoms represent more complex structures built up of protons and electrons, into the details of which we need not enter here. But this theory encourntered difficulties of theoretical as well as of experimental character. Niels Boh eliminated most of them by the application of the quantum theory to theatom ,

For simplicity of writing in that which follows, I will use a descriptive language omitting in each statement 'we assume' . , but the reader should be contnuously aware, that when we deal with the sub-microscopic levels we deal ony with inferential units the representation of which involves a great many assumptions For my purpose it is enough to stress (1) the *negative* fact, that the structure of materials is definitely different from that which was assumed before the advent of the quantum theories, (2) that *in science*, inferential units represent abstractions of higher order and are as reliable as the lower order abstractions which we gather on the macroscopic levels, *if* they are treated semantically as *hypothetical* units The layman should realize that his 'world-outlook' appears as full of assumptions as any scientific one, except that his assumptions are not conscious and cannot be verified, whereas most of the scientific assumptions are conscious and are *continually verified.*

In the older theory the orbits of the electrons were supposed to be arbitrary, in the Bohr theory the orbits have precedence, for which a definite magnitude, a whole number multiple of the elementary quantum of action is specified We posit one-quantum orbits, two-quantum orbits , to which definite values of the orbits, the velocity, the number of revolutions, and the energy correspond. In a one-quantum orbit, for instance, the velocity is supposed to be equal to $c/140$, that is one 140th of the velocity of light, and the number of revolutions equal to 6000 billions a second

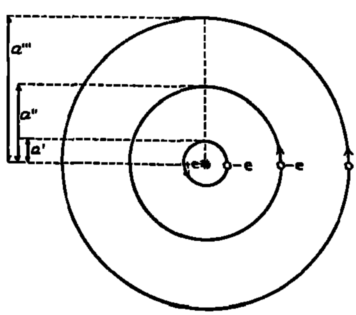


FIG 3

Bohr later modified his atomic model structurally by taking into account the movement of the nucleus. The electron was not supposed to revolve any longer around the proton, but both proton and electron were assumed to revolve around their common centre of gravity In the simplest form the Bohr atomie model is shown in Fig 3, representing the atom of hydrogen, which we assume to consist of a nucleus with one positive charge and of an electron that revolves about this nucleus The nucleus is designated by a star, the three circles represent the possible orbits for the electron The orbit of radius a' is the most stable, and usually the hydrogen electron is supposed to be found there, but through the action of heat or electric fields or collisions . , the electron may be removed to one of the outer orbits a'' or a'''. Such a condition is not so stable, and sooner or later the electron is assumed to return to the orbit a' During these transitions of the electron, energy is radiated This structural model is similar to the copernican planetary system, the planet-electrons revolving around the sun-nucleus [1]

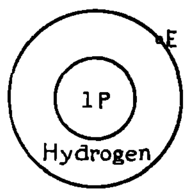


FIG 4

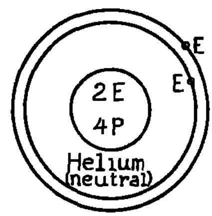


FIG 5

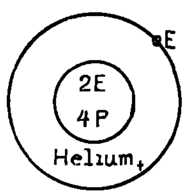


FIG 6

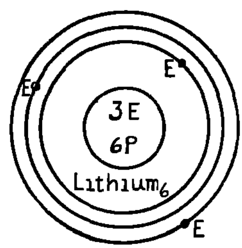


FIG 8

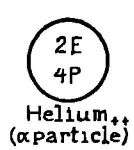


FIG 7

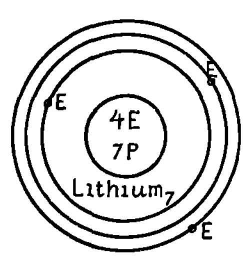


FIG 9

P = proton E = electron

The above diagrams show schematically the supposed structure of some of the simpler atoms Fig 4 represents the hydrogen atom, consisting of one proton and one electron revolving around the proton The mass of the proton is about 1845 times the mass of the electron, and we assume that the proton effectively gives us the mass of the atom

Fig 5 represents the neutral atom of helium Its nucleus consists of four protons and two electrons, and it has two revolving electrons, in all, four protons with four positive charges and four electrons with four negative charges, the eight charges just neutralizing each other

Fig 6 represents a helium atom which has lost one electron It has, therefore, four positive charges and only three negative charges Such an atom has a resultant positive charge, and is denoted by He_+ If the helium atom loses two electrons it is doubly charged with a positive charge (He_{++}) The helium nucleus He_{++}, as shown in Fig 7, represents the particle emitted from radio-active materials

Lithium consists of two isotopes, that is, two elements which appear extremely similar to each other in physical and chemical characteristics, but differ from each other in the number of electrons and protons In Fig 8 is shown lithium$_6$, with 6 protons and 3 electrons in the nucleus, and 3 revolving electrons In Fig 9 is shown lithium$_7$, with 7 protons and 4 electrons in the nucleus and 3 revolving electrons [2]

In general terms, Bohr tried to account for all other atoms on the base of the structure of the hydrogen atom The next important generalization and extension of the Bohr theory was accomplished by Sommerfeld about 1915 The achievement of Sommerfeld can be compared with the advance which Kepler made over the copernican theory of planetary motions Copernicus considered the planetary orbits as circular. Kepler* considered them as elliptical, and thus introduced a tremendous structural advance in astronomy Sommerfeld replaced the circular orbits of Bohr by elliptical ones The theory became much more complicated, because a circle is given by one magnitude, namely, its radius, while an ellipse needs two data, its major and minor axes, and so two quantum numbers for the specification of an orbit. Sommerfeld also introduced some of the results of the Einstein theory, for example, that the mass of a body also depends on its velocity Since the velocity of the negative electrons in the atom is supposedly very large it was quite probable that the relativity considerations should be appreciable According to the Einstein theory, the faster a body is moving the greater is its mass In an elliptical orbit the electron should have a larger mass at the perihelion than at the aphelion, and so the orbit would not be exactly an ellipse but the perihelion would advance slightly at every revolution

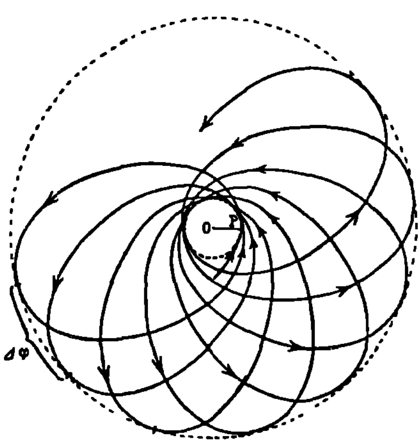


FIG. 10

Fig 10 gives us the relativistic Kepler orbit as introduced by Sommerfeld O is the fixed focus in which the nucleus is situated and P is the initial position of the perihelion The motion of the perihelion occurs in one sense with that of the orbit [3]

The last analogy in the structure of the atom taken over from astronomy was introduced in 1925, when Goudsmit and Uhlenbeck proposed their theory of the spinning electron The electron was supposed to be spinning about its axis like a planet or a top A similar notion was used by Compton in 1921, in connection with the magneton, but the notion of using the spinning electron for

*Kepler's first law states 'The planet moves in an ellipse, at one focus of which the sun is situated Perihelion is that point of a planet's orbit at which it is nearest the sun Aphelion is that point of a planet's orbit at which it is farthest from the sun.'

the solution of a structural difficulty in the quantum theory, and thus assigning a fourth degree of freedom to the electrons, originated with Goudsmit and Uhlenbeck independently of the work of Compton[4] It is not necessary for our purpose to follow all the further refinements of the classical theories Suffice it to say that scientists work under uniquely severe mutual supervision, and that any theories advanced in science are taken under consideration only when the new theories agree better with experiments, and when they also prove structurally fruitful in predicting new experimental facts, which again must stand the test of experiment

The Sommerfeld orbits have proved to be an advance over the older Bohr orbits, but they had also to be refined to take into account that the electron does not seem to revolve around a simple nucleus but around a *core* consisting of the nucleus and one or more electrons, and so again we had more complex orbits

For our semantic purpose it is enough to say that to the best of our knowledge (1933), this world appears entirely *different* from what our primitive ancestors knew thousands of years ago, and perhaps from what the average layman knows today As the problems of 'sanity' represent problems of semantic *adjustment*, and adjustment means adjustment to something—in this case to the structures of the world around and in us—it appears imperative that we should take into account the best knowledge we have of these structures

The few remarks given above about the structure of 'matter' already show unmistakably that the old 'matter' is not so very 'material', so very 'solid', so very definite, as we once assumed it was, but it represents a *process* We see that our nervous system, because of its gradual growth and evolution, has developed different levels or strata, our 'knowledge' also has different levels or strata, operative as-a-whole, although different aspects of it can be analysed in terms of order The reader should realize that because of the old *s r* we still 'need' some 'bits' of something to speak about It is a *linguistic* semantic consequence of our pre-scientific, *el* language, which posits absolute 'matter', 'space', and 'time' Thus, through a process of identification, we ascribe to these terms objective existence In the old manner of speaking the term 'is' of identity played the main semantic havoc

In the older days electrons were often taken as 'bits' of something or other For the layman a 'bit' was identified with 'matter', and here a great deal of confusion comes to light Even a 'bit' of something is not necessarily material Materials, by *definition*, are supposed to exhibit colour, temperature, hardness , . A 'bit' which did not have these characteristics would *not* be material by definition

Although the 'electron' is defined as an electrical charge, in the older days we had the habit of considering the electrons as some definite 'bits' of something, some kind of 'matter' Through a process of objectification we made them revolve in definite 'orbits', with definite 'velocities' , which implies the definite application of *terms* such as 'space' and 'time', derived from *macroscopic gross experience*, but not necessarily applicable to the sub-atomic levels

What has been said here about the structure of 'matter' is quite sufficient for our purpose Here, as always, the *negative*—the 'is not'—results count We are in a position to realize by now that the overwhelming evidence which science gives and which would be impossible to repeat here, shows us a structural picture of the world of tremendous complexity, beauty, and mystery of a structure undreamed of by our primitive ancestors who formulated the current mythological structures which moulded our older *s r* and languages

We can sum up for our purpose, what we know about the structure of 'matter' somewhat as follows The bits of materials visible and invisible to the unaided eye seem to be less simple than we assume them to be and to the best of our knowledge (1933) represent extremely complex processes of a dynamic structure. It appears also that our usual forms of verbal representation which were built by our primitive ancestors are not similar in structure to the world and so are not fit to represent the happenings going on on the un-speakable levels. As all our knowledge is due to the structure and function of our nervous system, which represents an abstracting mechanism, all our knowledge therefore, appears as some kind of abstractions of different orders, on different levels, of different character, and of varying precision and intensity, resulting in various definite general or individual *s r*

To bring what is said here to the lower level of abstracting, namely, to the level of structural visualization and feeling, we may use the rough analogue of an electric or mechanical fan When such a fan rotates we *see a disk*, simply because our nervous system was evolved under natural conditions necessitating integration, and so does not discriminate between the rotating blades The separate rotating blades are visually abstracted by us as a single solid disk, although there is no disk present

To the best of our knowledge atoms represent very minute energetic configurations or dynamic structures where extremely rapid processes are going on, which our nervous system abstracts as 'solid' Judging by our present standards in science and the amount of knowledge we have we may consider that science in the days of Newton (1643-1727) was in its infancy During that period we knew a little about the shining specks we see in the skies, and more about the rough macroscopic facts of our daily experience The genius of Newton not only advanced the detailed knowledge of his day in many branches of science, but also formulated two general theories One was the differential and integral calculus, which he discovered independently of his contemporary Leibnitz, the other was what we call mechanics

In Newton's era the problems of macroscopic, microscopic, and sub-microscopic levels of investigation had not yet arisen in the modern sense, although in formulating the differential and integral calculus a theoretical structural step was taken toward the analysis of the processes on the subtler levels Quite naturally we applied the wisdom we derived from Newton to all phases of life and knowledge With the advent of more detailed structural knowledge of electromagnetic phenomena which occur on sub-microscopic, as well as macroscopic levels, difficulties began to appear It seemed as if the newtonian mechanics were not entirely applicable to these new and smaller scale phenomena Finally Maxwell (1831-1879) produced his famous theory of electromagnetism This theory appears structurally at variance with the classical mechanics Attempts were made to reconcile both kinds of phenomena in one theory The problems of macroscopic, microscopic, and sub-microscopic structure and levels came to the foreground

With the advent of the quantum theory further difficulties made their appearance. It became quite obvious that neither the classical 'continuous' mechanics, nor the classical electromagnetic theory could fully account for the 'discontinuous' quantum facts The situation became acute and bewildering The Einstein theory with its profound structural semantic and methodological revolution liberated us from our semantic delusions of the uniqueness, absoluteness, and 'objectivity' of 'matter', 'space', and 'time' It built up a new semantic attitude in the younger generation of scientists already educated on this new structure, and therefore unhampered by the old prejudices New theories are now being formulated along increasingly more constructive and creative lines

It is true that as yet neither 'psychologists' nor 'philosophers' have paid enough attention to the subjects discussed here, and so have not made us conscious of the structural and semantic problems involved However, the Einstein theory has had a profound structural influence on the semantic attitudes of the younger scientists, though in the main they are unconscious of this fact

The main issues at hand are twofold One is semantic, namely, to inculcate the permanent structural feeling that words are *not* the things they stand for If applied habitually, this leads to the rejection of the term 'is' of identity The other is to replace old languages and methods by structurally new languages and new methods, in which when we describe ordered happenings, we describe the functioning, the behaviour , by speaking more in a language of what something 'does' than in the old language of what something 'is', which as we have seen *must* be always structurally fallacious and semantically dangerous

The reader should not take lightly these most general structural and semantic issues They are unusually important for sanity When they are formulated we can pass them on, and train children in the new *s r* quite easily It is much more difficult, after training a child thoroughly in the *old* vicious structural semantic habits of identification, eventually to have to appoint a guardian angel to watch him day and night to remind him that a word *is not* an object , Such a procedure would lay a terrific strain on us and on the guardian angel It would probably be also very expensive, judging by our present earthly substitutes for the 'heavenly powers'

Once this is realized and applied, the second issue becomes a purely structural linguistic one There is no *a priori* reason why a language which applies to one level should apply to another

With these two main issues in view, it is readily understood why modern science tries so hard to develop functional languages and methods in order to be able to describe in terms of order happenings and processes which are observed. Something similar could be said about *all* theories which postulate too much of a definite mechanism, usually involving some identification somewhere.

The slightest discrepancy between such a theory and observation eliminates the theory as structurally unsatisfactory, while theories which succeed in not postulating mechanisms, and so are formulated in a functional language, last much better. One of the enormous advantages of the Maxwell electromagnetic theory is the fact that it describes the behaviour of electricity and magnetism while hardly positing any mechanism at all. A similar statement applies also to the Einstein theory.

The above general remarks are extremely well illustrated by the newer quantum mechanics.

The classical theories, as usual with scientific theories, were very satisfactory in many respects, but not in all, which is an unattainable ideal always demanded from a good scientific theory. They also postulated too much of a definite mechanism, which was the result of, and led to, the semantic disturbance called identification. Indeed, I have read an address by a prominent physicist in which he claims to have 'seen', and invites everybody else to 'see', an 'electron'. He challenges his critics, and seems to feel like fighting—a quite usual result of identification. Electrons represent *inferential entities*, and as such cannot be 'seen', but only inferred, which does not detract at all from the importance of the 'electrons'. The 'seeing' business was good enough in the infancy of science, but not in 1933. We 'see' the stick broken in water, the camera records it as broken, and yet it is not broken. We 'see' the fan as a disk, the camera records it so, but there is no disk. We 'see' a 'solid' piece of wood or stone, which under the microscope proves to have a very different structure, ...

In the older days the electrodynamics of moving bodies presented difficulties quite similar to the difficulties encountered in the quantum mechanics. Einstein by an epoch-making stroke of genius solved the problem by observing that, in the languages in question, we operated with a notion of 'simultaneity' which did not correspond to any observable structural phenomenon in the physical world. He discovered that it is impossible to establish the simultaneity of two events occurring at different places, and that a thorough revision of our old theories is necessary in this connection. Einstein formulated a procedure, a method for measurements, taking into account the known laws of the propagation of light and electromagnetic phenomena. He once more established the most important semantic thesis that the laws of nature are relations which are discovered between events which are actually observed, or which are *fundamentally observable.*

It appears that in the older quantum mechanics there were introduced some objectified entities which were never observed, as, for instance, the positions, velocities, and periods of 'electrons' inside the atom. How indeed could we find lengths and 'times' *inside* the atom? Such a procedure requires the introduction of rods and clocks, which themselves consist of atoms, so that *inside* the atom such a procedure cannot be applied. We see clearly that all such conclusions are of an indirect character, but of course such conclusions should be based on some observable facts, and not only on our freedom to use words in any manner whatsoever It follows that we must give up a language that speaks in terms of the 'position' of an electron at a given 'time' , and use instead a language that describes observable characteristics, as, for instance, energy levels which are directly measurable by electron impacts and the frequencies which are derivable from them, the intensity and polarization of the emitted waves . , instead of electronic 'motions' inside the atom, which never are and never *can be* actually *observed* It is structurally indispensable to look for such data which are actual or at least can be observed

As words are not the things we speak about, and structure is the only link between them, structure becomes the only content of knowledge If we gamble on verbal structures that have no observable empirical structures, such gambling can never give us any structural information about the world Therefore such verbal structures are structurally obsolete, and if we believe in them, they induce delusions or other semantic disturbances.

CHAPTER XLI

THE NEWER 'MATTER'

The twofold nature of light as a light-wave and as a light-quantum is thus extended to electrons and, further, to atoms their wave-nature is asserting itself more and more, theoretically and experimentally, as concurrent with their corpuscular nature (481) A. SOMMERFELD

The concepts of wave amplitude, electric and magnetic field strengths, energy density, etc, were originally derived from primitive experiences of daily life, such as the observation of water waves or the vibrations of elastic bodies (215) W HEISENBERG

The problem of quantum theory centers on the fact that the particle picture and the wave picture are merely two different aspects of one and the same physical reality (215) W HEISENBERG

To me it seems extraordinarily difficult to tackle problems of the above kind, as long as we feel obliged on epistemological grounds to repress intuition in atomic dynamics, and to operate only with such abstract ideas as transition probabilities, energy levels, etc (466) E SCHRODINGER

. . for visualization, however, we must content ourselves with two incomplete analogies—the wave picture and the corpuscular picture (215) W. HEISENBERG

Not every physicist is an epistemologist, and not everyone must or can be one Special investigation claims a whole man, so does the theory of knowledge (326) E MACH

The following chapter was written in 1928 and since then the newer quantum mechanics has been developed much further, proved enormously fruitful, and has been repeatedly supported by experiments The literature on this subject is steadily accumulating, the most important classical memoirs by the originators of this new scientific trend have been collected into book form and are now easily accessible There is also a large number of excellent technical, as well as non-technical presentations On reading in December 1932 what I had written in 1928, I find that although from some aspects the presentation may be considered unsatisfactory and antiquated, yet the epistemological side of the older presentation remains valid. So it seems advisable to retain this chapter and add only a few further $\bar{A}$ suggestions

It is known that practically all creative and constructive physicists, who have produced revolutionary and lasting works, were interested in epistemology There are many physicists who know as much physics as an Einstein, for instance, yet Einstein remains quite unique and his work is to a large extent responsible for the present revolutionary developments of physics The reason is simple Einstein has corrected a long established epistemological fallacy, which can be expressed in my language as the rejection of the structural fallacy of elementalism in a limited yet very important field of physics He also established and applied new fundamental epistemological principles, which is another way of saying that he established new standards of evaluation in physics, as for instance, that we should never postulate entities which cannot possibly be observed, that the 'laws of nature' should be formulated in terms of generally invariant relations expressed in tensor equations, .

The weakness of the system of Einstein, resulting in many futile criticisms, lies in the fact that he eliminated elementalism in one vital region of physics, but he did not formulate the *general epistemological principle* of *non-elementalism*, which should be applied everywhere, daily life included This he could not have accomplished without a still deeper enquiry into the mechanism of time-binding, which produces all science, and which leads to the discovery of the fundamental fallacy in the use of the 'is' of identity Only after the elimination of this remainder in us of the primitive man, does *structure* become the only possible link between the objective and verbal worlds, and becomes also the only possible content of 'knowledge' 'Similarity of structure' then demands the complete and general elimination from science and life of any elementalism

The strength of the newer quantum mechanics lies in the fact that the younger physicists have accepted the new epoch-making einsteinian standards of evaluation or epistemological principles, the weakness lies in the fact that the scientists do not realize that the fallacy of elementalism is entirely general and vitiates *all* scientific outlooks No one can produce satisfactory theories, nor evaluate, nor interpret them properly as long as he continues to use the few-valued and *elementalistic* 'logics' and 'psychologies', which are at present always found at the bottom of any 'evaluation' or 'interpretation'

The latest work of Dirac goes very far in the direction of building $\bar{A}$ physics by establishing his language of transformations, states, observables , ascribing *structure* to protons, magnetic poles , but even Dirac does not seem to realize the general fundamental $\bar{A}$ issues involved Dirac says 'The description which quantum mechanics allows us to give is *merely* a manner of speaking which is of value in helping us to deduce and to remember the results of experiments and which never leads to wrong conclusions *One should not try to give too much meaning to it*'[1] (Italics are mine) The italicized words show that even Dirac does not realize fully the mechanism of identification, as otherwise he would not have used these words in this form If we entirely abandon identification, then a theory or a book, being verbal, represents nothing else but special language, there is no 'merely' about that either, structure being the only possible link between the non-verbal and verbal worlds Instead of warning the reader 'that one should not try to give too much meaning to it', we must simply *insist* that the *only* 'meaning' should be looked for in structure , the 'too much meaning' always indicating inappropriate evaluation and ultimately semantic disturbances

Current physical literature shows that the main problems of 'interpretation' depend on the *solution* of the *m o* problems of 'observation', 'reality', 'fact' , and border on the *scientific* solution of the problems of pathological 'delusions', 'illusions', and 'hallucinations', all of which involve the fundamental issues of the elimination of *identification* But once the $\bar{A}$ issues are structurally formulated and applied in practice, they result in a $\bar{A}$, *non-el*, ∞-valued orientation which involves the recognition of the *multiordinality* of terms, which also solves the problems of quantum 'interpretations', the details of which I cannot enter into here.

Originally the quantum writers had an inclination to ascribe 'physical meaning' to waves. The present tendency of specialists is to regard the waves as 'purely symbolic', forgetting that experimentally something else besides the symbols 'bends around the corners'. From a $\bar{A}$ point of view, when the problems of the multiordinality of such terms as 'observation', 'fact', 'reality', are understood we will have to ascribe 'physical reality' to the waves, ascribe finer *structure* to the 'electron'. We would also have to abandon the A 'particle'-orientation and treat the 'electron', 'proton', in a $\bar{A}$, ∞-valued way as minute *fields*, which under the present experimental conditions behave as 'particles'. This $\bar{A}$ field-orientation suggests a great many possible interpretations impossible in the A 'particle'-orientation.

Mathematically, the geometry of 'space filling' curves would have to be elaborated further so that we would better understand the *structure of plenums*, and this knowledge should be applied to physics.

We should also perform a direct series of experiments with a more elaborate Faraday box. A small wooden laboratory should be isolated from the rest of the world by every available *energetic screen* and physical experiments repeated under such new dynamic conditions. Technically the winding of, say, a foot thickness of insulated wires for different currents would not present any difficulties except for the door which should be also in the circuit. The eventual probable results of experiments in such a laboratory under different conditions could be calculated in advance, and it may be fairly well anticipated that at least significant discrepancies between the calculations and actual experiments would appear, throwing new light on the structure of the space-time plenum, the eventual connection between gravitation and electromagnetism. Arguments alone will not help in this field and only experiments will point the pathway.

Because of the extensive literature dealing with the new quantum mechanics it seems unnecessary to dwell upon it any further, except by expressing the hope that some mathematicians and physicists will master the $\bar{A}$ ∞-valued orientation and will revise the theories now existing.

Section A. Introductory

The new researches in the structure of the materials of the universe proceeded under unique conditions. On the one hand, since Planck in 1900 originated the quantum theory, which earlier elaboration we now call the classical quantum theory, the amount of experimental facts pointing in the direction of *some* quantum theory had become very convincing, yet, on the other hand, the lack of a structurally satisfactory theory to co-ordinate these new experimental facts was becoming distressing.

It appeared as though there were either a lack of 'geniuses' able to produce the required new theories, or the geniuses existed, but were unable to function properly They seemed to suffer from a semantic blockage due to some identification which successfully prevented broader and unhampered vision

Presently the co-ordinating theories *were* produced In the production of the new quantum mechanics we see at work the unconscious semantic liberating influence of the $\bar{E}$ and $\bar{N}$ systems which had been developed and which we have already analysed The classical theories had come to a structural impasse, but with the advent of younger scientists, who were educated in the theoretical semantic freedom of these new systems, the semantic blockage due to ascribing 'objectivity' to 'matter', 'space', and 'time' was removed, creative forces were released, and these young scientists proceeded to construct the structural formulations needed. They are now rightly hailed as geniuses

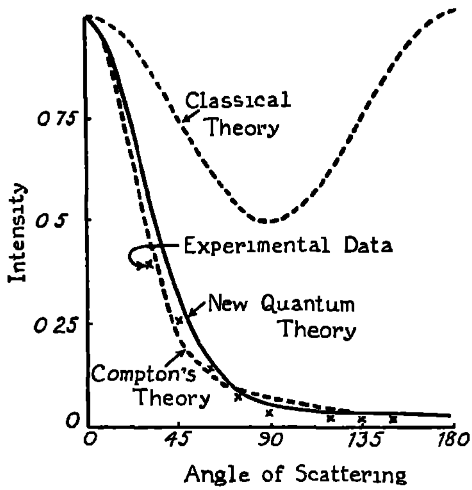
It is astonishing how these young post-einsteinian scientists of various lands and different inclinations produced, independently and practically simultaneously, various new quantum theories, using different methods and different mathematical, as well as national, languages When these different theories were studied and compared they were found to amount practically to one theory, but expressed by different mathematical languages Now the use of different mathematical languages to express one group of experimental facts brings an additional benefit in that it gives diversified verbal structural information about the problems at hand Since these developments are very recent and progress has been extremely rapid it is hard to keep track of the status of the problem.

In this account of the newer quantum mechanics I will emphasize only the structural and semantic side, treating the different theories as behaviour of their respective authors, and as illustrating the issues above mentioned From this point of view, we are not interested in discussing to what extent the given theories are 'true' or 'false', which means no more than similar or dissimilar to the world in structure We are interested directly in those semantic aspects of human behaviour which have been neglected When 'Smith' puts a black mark on white paper, it is to be called *human behaviour*, and behaviour *unique* for man Our analysis does not involve the question of the validity of his doings But since he did it, let us analyse his doings

In speaking about theories as complex and technical as the newer quantum mechanics, it is practically impossible to give a satisfactory account in a non-technical way Such theories have not yet been worked out thoroughly enough, many points are still not clear, and no proper evaluation is possible at present These difficulties are really immaterial to our present purpose, because these theories are *empirical facts on record*, and they throw significant light on human behaviour It is not here proposed to make reflections on the world around us, but to analyse a certain linguistic, structural form of human behaviour

From the point of view of structure and *s r*, the classical quantum mechanics would be quite enough for *sanity* and adjustment, it is enough to realize that the still older theories of 'matter', 'space', and 'time' are *el*, *structurally fallacious* and represent only primitive identifications

The advent of such a crop of geniuses and of several theories expressed quite differently, yet nearly equivalent, is an event of deep human semantic significance It helps to understand the working of the human nervous system, and is in accord with the present general theory



(After *The New Quantum Mechanics* by G Birtwistle, p 192)

FIG 1

From the point of view of the physicist, these new theories are a marked structural improvement over the classical theory, a fact which can best be illustrated by a diagram of a special case In Fig 1, the crosses indicate the experimental data, the curves indicate the results as predicted by the classical theory, by the Compton theory, and by the new quantum mechanics It should be noticed that the new quantum theory appears much more in accordance with the experimental data than the older theories This fact is of great structural and semantic importance to us as well as to the physicist

Section B The nature of the problem

At this point we may explain briefly the nature of the problem that was demanding solution

We have become familiar with the use of co-ordinates This procedure has been generalized and has given rise to 'generalized co-ordinates' These are defined as *arbitrary* variables which represent not merely lengths but may also represent angles, surfaces, volumes , though they must be capable of representing the $3n$ orthogonal co-ordinates If, in a special case, we make the number of generalized co-ordinates equal to the number of degrees of freedom which the system has, these s generalized co-ordinates can be regarded as independent of each other If we denote by q_i the generalized co-ordinate, q_i ($i = 1$ to s) then the orthogonal co-ordinates of any of the n particles can be represented as definite functions of the generalized co-ordinates, so that

$$x_h = f_h\,(q_1, q_2, q_3, \quad q_s) \qquad (1)$$

We know that kinetic energy is represented by $mv^2/2$ where m represents the mass and v the velocity, or the 'time' derivative of the 'space' travelled

If we want to find the value for the energy we must differentiate each of the $3n$ equations (1) with respect to 'time', which gives the components of the velocity, square them, multiply them by the corresponding masses and add them together to find the double value for the energy

For simplicity we will denote the 'time' derivatives by the chosen letter, but with a dot over it (newtonian method). Thus,

$$x_h = \frac{\partial x_h}{\partial q_1} q_1 + \frac{\partial x_h}{\partial q_2} q_2 + \cdots + \frac{\partial x_h}{\partial q_s} q_s \tag{2}$$

Squaring (2), we have

$$\left.\begin{aligned} x_h^2 = \left(\frac{\partial x_h}{\partial q_1}\right)^2 q_1^2 + \left(\frac{\partial x_h}{\partial q_2}\right)^2 q_2^2 + \cdots + \left(\frac{\partial x_h}{\partial q_s}\right)^2 q_s^2 \\ + 2\frac{\partial x_h}{\partial q_1}\frac{\partial x_h}{\partial q_2} q_1 q_2 + 2\frac{\partial x_h}{\partial q_1}\frac{\partial x_h}{\partial q_3} q_1 q_3 + \cdots \\ + 2\frac{\partial x_h}{\partial q_2}\frac{\partial x_h}{\partial q_3} q_2 q_3 + \cdots \end{aligned}\right\} \tag{3}$$

The last expression (3) can be simplified

$$x_h^2 = \sum_{i=1}^{i=s} \sum_{k=1}^{k=s} \frac{\partial x_h}{\partial q_i} \frac{\partial x_h}{\partial q_k} q_i q_k . \tag{4}$$

It is easy to see that if in (4) we put $k = i$ we will have square members, and when $i \neq k$, every term will occur twice and so the above abbreviation (4), covers the formula (3)

If we write similar expressions for all of the $3n$ orthogonal co-ordinates, multiply by the corresponding masses and then add them together we obtain twice the value for the kinetic energy $2L$

$$2L = \sum_{i=1}^{i=s} \sum_{k=1}^{k=s} c_{ik}\, q_i\, q_k, \text{ where}$$

$$c_{ik} = \sum_{h=1}^{h=n} m_h \left[\frac{\partial x_h}{\partial q_i}\frac{\partial x_h}{\partial q_k} + \frac{\partial y_h}{\partial q_i}\frac{\partial y_h}{\partial q_k} + \frac{\partial z_h}{\partial q_i}\frac{\partial z_h}{\partial q_k} \right]$$

The coefficients in the expansion of c_{ik} depend only on the values of the generalized co-ordinates and are independent of the value of the time-derivatives The time-derivatives can be properly called *generalized velocities*, and we may denote them by q_i [2]

In establishing formulae for the quantum theory we want to be as general as possible and not restrict ourselves to vibrational energy only But we want to take into consideration *any* arbitrary point-mass, independently of whether we assume this point to be charged or not

We define the momentum or impulse as the product of the mass and the velocity, or, $p = mv$ If, instead of denoting our co-ordinates by x, y, and z, we use the generalized co-ordinates q_i, we would have for the magnitude and direction of the velocities the time-derivatives of the co-ordinates, namely, q_i, where $q_1 = x = dx/dt$, $q_2 = y = dy/dt$, .

If p_1, p_2, p_3, represent the corresponding components of the momentum or impulse, then we would have $p_i = mq_i$ (5)

We should notice that the dynamical triplet of impulse co-ordinates occurs conjointly with the geometrical triplet of the co-ordinates of position. The second law of motion tells us that 'the change in momentum is proportional to the impressed force and takes place in the direction in which that force acts'. If we assume that the force K is derivable from the potential energy E_{pot}, (a function of q_i), then we have $\dot{p}_i = K_i = \dfrac{-\partial E_{pot}}{\partial q_i}$ (6)

The kinetic energy (E_{kin}) is represented by

$$E_{kin} = \frac{m}{2}(\dot{q}_1^2 + \dot{q}_2^2 + \dot{q}_3^2) = \frac{p_1^2 + p_2^2 + p_3^2}{2m}$$

where by (5), $\dot{q}_i^2 = p_i^2/m^2$ We call the total energy, which is represented as the sum of the kinetic and the potential energy, as expressed in terms of the generalized co-ordinates and momenta, the hamiltonian function H Then we have.

$$H(q, p) = E_{kin} + E_{pot}, \quad \frac{\partial H}{\partial q_i} = \frac{\partial E_{pot}}{\partial q_i}, \quad \frac{\partial H}{\partial p_i} = \frac{\partial E_{kin}}{\partial p_i} = \frac{p_i}{m}. \tag{7}$$

From (5), (6), and (7), we get the fundamental equations of motion,

$$\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \text{ and } \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$$

The above hamiltonian, or canonical, form of the equations is remarkable because it preserves its form if any arbitrary co-ordinates are introduced, it is invariant under the transformation of co-ordinates. The equations hold not only for an individual point-mass but also for any arbitrary mechanical system. For arbitrary co-ordinates and systems the momentum or impulse p is defined by

$$p_i = \frac{\partial E_{kin}}{\partial \dot{q}_i},$$ so that the

kinetic energy is expressed as a function of the q_i's and their derivatives the $\dot{q}_i$'s

To help visualization we can construct and consider the p and q as rectangular co-ordinates in two dimensions in the phase plane of our system. In this plane the sequence of those graph-points that correspond to the successive states of motion of the system represent the phase paths or phase-orbits The characteristic structural feature of the quantum theory is that it selects a discrete family of phase-orbits from the infinity of possible orbits

We next consider a point-mass m that is bound elastically to its position of rest, and which can move to either side of the central position only in the direction $x = q$, or its reverse, when experiencing a restoring force We call such point-mass a linear oscillator If we wish to emphasize that our oscillator is capable only of definite vibrations, on account of its elastic attachment, we call it a 'harmonic oscillator'. If the vibration number, or the frequency of the oscillator, which is represented by the number of its free vibrations per unit of 'time', is denoted by ν, then the vibration is represented by $x = q = a \sin 2\pi\nu t$

The impulse becomes $p=mv=m\dot{q}=2\pi\nu ma \cos 2\pi\nu t$. The phase-orbit is represented by an ellipse in the p-q-plane and is given by the equation $\frac{q^2}{a^2}+\frac{p^2}{b^2}=1$, where the minor axis $b=2\pi\nu ma$

In our family of orbits the phase area between two orbits is equal to the quantum of action h Sommerfeld regards h as an elementary region or element of the phase area, and considers it as the definition of the Planck quantum of action h If W_n represents the energy of the oscillator when it describes the n-th orbit, then $W_n=nh\nu$ In these orbits the energy appears as a whole multiple of the elementary quantum of energy, $\epsilon=h\nu$, and $W_n=n\epsilon$

We call *stationary states* of the oscillator those states which the oscillator may pass through without cessation and without loss of energy, or, without radiation.

When an oscillator retains its stationary state, its energy is constant and its graph appears as an ellipse of the family in the phase plane However, when the energy of the oscillator changes and jumps over to a smaller orbit, it emits energy When it passes to a larger orbit it absorbs energy. The emission and absorption of energy occurs in multiples of the energy quantum, ϵ

The graphs of the system in the phase plane are restricted to certain 'quantised' orbits Between each orbit and its successor there is an elementary region, of area h. The n-th orbit, if closed, has as area nh Or, expressed symbolically, $\int pdq=nh$ This integral is called the phase integral and is taken along the n-th orbit

The quantum hypothesis can be structurally formulated so that the phase integral must be a whole multiple of the quantum of action h This form of the classical quantum postulate is more general than the original formulation of Planck, although it includes the latter as a particular case

In case of a rotating point-mass, a similar analysis gives us $E_{kin}=\frac{p\dot{q}}{2}$ and when $\nu=\frac{\dot{q}}{2\pi}$, $E_{kin}=\frac{nh}{2}\frac{\dot{q}}{2\pi}=\frac{nh\nu}{2}$ where ν represents the rotation frequency of the rotator, or the number of full revolutions per unit of 'time', and takes the place of the vibration number of the oscillator

In the classical theory, the quantised states were distinguished from all other possibilities by the characteristic whole numbers, and so we had a network In a quantum orbit the 'electron', if undisturbed, was supposed to move permanently without resistance and not to emit radiation The phase-space, representing the manifold of the possible states, including non-stationary states, is crossed, mesh-like, by the graph curves of the stationary orbits The size of the meshes is determined by Planck's constant h [3]

Section C Matrices.

The older quantum mechanics forms an elaborate system, and we have a large accumulation of numerical data on record Some of these data corroborated he older theories nicely, but some data were in contradiction to the classical theory The problem was not to discard the numerical data, which, whatever they mean structurally, represent quite solidly established data, but to find new equations which would be satisfied by these facts Now 'new equations' really mean *languages of new structure*, and therefore new formulations had to be discovered

Dealing with tables which give special theoretical data, it was natural to start with a calculus which deals with such numerical special tables Such a calculus had been developed long ago, and was called the matrix calculus Later on, when matrices themselves were treated as complex quantities, and still later, as operators, we were enabled to pass to the more developed calculi which use ordinary differential equations The new quantum theories give us a unique case, in which several mathematical methods have been used at once and of which the results are fairly in accord

At this point it is advisable to give a few structural explanations of these mathematical notions, including the matrix calculus If we have two equations of the first degree with two variables, namely, $a_1x+b_1y=c_1$ and $a_2x+b_2y=c_2$, the solution of these equations takes the form $x=\frac{b_2c_1-b_1c_2}{a_1b_2-a_2b_1}$, $y=\frac{c_2a_1-c_1a_2}{a_1b_2-a_2b_1}$ The common denominator of the two solutions can be written in a two-dimensional table

$$\begin{vmatrix} a_1, & b_1 \\ a_2, & b_2 \end{vmatrix} \tag{1}$$

which is understood as the product of the upper left-hand number and the lower right-hand number, minus the product of the lower left-hand and upper right-hand numbers

Similarly, the numerators of these solutions can also be represented in the form of two-dimensional tables, namely,

$$b_2c_1-b_1c_2=\begin{vmatrix} c_1, & b_1 \\ c_2, & b_2 \end{vmatrix} \quad (2) \text{ and } \quad c_2a_1-c_1a_2=\begin{vmatrix} a_1, & c_1 \\ a_2, & c_2 \end{vmatrix} \tag{3}$$

to which the above-mentioned rule applies Expressions like (1), (2), (3), are called determinants of the second order

The numbers in the first, second , horizontal lines are called the first, second , *rows*, respectively, the vertical lines are called first, second , *columns*

The above definitions and method can be applied to any number of equations with an equal number of variables, and in each case our determinant would have n^2 numbers, n rows and n columns.

We may use another notation which employs one letter for the coefficients of our variables, with indexes or suffixes to indicate that their values are different Let us consider n^2 elements in the table

$$\begin{vmatrix} a_{1,1}, & a_{1,2}, & a_{1,3}, & \dots, & a_{1,n} \\ a_{2,1}, & a_{2,2}, & a_{2,3}, & \dots, & a_{2,n} \\ \cdot & & & & \\ a_{n,1}, & a_{n,2}, & a_{n,3}, & \dots, & a_{n,n} \end{vmatrix} \tag{4}$$

The expression (4) is called a determinant of the n-th order.

The notation by suffixes is very convenient and is very much used these days The first suffix denotes the row, the second the column in which the element is situated Usually the comma dividing the two numbers in the index is dropped and the coefficients are written simply a_{11}, instead of $a_{1,1}$ In general the element $a_{i,k}$, or a_{ik}, represents the element in row i and column k

The elements lying in the diagonal joining the upper left-hand to the lower right-hand number are called the principal diagonal In our example we notice that the elements in the diagonal are such that $i = k$

We have definite rules by which we can arrive at the solution of our equations, once the coefficients, which are the elements of the determinant, are given In general, the determinants are treated as a functional form

If m and n are positive integers, a manifold, or system of mn ordered quantities or elements arranged in m horizontal and n vertical rows, will be called a *rectangular matrix* and we may use the notation $A = (a(mn))$

$$\begin{pmatrix} a(11) & a(12) & a(13) \\ a(21) & a(22) & a(23) \\ a(31) & a(32) & a(33) \end{pmatrix}, \text{ or } (a(nm))$$

The numbers m and n are called the orders of the matrix If $m = n$ the matrix is called a *square* matrix Without loss of generality we can treat any rectangular matrix in which $m \neq n$ as a square matrix by supplementing the missing rows and columns with zeros.

A matrix of the type,

$$1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ or } (\delta(nm)),$$

where $\delta(nm) = 1$ for $n = m$, and $\delta(nm) = 0$ for $n \neq m$, is called a unit matrix The matrix

$$\begin{pmatrix} a(11) & 0 & 0 \\ 0 & a(22) & 0 \\ 0 & 0 & a(33) \end{pmatrix} \text{ or } (a(nm)\ \delta(nm)),$$

is called a diagonal matrix In the new quantum mechanics a diagonal matrix is independent of t and represents a constant of the classical theory The reverse is not necessarily true. The operation of differentiation can be expressed in terms of multiplication of matrices with the aid of the unit matrix [4]

Equations in which matrices are equated are called matrix equations If the equations involve only one unknown matrix, which does not occur more than once as a factor, such equations are called matrix equations of the first degree.

The m scalar equations

$$\begin{aligned} a_{11}x_1+a_{12}x_2+\ldots &+a_{1n}x_n=c_1\\ a_{21}x_1+a_{22}x_2+\ldots &+a_{2n}x_n=c_2\\ &\ldots\\ a_{m1}x_1+a_{m2}x_2+\ldots &+a_{mn}x_n=c_m \end{aligned}$$

are together equivalent to one single matrix equation There are several ways in which the notation can be simplified

The difference between a determinant and a matrix is subtle, but important. By a determinant we understand, by definition, a certain homogeneous polynomial of the n-th degree, in the n^2 elements a_{ij} Accordingly, a determinant gives a definite number when calculated

But in many instances we are interested in the *table*, or the n^2 elements arranged in a certain order but *not* combined into a polynomial Such an array, or table, is called a matrix Thus, from this point of view, a matrix does not represent a definite quantity, but a system of quantities, and so a matrix is *not* a determinant.

We can illustrate this difference by an example If we take a determinant of the second order

$$\begin{vmatrix} a_1, & b_1 \\ a_2, & b_2 \end{vmatrix}$$

and change the rows into columns, or vice versa, thus

$$\begin{vmatrix} a_1, & a_2 \\ b_1, & b_2 \end{vmatrix}$$

the value of both determinants will be equal, namely,

$$a_1b_2-a_2b_1,$$

by the definition rule already given, yet the *matrices* of the two determinants are *different*

Although different, a determinant nevertheless defines a matrix, called the matrix of the determinant, conversely, a matrix defines a determinant, called the determinant of the matrix

We have said that a matrix does not represent a quantity, while a determinant does At this stage, and from this point of view, we may say so legitimately However, we might eventually treat a matrix as a quantity also, but for this purpose we should have to enlarge the meaning of the term 'quantity'

In our present use of the term 'quantity' we mean the real and complex quantities of ordinary algebra

It may be said that mathematicians have had a peculiar tendency, which has proven of great value in the development of mathematics, gradually to extend the meaning of terms in order to embrace new notions as they arise For instance, we have enlarged the primitive meaning applied to positive integers to embrace negative numbers, which formerly would not have been considered as quantities Similarly, if we here use the ordinary notion of algebraic quantity, then a matrix is not a quantity but a system of quantities The problem is, how shall we enlarge this meaning to include the matrices?

Mathematics recognizes that this generalization of *mathematical* notions is extremely useful and *legitimate* This structural issue appears to be of very general application, as all of us exhibit a tendency towards it It is a purely mathematical and useful tendency in mathematics, but it leads to disastrous results when applied to daily-life abstractions, as explained in Part VII In this connection we should recall the difference between the mathematical contentless abstractions and the abstractions with physical content, with which we are generally concerned in science and life.

Let us now follow up the method by which a matrix can be considered as a quantity If we have objects of two or more kinds which can be counted or measured, and if we consider an aggregate of such objects, say 5 horses, 3 cows and 2 sheep, we could denote such a complex quantity by the symbol (5,3,2) In this case, the first place in our symbol would be reserved for horses, the second for cows, and the third for sheep

In mathematics, we do not specify horses, or cows, or sheep, but consider sets of quantities, and distinguish them by the position which they have in our symbolism We may denote such a complex quantity by a single letter, $A=(a,b,c,)$ (For instance, we denote a fraction by a single letter, although a fraction is specified by *two* numbers)

In such an instance, we should call a complex quantity equal to another when, and only when, the components are respectively equal And a complex quantity is said to vanish only in case all the components vanish

Ordinary mathematical operations can be applied to such complex quantities For instance, we may define a sum or difference of two complex quantities

$$A'=(a_1, b_1, c_1) \text{ and } A''=(a_2, b_2, c_2) \text{ as}$$

$$A'\pm A''=(a_1\pm a_2, b_1\pm b_2, c_1\pm c_2) \quad ,$$

a definition which is entirely satisfactory theoretically, and also practically, as can be verified from our example

From this point of view we may consider a matrix as a complex quantity with mn or m^2 components A matrix would then represent a complex quantity, as a special case under the general method sketched above

We could then define our further operations A matrix would be said to be zero when all elements are equal to zero Two matrices would be said to be equal when they have equal numbers of rows and columns and every element of one is equal to the corresponding element of the other

By setting up some such rules we could develop a calculus of matrices, and matrices would be considered as complex numbers In general, the algebraic rules would be found to be applicable to matrices, which would further justify us in treating matrices as complex numbers

One of the notable exceptions in our operations would be found in the application of the classical operation of multiplication and its dependencies In ordinary algebra and arithmetic, multiplication is what is called 'commutative' which means that $2\times 3=3\times 2=6$, or $a\times b=b\times a$.

In defining the multiplication of matrices we have no *a priori* grounds for determining why one definition or restriction should be preferable to another.

Only practice can show which definition is more workable or more fruitful in results In the matrix calculus the definition of Cayley is generally accepted, as it has led to the most workable results It was based on considerations of the composition of *linear* tranformations

The definition is approximately as follows The product **ab** of two square matrices of the n-th order gives a square matrix of the n-th order in which the element which lies in the i-th row and j-th column is obtained by multiplying each element of the i-th row of **a** by the corresponding element of the j-th column of **b** and adding the results

If we denote by a_{ij} and b_{ij} the elements in the i-th row and j-th column of **a** and **b** respectively, then by definition the element (i,j) of our product **ab** would give,

$$a_{i1}b_{1j}+a_{i2}b_{2j}+ \quad +a_{in}b_{nj}, \tag{5}$$

and the (i,j) element in the matrix **ba** would be

$$a_{1j}b_{i1}+a_{2j}b_{i2}+ \quad +a_{nj}b_{in} \tag{6}$$

In general, the quantities (5) and (6) are not equal and therefore we see that the multiplication of matrices is, in general, *not commutative* The *order* in which we perform our multiplication is of importance and **ab** *is not* generally equal to **ba**, $(\mathbf{ab} \neq \mathbf{ba})$ [5]

It should be noticed that the vector calculus has made us familiar with new operations which differ from arithmetical operations For instance, the sum of two vectors differs in general from the arithmetical sum and is defined by the law of the parallelogram (see Chapter XXXIII) This definition is more general, and the arithmetical definition expresses only the particular case in which the vectors have one direction. Similarly, the non-commutative law of multiplication corresponds more closely to vector multiplication than to arithmetical multiplication

We will not go further into the details of the matrix calculus, which is a well-developed mathematical discipline with a large literature, but will emphasize some methodological points of importance

One of the main applications of the theory of matrices is found in the subject of *linear transformations*

In mathematics, instead of using the given variables, we very often introduce new variables which are functions of the old Such transformations, or change of variables, are particularly simple and important when the functions in question are homogeneous and *linear*

If $x_1, x_2, \quad , x_n$ represent the original variables, and $x_1', \quad , x_n'$ the new variables, we have, by definition, the formulae of transformations

$$\begin{aligned} x_1' &= a_{11}x_1 + \cdots + a_{1n}x_n \\ &\cdot \qquad \cdot \\ x_n' &= a_{n1}x_1 + \quad \cdot \quad + a_{nn}x_n \end{aligned}$$

The square matrix made up of the coefficients is called the matrix of the transformation and the determinant is called the determinant of the transformation and is completely determined by the matrix. We have already seen the importance of linear equations and linear transformations in physics and therefore in the investigation of the world around us The theory of matrices is connected with such transformations, hence the importance of the theory of matrices for physics

For our purpose another characteristic of the matrix calculus is of interest and that is the fact that in physics we usually have a large number of empirical numerical data which enter as coefficients in equations and which can always be put in the form of a two-dimensional array of numbers, or a table, which we have just called a matrix

It appears that every physical quantity, however complicated, can be represented by such a table giving the values of the parameters which determine its character From the definition of the term 'variable' as *any* value out of a possible range of values, we might treat our variables in two distinct ways, one from the point of view of *function* or operations, the other from the *extensional* point of view, when the function or operations are unknown, although the particular values of the variable are given The matrix calculus takes this last point of view

In physical research work we deal for the most part with arrays of numbers or unique and specific, mostly asymmetrical, relations which the experiments give us Our usual problem is to find the structure, the function, and the operations which are satisfied by the given experimental relations

We see that the dual approach to our solutions is due entirely to the definition which we have accepted for the variable We have two issues Either to find the values of the variable which satisfy the given function and operations or, having particular values of the variable (experimental), to find the function and operations

Obviously every physical quantity can be represented by a matrix, which may be a sequence, and every mathematical theorem can be reduced to a property of matrices Once the proper mathematical theories are worked out it will be always possible to pass from one form of representation to the other [6]

In the older mechanics the functions were rather obvious and so the use of the matrix calculus was not so imperative In the newer mechanics, the opposite is the case We have a large amount of numerical experimental relations, but the functions and operations connecting these variables are unknown and the problem is to find them From this point of view the matrix calculus represents an *extensional* calculus, a calculus of *observation* In using the descriptive term 'observation' we must add that some objections have been advanced to such a use of the term The answer is that the term 'observation', like most of our most important terms, must be considered a *multiordinal term*. Once this is understood the objections to the use of the term do not hold

There is no limitation as to what the elements of a matrix may represent, they may be functions, functions of functions ,

Section D The operator calculus.

The use of the *operator calculus* is interesting, structurally and psychologically, in that attention is concentrated, not on the numerical quantities, but on the semantic *operations of combining* them The calculi used in the newer quantum mechanics are peculiar, because, while they retain the numerical data, and as far as possible, the classical equations, they alter the operations by which these quantities are combined, or the interpretation of the equations

As an illustration of such a procedure we can take two different formulae for the addition of velocities, one from the classical mechanics, where

$$V_{13} = V_{12} + V_{23}, \tag{1}$$

and the other, the formula for velocity as given by the Einstein theory, namely,

$$V_{13} = \frac{V_{12} + V_{23}}{1 + \dfrac{V_{12} \times V_{23}}{c^2}} \tag{2}$$

In these formulae V_{12} represents the velocity of body 1 relative to body 2 , . In formula (1) the sign '+' symbolizes the ordinary arithmetical operation of addition As we already know, this formula has proven too simple to represent accurately the experimental data, and Einstein has replaced it by the more elaborate formula (2)

The above statement is the usual way of speaking about the modification in formulation which has taken place in physics since Einstein But we could equally well say that the formula has *not* been altered except that the '+' has no longer the old meaning and does not now represent the arithmetical operation of addition Both points of view lead ultimately to one value, V_{13}, and the *computations* are similar in both cases

We should notice especially the great freedom with which we can treat mathematical entities Our voluntary selection of the point of view becomes important A similar freedom of selection of interpretation appears to a still larger extent in all *verbal* problems, a fact of considerable structural and semantic importance in any theory of sanity, as we have already seen

Further illustration of this freedom can be seen in the way in which the ordinary notion of multiplication is re-interpreted in the operator calculus Let us denote by q and f, two numerical quantities, and qf as their product But we could view this problem differently We could say that *qf results* from a semantic *operation* q performed on f, or a semantic *operator* $(q \times)$ acting upon f which transformed f into qf We could denote the *operation* of multiplying by q or the operator $(q \times)$ by a single symbol Q Quite obviously the operator Q is not the number q, in other words, the semantic operation of multiplying, say by two, is *not* the number 2

The operation of multiplying integer 1 by integer 2 gives the result 2 Similarly, the operation of multiplying 1 by q, or, in our new language, the application of the operator Q to the integer 1, gives q In symbols, $Q1 = q$ If we take any arbitrary function f, the result of the operation of Q upon f is written $Qf = qf$ If we were to follow the operation Q by the differential operation d/dx the result would be*

$$\frac{d}{dx}(Qf) = \frac{dq}{dx}f + q\frac{df}{dx} = \left[\frac{dq}{dx}\times\right]f + Q\frac{df}{dx}$$

But as f is arbitrary, it may be omitted from the equations and the result written in the operator form as

$$\frac{d}{dx}Q - Q\frac{d}{dx} = \left[\frac{dq}{dx}\times\right] = \frac{dQ}{dX}.$$

The symbol $\frac{dQ}{dX}$, defined by this equation, should be read as 'the operation of multiplying by dq/dx' Similarly, $\frac{d(QP)}{dX} = \frac{dQ}{dX}P + Q\frac{dP}{dX}$.

In translating the ordinary equations into the language of operators nothing new is introduced This translation involves only a change of *'mental' focus* Instead of concentrating our attention on the numerical values we concentrate on the operations of combining them Since the great problems of the quantum mechanics consist in finding new methods of computation, or of combining numerical values, such a change of attitude may prove to be structurally useful

It should be noticed here that once matrices are considered, and treated, as quantities, or unique and specific relations, by similar reasoning they can be treated as operators This problem is of fundamental structural and semantic importance because in the quantum theory we deal with matrices which have infinite numbers of terms and since this complexity presents great technical difficulties it is of enormous advantage to be able to pass to some more developed methods of calculation

At the preliminary stage in the operator calculus we have assumed that multiplication was commutative, that $QP = PQ$, but in the further and more general development of the theory, this does not hold

In general we must assume in the operator calculus that multiplication is not commutative, that $QP \neq PQ$ For instance, if $Q = (q\times)$ and $P = d/dq$, the two operations are certainly not commutative Naturally, the validity of $2\times2 = 4$ is not doubted, but generalized non-commutative multiplication has a definite asymmetrical and so structural geometrical interpretation, to be found in the vector calculus When we associate with each numerical quantity its own operation of multiplication we thus obtain a more general calculus. Operators may be regarded as compound, or built up from the elementary arithmetical operations of addition and multiplication They represent, so to say, functions of these operations [7]

*The operator D_x, also written d/dx, is called a differential operator If applied to a product (uv) the results are given by the formula $D_x(uv) = uD_xv + vD_xu$

Section E The new quantum mechanics

The main problem of the quantum theory is to determine these functions of the operations, so that the solution of certain equations (hamiltonian) may represent the experimental facts The original equations of the new mechanics of Heisenberg, Born, and Jordan were frankly founded on an empirical basis As Dirac puts it, in seeking for the new equations, the classical equations were to be retained as far as possible and only the operations by which these quantities are combined were to be altered

To gain this freedom to alter multiplication the data were first interpreted as matrices Then it was found, by Born and Wiener, that the matrices could be interpreted as a special kind of operator, which furnished means to calculate the matrices Carl Eckart independently developed a simple operator calculus for the solution of the quantum problems In this present work I follow closely the paper of Eckart [7]

The origin of the new quantum mechanics was an epoch-making paper by Werner Heisenberg, in July, 1925 The older quantum theory had postulated the existence of stationary states of the atom, which were calculated with the aid of the older mechanics In the new mechanics the equations have similar form as in the classical theory, but the variables no longer obey the commutative law of multiplication In general pq is not equal to qp, ($pq = qp$ and $pq - qp = 0$ in the classical theory) but $pq - qp = \frac{h}{2\pi i}1$, where h represents the Planck constant, q the generalized co-ordinate, p the momentum, 1 stands for the unit matrix, and π and i have the usual meaning The fact that multiplication is not commutative in our calculus allows us to give a definite value to the above difference and by introducing the Planck constant h, we are enabled to introduce the quantum conditions in our calculations

The quantum conditions of the older theory led to an algebraic equation. By using the classical equation with a non-commutative multiplication law for the variables it is possible to perform calculations in the new and wider scheme of dynamics The difference between pq and qp is expressed in terms of the Planck constant h When h is made to approach zero, pq approaches qp, and so we pass to the classical mechanics Thus we see that the classical mechanics appear only as a particular case of this more general theory

In introducing his theory, Heisenberg pointed out that the older mechanics uses quantities which are ***never observable***, and ***can never be observed***, such as, for instance, orbital frequencies and amplitudes, or the position and 'time' of revolution of an 'electron' , which, as such, have no physical meaning He proposes to use ***observable*** data, such as the frequencies and intensities of the radiations , . Now these frequencies are always differences between two terms given by integers If T_n and T_m are two such terms, the observable frequency is theoretically represented by $\nu_{nm} = T_n - T_m$ Such numbers as ν_{nm} characterize the atom as far as it is observable It was natural that such a collection ('sum' in this case has no longer any physical meaning) of terms could best be represented by a matrix In the classical theory a dynamical quantity was represented structurally by a trigonometrical Fourier series, in the new, it is represented by a two-dimensional table of values, that is, by a matrix giving the frequencies and the intensities of radiations

An important and interesting structural issue now appears It is that the Heisenberg theory gives a new formulation for the hamiltonian equations of motion, whereby their form is preserved, yet they are made applicable both to periodic and to *non-periodic motion* It becomes possible to fuse the classical mechanics with the quantum mechanics The distinction between 'quantised' and 'unquantised' motion loses all meaning, and a fundamental equation, $pq-qp=\frac{h}{2\pi i}1$, is formulated which is valid for *all motion.*

The Heisenberg theory is also characterized by its thoroughly behaviouristic, actional, functional, and operational character The number of unjustifiable assumptions is the lowest in existence and most of the identifications are eliminated. According to Heisenberg, electrons and atoms do not have the 'same' kind of 'reality' as ordinary *objects* of lower order abstractions This conclusion, which underlies his whole work, is of particular importance structurally. As we know, differences in character separate different orders of abstraction and since the quantum phenomena belong to a higher order of abstraction they must differ from objects which belong to a lower order of abstraction In this theory the feelings of 'space' and 'time' are no longer applicable to the 'inside' of the atom—as might be expected

The distinction between 'inner' and 'outer' electrons in an atom becomes meaningless, since it is impossible to recognize a particular entity among a series of similar entities In accordance with the new 'space-time' outlook we gain a physical basis for the absolute individuality of some eventual unit

Because of its structure, the Heisenberg theory is a very fundamental one and there is little doubt that the Heisenberg *methods* will be elaborated further and will be kept as a permanent *checking method* in physics A theory which is thoroughly behaviouristic, with a minimum of assumptions, will probably remain both a most important instrument of research and an inspiration to physicists and mathematicians

The Heisenberg theory, again, because of its structure and method, does not lend itself easily to visualization This is not against the theory The pictorial representations of lower order abstractions are not to be relied upon. Besides, visualization depends on the *lower centres* and therefore must be represented by a *macroscopic* representation of a continuous (rather than a discrete) character, such as waves ,

If we were to try to describe the Heisenberg theory pictorially, which is obviously difficult to do, we would have to give a *negative* description. We should have to say that what we observe must be considered only as radiations from the location which the atom was supposed to occupy [8]

There remains but to mention some more characteristics of the Heisenberg theory which seem to have very far-reaching structural and semantic bearings

This theory appears frankly statistical and introduces fundamental probability assumptions. The moment we realize that the human organism is essentially an *abstracting* affair and that the abstracting is performed on different levels, or in different orders, it becomes obvious that statistical methods and probability notions become fundamental.

In the earlier days we used to assume that statistical laws were laws with exceptions. Such an outlook was conditioned by our dealings with macroscopic events. Now, we analyse such macroscopic events in terms of microscopic and sub-microscopic events, the statistical laws become accurate laws, not for individuals but for *groups* of individuals. Because we abstract in different orders, we deal *only* with statistical data, mass effects of different 'packets' of nervous excitement, as is best illustrated by different thresholds in different nervous tissues.

The processes in the higher centres, ***more remote from the external world,*** deal with a special material, no more with statistical data of 'packets' and averages, but with what we used to call 'inferences', 'inductions' . . , which give only the *probability* of happenings. But as we have already seen, probability has become a well-developed structural mathematical discipline, which has not yet made much effect upon our primitive-made macroscopic metaphysics and language. It should be noticed that the highest activities of the nervous centres are based on statistical data furnished by the lower centres. So we see that, to the best of our knowledge about ourselves and the world around us, a modern structural and semantic outlook, in science or in life, must be based on statistical and probability methods.

In space-time every point has a date, and therefore in the language of space-time all points are different and do not repeat themselves. Such structural outlook is, of course, again conditioned, and leads toward the statistical and probability methods. The main psycho-logical importance of the new methods is to diminish affective tension, which is always wasteful and harmful. Inferences may involve belief. When *belief* is *too strong*, although this is never justified according to the best modern knowledge, we very easily fall into identification, delusions, illusions, and the like. It should be emphasized that the last-mentioned pathological states are always compound. They involve at least two components. One of these consists of some ignorance somewhere, the other of strong affective belief in the 'truth' of our mistaken notions. The stronger the affective tension is, the more dangerous the semantic disturbance becomes.

The Heisenberg theory has succeeded in formulating (verbal) structural methods which are best suited to represent the experimental facts which underlie physics, as well as being structurally in accord with the working of the human nervous system. That is why I venture to assert that this theory will never be abandoned as a checking and research instrument.

In an hypothetical experiment in the quantum field, we may assume what may be called a gamma-ray microscope. If we were to illuminate an 'electron' by gamma-rays, the rays would disturb the experiment, and in our fundamental equation, $pq - qp = \frac{h}{2\pi i} 1$, by which the 'position' was to be determined, the 'momentum' would thereby be disturbed This change of 'momentum' would be greater the shorter the wave-length of the rays used, and the shorter the wave-length of the ray, the more accurate the determination of 'position' would be Hence, the more exactly a co-ordinate q could be found, the less exactly could its momentum p be found, and vice versa

So we have to introduce corrections for errors, and have to introduce 'mean values' and 'probability functions', which we can develop and compute. Lately, Bohr has further developed the probability aspects of the newer quantum mechanics but I have not seen this work Heisenberg introduces 'probability packets' which correspond to the 'wave packets' of Schrodinger

It is difficult to speak briefly, and yet in a satisfactory way, about these new developments, and particularly difficult to give credit properly to different authors All their works are interwoven and at present they all really work together in spite of the fact that historically some of these theories have been developed independently

What we call today, for the sake of brevity, the Heisenberg theory, because of its originator, has been further developed by Heisenberg, Born, Jordan, and others Later, when the wave-mechanics appeared, all the new theories were finally fused into a very elaborate and impressive structure

Historically, P A M Dirac worked at the theory from a different mathematical point of view, utilizing what is called the 'Poisson bracket' method In this treatment the difficulties of the matrix calculus were avoided He introduced dynamical variables which he called the q numbers These do not obey the commutative law of multiplication although the c numbers (classical) do. He also considered the difference of the non-commutative products $xy - yx$, where x and y are functions, respectively, of the co-ordinates $q_1 \ldots q_s$, and of the momenta $p_1 \ldots p_s$ of a multiple periodic system with s degrees of freedom

Dirac has generalized the matrix theory and the Schrodinger equations. His work seems to be most important, in physics and mathematics, but it is not possible for us to consider it here in any detail [9]

Let us recall once more that there are fundamental differences between the different orders of abstraction, and that we all have to abstract in different orders From this point of view it is natural that *every* theory, even if expressed at present in a form which cannot be *visualized*, like the Heisenberg theory or the original Dirac theory, has sooner or later to be expressed in a structural form which can be visualized These problems have really nothing, or at most very little, to do with the world around us They are concerned with the neurological structure which produces *all* theories

Theories of a structure such as that of the Heisenberg theory are extremely important, as already explained, but in them we lose the help of 'intuition' Now 'intuition' (lower centres) has two quite different effects—sometimes it leads us astray, but on other occasions it helps greatly.

An 'intuitive' theory has a creative aspect, but always ought later to be revised and scrutinized by non-intuitive means. In fact, because of our nervous structure, we should always strive to produce *both aspects* of theories—strive *consciously*—for thus we facilitate progress Historically, we can never completely avoid producing both types of theories, as they are inherent in our nervous structure and in the different orders of abstractions we produce.

It is precisely in the newer quantum mechanics that a typical example of this simple neurological fact is found The non-intuitive handling of data was introduced by Heisenberg the translation of the matrix calculus into operational and 'Poisson brackets' methods, and, finally, the new 'wave mechanics' of de Broglie, Schrodinger, and others, gives us a perfect translation into intuitive methods

It should be noticed that according to the old notions such two methods, the intuitive and the non-intuitive, were not supposed to be a *neurological necessity* We still assumed that they were separated 'absolutely', and even today in many quarters we argue as if they were absolutely separable. If we accept the principle of non-elementalism, we realize that this distinction is verbal only and that the invention of verbal means has little or nothing to do with the world around us, but that it depends on human structural ingenuity

Investigation of the ordered cyclic nerve currents shows unmistakably that such sharp differentiation is unjustifiable, and we must conclude, in accordance with historical experience, that translation from one method to the other must be a necessity, and so will be accomplished some day in every field. It is true that at present the Einstein theory has not been translated with entire success into terms of lower order abstractions This is a task which is facilitated by this present work The newer quantum mechanics gives us an unparalleled example of such translation, and hence our main interest should be concentrated on this structural aspect.

On neurological grounds it seems certain that visualization involves in some way the lower nerve centres, which again, by evolutionary necessity, involve macroscopic forms of representation Our macroscopic experience led us to *geometrical intuitions* These were framed three-dimensionally in '*absolute emptiness*', and were impossible in higher dimensions The old structure represented a static empty 'space' in which nothing could happen and which was thus unfit to represent this world around us where something is going on everywhere

The new structure represents '*fulness*' or a plenum We can visualize it as a network of intervals or world-lines and then, by the notions of the differential calculus, as already explained, we pass easily to the visualization of the many-dimensional space-time world of Minkowski-Einstein Now, in such a world, the curves are represented by functions, and, vice versa, functions represent curves Thus it is obvious that analytical 'non-intuitive' methods have 'intuitive' structural geometrical counterparts From this point of view the method of operators represents a passing step from the non-intuitive to intuitive methods As soon as we have functions, we can represent a functional calculus as an operational calculus This involves a more behaviouristic semantic attitude and so leads to the possibility of translation of either method into the other Ultimately these are psycho-logical transformations and translations.

We should not be surprised to find that in the development of the newer quantum mechanics the operational calculus plays just such a role

Now our older macroscopic experience, which affected our lower nerve centres, gave rise to the elementary geometrical structural notions of 'lines', 'surfaces', 'volumes', . For the building of physics we had to introduce 'time', 'motion', . In older days we did not realize that these give us forms of representation and that it is optional with us which forms we will accept as fundamental or use as a starting point.

The old descriptive apparatus posited structurally an absolute and immutable 'space' (emptiness), 'time', 'matter' out of which we built up a verbal definition for 'motion'. The semantic attitude of all of us, scientists included, depended upon identification We ascribed lower-centre significance to higher-centre abstractions We did not discriminate enough between the macroscopic and small scale events So we had 'geometrical optics' in which we 'perceived' a ray of light (in a dusty room, let us say) as a 'straight line' Further investigation disclosed that the 'rays' on one level of abstraction were *waves* on another, but they were not perceived as waves by the lower centres

But through our lower centres we had acquaintance with some waves, such as in water, so representation for waves was developed A wave-theory still remained intuitively workable, even when we dealt with waves which we could not see Now the equations of waves are well known. It is then possible to translate a non-intuitive matrix mechanics, when we treat matrices as operators, into a functional calculus which has an intuitive geometrical wave representation.

This is precisely what has happened and now, perhaps for the first time in human history, we have all the aspects of a theory being worked out simultaneously, with mutual co-operation of all workers and the use of methods which are mutually complementary from the neurological side There seems little risk in predicting that because of these neurological factors the newer quantum mechanics will give extremely rapid and far-reaching results When scientists become aware of the structural semantic and neurological issues involved, perhaps such achievements will be multiplied *consciously,* instead of being a kind of coincidence

Personally, I am convinced that these new achievements are not simply coincident. It seems that the abolishment of the old, *el*, static 'absolute space' and 'absolute time' *has relieved the younger scientists from a semantic blockage.* This release was due to the bold stroke of genius of Einstein in refusing to use the vicious aristotelian 'is' of identity As soon as we realize that words are not the objective levels, we gain an *unconscious semantic freedom* in handling words, as words At once this freedom is bound to produce many different forms of representation for events, according to the personal make-up of the individual workers. And of course these forms can be translated into one another

Section F The wave mechanics.

We have not sufficient space at our disposal to discuss more fully the new wave mechanics I found that short of a small volume, no explanations, readily intelligible, could be made

In mathematics and physics, which represent the most developed sciences, we consciously and unconsciously strive for more and more general formulations The work of Einstein, showing that the classical mechanics was only a particular case of a more general mechanics, has given a healthy stimulation to such a fruitful line of work

As the quantum phenomena could not be accounted for by the old mechanics, it was natural that physicists should try schemes of new mechanics which included classical mechanics as a particular case Thus, Sommerfeld, through his methods of the application of the Einstein theory to the quantum mechanics and his generalizations of phase-space, and his treatment of the relation of wave-optics to ray-optics and of the relation of mechanics to ray-optics, came close to the discovery of the wave mechanics [10]

The new wave mechanics originated in 1924 in Paris, with the thesis of Louis de Broglie, published in 1925, and republished in a book form in 1926

The controversy between the corpuscular light theory of Newton (emission theory) and the wave theory of Huygens is well known The emission theory had its support in the 'rectilinear' propagation of light, which followed from the inertia of light particles Also, it explained the reflection and the refraction , of light, but failed in other respects It is true that the wave theory also had its weak structural spots In it the 'rectilinear' propagation of light remained a complete mystery and it completely failed to account for the dispersion of light, until this was explained on the electron theory

Both theories assumed the periodicity of light phenomena, but the acceptance of one theory was generally held to mean the rejection of the other It did not occur to many that both theories might be correct but only partial structural aspects of a more general theory

With the advent of the quantum theory of Planck (1900) new methods were found In 1905 Einstein propounded his theory of 'light quanta' successfully He assumed that radiation occurs in discrete quanta of energy $h\nu$, where ν represents the frequency From this point of view the quantum had the characteristic discreteness of a corpuscle, and yet the frequency characteristic of a wave We see that the new theory involved a kind of a blend of the two older theories

De Broglie generalized still further the above notions His theory is in a way the result of the theory of Einstein As we already know, Einstein shows the connection of mass and energy, so that the conservation of mass becomes also the conservation of energy, and vice versa Starting with these premises de Broglie concluded that if any element, in the most general sense, be it an electron or proton or light quantum or what not, has energy W, there must be in the system a periodic phenomenon of frequency ν, defined by $W=h\nu$ From this point of view all forms of energy, radiation included, must have an atomic structure and the atoms of energy must be grouped around certain points, forming what we call 'electrons', 'light quanta', .

Applying the Lorentz-Einstein transformation, he finds a rather startling fact—that the frequency associated with any assumed mass m_0, namely, $\nu_0=\frac{m_0c^2}{h}$, represents no more and no less than a *periodic* phenomenon, analogous to a stationary wave, which spreads around the point of which the mass is a singularity [11]

In other words, a 'mass particle' at rest is the centre of a pulsation throughout the spread, or otherwise it is a singularity of the pulsation The quantity which pulsates is called ψ and $\psi\bar{\psi}$ is interpreted as the *electric density*, where $\bar{\psi}$ is the conjugate of the complex quantity ψ

In the Minkowski representation, the above astonishing result becomes quite simple, and we can see clearly how simultaneous pulsations become travelling waves In Fig 2, we give a two-dimensional diagram of space-time OX_0 is the 'space' co-ordinate, OT_0 the 'time' (ict_0) co-ordinate ψ_1, ψ_2, ψ_3, represent the traces of the surfaces of constant phase which are perpendicular to OT_0 The Lorentz-Einstein transformation is equivalent to the transformation from rectangular system X_0OT_0 to the rectangular system XOT, forming an angle θ. In the new system the lines ψ_1, ψ_2, ψ_3 . . , are no longer parallel to the X axis and so represent a moving *wave front.* In this new system the ψ-lines represent the moving wave front for different points, P, P' , on one phase line and have *different* values of t The smaller the velocity (v) of the particle, the smaller the angle θ and the greater the distance PQ travelled by the phase in a given 'time', $P'Q$ or Δt, which means the greater the phase velocity u.

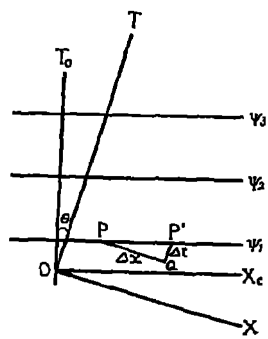


FIG 2
(From *Wave Mechanics*, by H F Biggs)

The frequency of these pulsations or waves is the 'total energy of the particle' divided by Planck's constant, h In symbols, $h\nu=mc^2+$potential energy, where

$$m=\frac{m_0}{\sqrt{1-\beta^2}},\ \beta=\frac{v}{c}$$

The problem before us is to connect structurally, the waves with two observations, one of the radiations, the other of what we call the 'material particles'

Our interest at present lies only in the connection with the latter. According to this theory, the 'region occupied by the particle' is only a region where a set of ψ-waves, which vary continuously in direction and in frequency in a small range, reinforce each other to give a *wave-group* travelling with what we usually call 'the velocity of the particle'

As the waves have different frequencies, they travel with different velocities and so we have to face the problem of the dispersing medium, where, according to classical theories, the region of reinforcement has a velocity different from that of the phase The ordinary expression for 'group velocity' gives, on the wave interpretation, the magnitude of the 'velocity of the particle' [12]

That which, in the classical theories, we called the 'motion of the particle' is represented by the motion of the region of reinforcement of ψ-waves The direction of motion is represented by the direction of a ray-, or wave-normal. The particular ray selected as the position of the 'path of the particle' is represented by the ray cut in coinciding phase by a set of ψ-waves with slightly varying directions The 'position of the particle' is given by the small region occupied by the *group* of waves of slightly different frequencies and velocities

In this connection we should notice an important point which is necessitated by the methods of generalization and of translation from macroscopic to sub-microscopic events and vice versa Schrodinger, by formulating the differential equations for the ψ-waves, has brought out clearly the important structural point that wave mechanics bears a similar relationship to classical mechanics of particles as wave-optics bears to ray-optics

Here again we have the macroscopic phenomena capable of being treated by mathematical methods different from those used for the small-scale phenomena.

In classical mechanics the state of a system whose co-ordinates were $q_1 \quad q_s$ and whose momenta were $p_1 \quad p_s$, was represented by a point in a $2s$-dimensional q-spread and the changes in the system were represented by the passage of the point along some curve, a 'ray', so to say

Schrodinger regards the classical mechanics as only an approximation, while rigorous treatment must be made by the aid of wave mechanics

The large-scale, or macroscopic, mechanical processes correspond to a wave signal in the q-spread and can be regarded as a point in comparison with the geometrical structure of the path In small-scale phenomena, such as the atomic processes, a rigorous wave formulation must be used

This analysis can be carried further and Hamilton knew well and used the analogy between mechanics and geometrical optics The Hamilton variation principle, $\delta\int L dt = 0$, is Fermat's principle for a wave motion, the Hamilton-Jacobi equation expresses Huygens' principle for wave motion, and the new wave mechanics expresses the Kirchhoff analysis of physical optics As Huygens' principle could deal with the problems of physical optics up to a certain point, so the Hamilton-Jacobi equations could deal with atomic problems up to a certain point At the exact wave analysis of Kirchhoff was needed to clear up the finer points of physical optics, so the new wave mechanics is required for the exact solution of the atomic problems [13]

A detailed analysis shows that classical mechanics was associated with geometrical optics (ray-optics). Obviously, a more exact system of mechanics would be one associated with wave-optics, which would give the classical results in all cases where the wave-length was negligible in comparison with the dimensions of the path Schrodinger suggests that a correct extension of the analogy would be to regard the wave system as *sine-waves* In this connection it should be recollected that Fourier has shown that any given form of waves can be represented by the superposition of *sine-waves*, and that therefore a *sine-wave* (see Chapter XXXII) may be considered as a general formulation

The ray methods in physics worked only to some extent, that is to say, in the cases where the radii of curvature of such rays and the dimensions of the spreads were large in comparison with the wave-length When this is not the case we have to consider waves and not rays Naturally, dealing with atomic dimensions, which are very small, instead of using the paths of the particles or the ψ-rays, we have to use the ψ-waves It appears that this difference was the main, rather puzzling distinction between the classical mechanics and the quantum mechanics, between the macroscopic theories and the sub-microscopic ones

The above realization, and its formulation into a mathematical theory, seems to be an important and extremely fruitful generalization, which probably will be retained as a method

One of the puzzling features of the quantum theory was the structural appearance of the whole-number laws of the 'orbits' That some such whole-number relation is justified seems to be well established, yet it contradicted the older 'continuous' mechanics A new theory, to be at all satisfactory, should be able to fit these whole-number empirical data The first test of the new wave mechanics, and also its first success, was precisely in this field

If a ray of the ψ-wave was supposed to run around in a circle for a stationary state, the circumference must be a *whole multiple* of the wave-length, or

$$2\pi r = n\lambda = n\frac{h}{mv}, \text{ and } mvr = n\frac{h}{2\pi}$$

where n is an integer We see that the quantum condition of the Bohr theory, that the angular momentum must be a whole multiple of $h/2\pi$, is only the result of the requirement that the wave-function ψ shall be single-valued, which is another way of saying that the circumference ($2\pi r$) must contain a whole number of wave-lengths It may be compared perhaps with waves travelling around a circular loop of string If they travelled both ways we would have stationary waves

At this point a very important structural feature of the new wave mechanics makes its appearance In the above interpretation the 'velocity of the electron' has lost its physical meaning, it becomes simply the *wave length* of the ψ-waves In the wave mechanics, as well as in the matrix mechanics, there is no meaning to the older 'position of an electron on its orbit' So the wave mechanics again embodies the advantages of the matrix mechanics by not postulating entities which can never be observed The whole numbers, as Schrodinger remarks, 'appear as naturally as do "integers" in the theory of vibrating strings'. In the theory of string vibrations these whole numbers are determined by certain boundary conditions which have to be satisfied by the solution of a differential equation. In the new wave mechanics there is also a differential equation representing the Schrodinger wave equation [14]

Section G. Structural aspects of the new theories

We should notice the important distinction between the two structural types of these theories The extensional matrix theory can hardly be visualized, with all the consequent advantages and disadvantages. The wave mechanics can be visualized From what we already know of the structure and working of the nervous system, we see that the wave mechanics will have a *creative* element and the matrix mechanics will remain an important *checking* method . At present, all these new theories seemingly have blended or perhaps it would be better to say that they have been translated from one language to another and all the workers in this field work from all angles.

It should be mentioned also, that Einstein, Bose, Jordan, and others, work from the point of view of *statistics*, and that these methods, too, are being retranslated and connected with the rest of the new theories.

The new wave mechanics evades the difficulties of the matrix calculus and brings the new mechanics within the scope of the highly developed analysis of the theory of differential equations It also enlists the creative aspects of 'intuition', 'visualization',

Concluding our consideration of the subject, three remarkable aspects of the wave mechanics must be referred to We are already acquainted with the term 'action'. It appears that the main point of the passing from the old mechanics to the new was the stroke of genius of de Broglie, when he divided action by the fundamental constant h with some definite numerical factor which then gives us the *phase*. In the expression for ψ, the energy appears as the 'time' component of a space-time vector whose 'space' components are those of the momentum When this vector is divided by h, its components become the *frequency*, or the number of waves which each axis cuts per centimetre.

These are the methods by which we can use differential equations, whereby the older discontinuities disappear and the particle is represented as a group of reinforcing waves [15]

From this point of view we also come to the conclusion that the 'conservation of energy', which was very valuable in the old days, is perhaps only a gross macroscopic generalization and will give place to a newer and more fundamental notion of the conservation of *frequency* or 'times' [16]

It has been already mentioned that the newer mechanics must be represented in accordance with *statistical* data, *probabilities*, with due attention paid to the theory of errors , . While these requirements have very little to do with the world around us, they are unconditionally required by our nervous structure, which is, after all, the general author of all our 'knowledge' and 'theories' Let us be candid about it, there is no such thing as 'knowledge' outside of a nervous system, and therefore the neurological requirements, as already mentioned, become paramount The newer theories brilliantly satisfy this requirement

As an example, we may perhaps mention an aspect of the wave mechanics theory which is not at present settled, but which remains just as interesting

Schrodinger shows that in highly excited states a suitably chosen group of waves represents a '*wave packet*', which behaves like a point-mass of the ordinary mechanics It oscillates with frequency ν_0 in a rectilinear path. The number and breadth of the waves which form the packet vary with the 'time', but the width of the packet remains constant The remarkable part about the shape of the curve is that it represents the *Gauss error curve*

Heisenberg has shown that this result is only accidentally true, but for our purpose of illustration this is quite enough

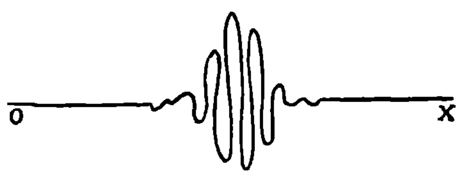


FIG 3

(From *The New Quantum Mechanics*, p 115, by George Birtwistle)

The newer quantum mechanics has shown once more the necessity for a re-analysis of our fundamental notions From a space-time point of view, which seems to be a permanent acquisition of science, since it is a language and method of structure closer related to the external world than the older languages and methods, it seems beyond dispute that even macroscopic phenomena are the results of *repeated observation*. Now, such a point of view, although it is extremely plausible, and close to neurological and physical data, necessitates a complete reconstruction of our describing apparatus, which is not adapted to such an outlook

The problem of 'observation' enters Bohr suggests, and rightly, that this vocabularly is strictly connected with the older 'causal' vocabulary One of the main points of the present work is to draw attention to the *multiordinal mechanism and terms* and to show that the analysis of these problems cannot be even attempted without first analysing the structure of our languages, of our 'knowledge', and the neurological and semantic aspects which such analysis involves

When this analysis is carried through we see that the problems of 'continuity' and 'discontinuity' lose their absolute character They become *verbal* problems, to be solved through the ingenuity of some one who will suggest the solutions.

The newer quantum mechanics give us ample material for work on these problems, but they also illustrate a much more general and important problem, which is the subject matter of the present book, namely, that all 'knowledge' is *structural*, strictly dependent on the nervous structure and functioning, and the language we use 'Method' is that aspect of the search for structure which deals with the most expedient means for finding structure Since words *are not* the things we speak about, the study of linguistic structure becomes a most important research method. The more languages (theories) we have for analysis and structural comparison, the more glimpses do we get at the structure of the world. The newer quantum mechanics give us enormous material of a linguistic, structural, and semantic character.

It is natural that this wisdom can come only from the study of the structure of the highest developed languages in existence, which are mathematical languages. If we want progress in any line of human endeavour, this progress is always dependent on the languages we use, since what we call 'progress' is a co-operative affair and therefore dependent on means of communications and languages.

From the point of view of structure, we deal with a world of *absolute individuals* and therefore our languages must be such as to reflect such individuality. We already know that this involves an extensional attitude and methods, which historically have produced mathematics as the only language which as yet reflects in structure the world around us.

With the newer quantum mechanics, the old 'discontinuity' resolves itself into an essential *individuality*, as noticed by Bohr, perfectly foreign to the older theories.

History proves that we were slow in arriving at that point. Our tragedies began when the 'intensional' biologist Aristotle took the lead over the 'extensional' mathematical philosopher Plato, and formulated all the primitive identifications, subject-predicativism , into an imposing system, which for more than two thousand years we were not allowed to revise under penalty of persecution. Mathematics was not particularly encouraged, but at least, not persecuted, so that it was developed into the present day great linguistic system. The theory of function involves semantic factors of non-identification.

The invention of the differential and the integral calculi, represents the two great structural and psycho-logical aspects of analysis and differentiation, versus synthesis and integration.

The application of these methods led us to differential geometries, to methods of treating 'fulness', and to 'contact' methods. 'Fulness' necessitated geometries of higher dimensions, impossible in 'absolute emptiness', and so the fusion of geometry with physics became possible. The four-dimensional world of Minkowski and the theory of Einstein finally achieved this fusion. The next step was the invention of the new quantum mechanics, where all these important, nay, all-important, structural, semantic, and linguistic achievements find their culmination. The old primitive metaphysics become too 'materialistic' for an enlightened age.

Without legislating about the 'truth' or 'falsehood' of the newer mechanics, as a matter of *human behaviour* these theories are the best indications and examples of the structure of human 'knowledge', which I have attempted to formulate in this work as a general theory.

The A-system was strictly interconnected with primitive-made structural assumptions or metaphysics, reflected in the structure of the older languages and in the *el* notions about language, 'psychology', 'logic', and the pre-scientific anthropomorphic astronomy, physics, and other disciplines Of late, science has developed in spite of all handicaps and persecutions, and has begun to depart structurally and semantically from the path of aristotelianism and the dark ages. Every science has had to build its own language and this fact completely condemns the A language, which, it is shocking to notice, we continue to preserve in our daily life

Should we wonder that we have shown hardly any progress at all in our purely human affairs and notions? We should wonder, rather, that we have been able to survive until now—though with needless difficulties and suffering More wars, more revolutions, more insanity, more morons, more struggle and competition, and more unhappiness are what we are entitled to *expect and predict* as the outcome of this structurally and semantically impossible situation.

As the organism works as-a-whole, such things as 'pure intellect' or 'pure emotions' represent structurally *el* fictions and scientists should realize that their professed detached scientific attitude is profoundly and fundamentally unjustified All science has 'emotional' components, which play most important roles in life If we live in a modern world, but keep the 'emotional attitudes' of primitive bygone days, then naturally we are bound to be semantically unbalanced, and cannot be adjusted to a fundamentally primitive 'civilization' in the midst of great technical achievements * When scientists understand that, then the layman will have a different attitude toward science He will understand that science is not a privilege of the few, something without effect upon all and every one He will realize that while he lives in a modern world, *made so* by science, *structural ignorance* of the fundamentals as discovered by science leaves him with primitive structural assumptions or metaphysics, which by necessity build for him a delusional world leading to semantic unbalance and ultimately to 'mental' and nervous ills.

From the *non-elementalistic* point of view, the only escape is to realize that ignorance in an adult is, and must be, pathological, because 'knowledge' is to be considered as a normal characteristic of human *nervous tissue*

A special structural and methodological brief and simplified account of scientific achievements, such as I have attempted in this work, must be a part of a theory of *sanity* Sanity means *adjustment* and without the minimum of the best structural knowledge of each date concerning this world, such adjustment is impossible

It is not necessary that the reader should fully understand all technical details of a theory, to be *aware* (instinctive, affective ,) of the *existence* of the structural and semantic problems and to realize that some of the most competent and skilled professionals are working at these theories Such awareness has great pacifying semantic influence, it eliminates the older affective tensions which were due to identifications, absolutism, dogmatism, flights into mysticism, and other similar pathological disturbances

*See my *Manhood of Humanity* (E P Dutton, New York).

A *non-aristotelian system* must deal with all these structural and semantic issues. It is hoped that once a $\bar{A}$-system is presented to the public, scientists and laymen will become more interested in the structural and semantic issues emphasized here, and that new and wider researches will be undertaken.

History shows that such hopes are not illusory. The greatest men of science have always had wide human aims and interests. From the *non-elementalistic* point of view, they probably became productive geniuses because of this broad human urge. From the point of view of psychiatry, it is well known that 'mental' ills involve usually anti-social affective attitudes. When we see men with distinctly anti-social tendencies, no matter how they rationalize them, they are invariably ill in some way. A fully healthy individual is never anti-social.

That science should include structural and semantic factors of sanity may be a startling notion, but only at first! In the present analysis this turns out to be, rather unexpectedly, a necessity. But on second consideration we should rather expect it. Science and mathematics show the working of the 'human mind' at its best. Accordingly, we can learn from science and mathematics how this 'human mind' should work, *to be at its best*. Then we should make an analysis of science and mathematics from some wider structural and semantic point of view,—the task which has been undertaken in the present work.

At this early stage it is, of course, of comparatively little importance to what extent this analysis turns out to be satisfactory. The main point is that it has been *originated.* If the present author fails, others, perhaps even because of his failure, may be stimulated to do it better. The great and vital thing is that it should be done, by someone.

SUPPLEMENT I

THE LOGIC OF RELATIVITY

BY R. D CARMICHAEL

In order to be able to deal with such quantities as are involved in the measurement of motion, time, velocity, etc, or indeed in the quantitative analysis of any physical phenomena, it is necessary to have some system or systems of reference with respect to which measurements can be made Let us consider any set of things consisting of objects and any kind of physical quantities whatever, as electric charges or magnets or light-sources or telescopes or other objects and instruments, each of which is at rest with respect to each of the others Let us suppose that among the objects are clocks, to be used for measuring time, and rods or rules to be used for measuring length, and that time and length may be measured at any desired instant and any assigned place. Such a set of objects and quantities and instruments, including the equipment for measuring time and length, all being at rest relatively to each other, we shall call a system of reference Such a system we shall denote by S. In case we have to deal at once with two or more systems of reference we shall denote them by S, S', S_1, S_2, . .

In this definition of systems of reference nothing specific has been said about the units of length and of time If we were dealing with our usual principles of mechanics we might pass over such a matter without any feeling of difficulty about it, it would be sufficient to proceed in accordance with our intuitive conceptions of time and length But in the theory of relativity these appear in a new light We can not proceed with confident dependence upon our intuition. On the other hand we shall not attempt to give explicit definitions of units of time and length We shall proceed from certain principles or postulates, presently to be stated, to an analysis of time and length and so arrive at a suitable precision of these conceptions by means of certain guiding principles. It will be seen that it is not far from the truth to say that our fundamental terms are defined implicitly and indirectly by means of the statements made about them and accepted initially as valid and that they may mean anything which is consistent with the truth of these fundamental principles and postulates

The restricted principle of relativity may now be stated in the following form.

RESTRICTED PRINCIPLE OF RELATIVITY. *If S_1 and S_2 are two systems of reference having with respect to each other a uniform unaccelerated motion, then natural phenomena run their course with respect to S_2 in accordance with precisely the same general laws as with respect to S_1*

This principle says nothing about the suitability of any particular system of reference for the convenient expression of the laws of nature, but it does say that if either S_1 or S_2 is suitable the other is equally suitable, the relative motion of the two being unaccelerated

In order to bring into suitable relations the measurements made on one system of reference and those made on another it is necessary to have some agreement as to the correspondence of units on the two systems. Accordingly we shall make the following assumption concerning the correspondence of units

PRINCIPLE OF CORRESPONDENCE OF UNITS *The units of any two systems S_1 and S_2 are such that the same numerical result will be obtained in measuring with the units of S_1 a quantity L_1 and with the units of S_2 a quantity L_2 when the relation of L_1 to S_1 is precisely the same as that of L_2 to S_2*

We shall agree that the restricted principle of relativity is to be understood in a sense which implies this assumption concerning the correspondence of units, that is, the latter will be taken as a more precise formulation of a part of the content of the former It is clear that the possibility of realizing this latter is taken for granted in the Galileo-Newtonian mechanics, it is often passed over without remark although it is a profound fact and is a part of the essential basis of any theory of motion

It is a grave question whether the restricted principle of relativity can be maintained in the interpretation of natural phenomena Indeed in the more general theory of relativity, to be taken up later, it is treated merely as a sort of approximation to a more comprehensive principle—an approximation strictly valid only in the absence of a gravitational field but very close to the truth for a wide variety of phenomena including most of those which are purely terrestrial.

There are two particular characteristic postulates, or 'laws of nature', lying at the base of the restricted theory of relativity These may be stated as follows

POSTULATE M *The unaccelerated motion of a system of reference S can not be detected by observations made on S alone, the units of measurement being those belonging to S*

POSTULATE R *The velocity of light, in free space, measured on an unaccelerated system of reference S by means of units belonging to S, is independent of the velocity of S and of the unaccelerated velocity of the light-source.*

For these two particular postulates there is the strongest possible experimental evidence Everything known points toward their truth, and there is nothing known which in any way seems to be in disagreement with them. It is to be observed that they apply only to the ideal case, that is, the case in which there is supposed to be no gravitational field

For the development of the restricted theory of relativity there are three additional necessary postulates, or 'laws of nature,' those that theory shares in common with the Galileo-Newtonian mechanics Such assumptions in some form are essential to the initial arguments and to the conclusions which are drawn by means of them To the present writer it seems to be preferable to have these assumptions explicitly stated. They may be put into the following form·

POSTULATE V *If the velocity of a system of reference S_2 relative to a system of reference S_1 is measured by means of the units belonging to S_1 and if the velocity of S_1 relative to S_2 is measured by means of the units belonging to S_2 the two results will agree in numerical value.*

POSTULATE T *If two systems of reference S_1 and S_2 move with unaccelerated relative velocity and if a body moves relatively to one of the systems in a straight line with unaccelerated velocity then it also moves in a straight line relatively to the other and with unaccelerated velocity*

POSTULATE L. *If two systems of reference S_1 and S_2 move with unaccelerated relative velocity and if a line segment l is perpendicular to the line of relative motion of S_1 and S_2 and is fixed to one of these systems, then the length of l measured by means of the units belonging to S_1 will be the same as its length measured by means of the units belonging to S_2*

We now have before us the logical basis upon which may be built the restricted theory of relativity in all its details It has been put in essentially the same form as that employed in my "Theory of Relativity" (published by Wiley and Sons, New York) and in my earlier articles in "The Physical Review". Reference may be made to the book named for the detailed development of the theory. Here we shall attempt to sketch only the progress of ideas and to indicate the main conclusions

The first thing to be done in developing the theory on this basis is to consider carefully the relation between the time units of the two systems The following remarkable conclusion is reached by a process of reasoning which is fully cogent in character

If two systems of reference S_1 and S_2 move with a relative velocity v and β is the ratio v/c of v to the velocity c of light as measured on either system, then to an observer on S_1 the time unit of S_1 appears to be in the ratio $\sqrt{1-\beta^2}$ 1 to that which is described to him as a unit by an observer on S_2 while to an observer on S_2 the time unit of S_2 appears to be in the ratio $\sqrt{1-\beta^2}$ 1 to that which is described to him as a unit by the observer on S_1

Thus we have the extraordinary conclusion that the time units of the two systems of reference S_1 and S_2, not at rest relatively to each other, are of different lengths in such a way that an observer on either system thinks that the time unit of the other system is greater than his own It is evident that no simple change of the unit on either system (or both) will bring the units into agreement for observers on both systems As postulates V and L and T are generally accepted and have not elsewhere led to such strange conclusions it is natural to suppose that the strangeness here is not due to them In the argument the restricted principle of relativity needs to be used only in so far as it is involved in the conclusion that the units of any two systems of reference S_1 and S_2 are such that the same numerical result is obtained in measuring with the units of S_1 a quantity L_1 and with the units of S_2 a quantity L_2 when the relation of L_1 to S_1 is precisely the same as the relation of L_2 to S_2 But this principle is accepted in the classical mechanics and has not elsewhere led to strange results The conclusion in postulate M appears to be demanded by the strongest experimental evidence, it is generally accepted, if the strange element in the result concerning units of time is due to this postulate, it appears that we must accept it as being required by such experience as has already been tested with due care Hence the conclusion seems to be inevitable that the strangeness in our result is due principally to postulate R

We shall presently see that the same basis of postulates leads to the conclusion that corresponding units of length in the two systems are also different when taken in certain directions From the transformations of time and space which result from the conclusions thus obtained the whole restricted theory of relativity may be deduced (as is shown in the book mentioned) Therefore this theory depends essentially on the principle of correspondence of units in two systems of reference and on the propositions set forth explicitly in the postulates; and all of these are either generalizations from experiment or statements of laws which have usually been accepted Hence we conclude *The restricted theory of relativity may be developed by logical processes from the generalized results of certain experiments together with certain laws which have for a long time been accepted.*

The main result concerning the relation of units of length may be put in the following form

If two systems of reference S_1 and S_2 move with a relative velocity v and if β is the ratio v/c of v to the velocity c of light as measured on either system, then to an observer on S_1 the unit of length of S_1 along the line of relative motion appears to be in the ratio $\sqrt{1-\beta^2}$ 1 to that which is described to him as a unit by an observer on S_2 while to an observer on S_2 the unit of length of S_2 along the line of relative motion appears to be in the ratio $\sqrt{1-\beta^2}$ 1 to that which is described to him as a unit by the observer on S_1

These remarkable conclusions concerning units of length in two systems of reference rest on just those postulates which led to the strange results as to the units of time

What often impresses one as the most remarkable conclusion in the theory of relativity is one which implies that the notion of simultaneity of events happening at different places is indefinite in meaning until some convention is adopted as to how simultaneity is to be determined In fact, *there is no such thing as absolute simultaneity of events happening at different places* With respect to the measured time and space of physics we must conclude that time does not run its course independently of space Measured time and space are indissolubly bound together. The theorem which sets this forth most concretely may be stated in the following way.

Let two systems of reference S_1 and S_2 have an unaccelerated relative velocity v Let an observer on S_2 place two clocks in the line of relative motion of S_1 and S_2 and adjust them so that they appear to him to mark simultaneously the same time. Then to an observer on S_1 the clock on S_2 which is forward in point of motion appears to be behind in point of time by the amount

$$\frac{v}{c^2} \cdot \frac{d}{\sqrt{1-\beta^2}},$$

where c is the velocity of light, $\beta = v/c$, and d is the distance between the two clocks as measured by the observer on S_1

By means of the foregoing theorems we may readily obtain the formulae for the celebrated Lorentz transformation of space and time coordinates (The non-mathematical reader may omit the remainder of this paragraph) Let two systems of reference S and S' have the relative velocity v in the line l. Let systems of rectangular coordinates be attached to the systems of reference S and S' in such a way that the x-axis of each system is in the line l and that the two x-axes have the same positive direction, and let the y-axis and the z-axis of one system be parallel to the y-axis and z-axis respectively of the other system and have their positive senses in the same directions Let these two systems of axes coincide at the time zero Furthermore, for the sake of distinction, denote the space and time coordinates on S by x, y, z, t, and those on S' by x', y', z', t' Let us suppose that S' moves with respect to S in the direction of increasing values of x. Then it turns out that the foregoing theorems imply the following relations between the two systems of coordinates:

$$t' = \frac{1}{\sqrt{1-\beta^2}}\left(t - \frac{v}{c^2}x\right),$$

$$x' = \frac{1}{\sqrt{1-\beta^2}}(x - vt),$$

$$y' = y,$$

$$z' = z,$$

where $\beta = v/c$ and c is the velocity of light.

The foregoing theorems, or (in more compact language) the foregoing equations of transformation, furnish the effective means for developing the whole of the restricted theory of relativity Our purpose does not require us to follow that development further in detail But we may mention a few of the remarkable conclusions which now emerge readily If two velocities, each of which is less than c, are combined the resultant velocity is also less than c The mass of a body increases with an increase in its velocity relative to the system on which the mass is measured The mass of a body at rest appears to be the measure of its internal energy Mass and energy in general appear to be essentially convertible terms The velocity of light is a maximum which the velocity of a material body may approach but can never equal or exceed.

The development by Einstein in 1905 of the foregoing restricted theory of relativity led to a fresh analysis of the whole foundations of physics. This was made inevitable by its effective attack upon such fundamental notions as those of length and time and mass and velocity. Einstein himself succeeded in 1915 in greatly extending the range of his theory, developing what has since been called the general theory of relativity We shall now speak briefly concerning the foundations underlying the latter

Already in the restricted theory time and space had become essentially blended so that we could no longer speak of a three-dimensional space as separate and apart from the one dimension of time A sort of combination of the two came into our conception and we began to realize that they can not be disentangled by the measurements of physics We are forced to consider a four-dimensional continuum of space and time It is with this space-time extension of four-dimensions that the general theory of relativity has essentially to do, and its problems are intimately connected with the relations of two systems of reference of the generalized sort which this makes necessary The Lorentz transformation was a great psychological (and even logical) aid in the formation of the new theory.

Let us consider a four-dimensional extension in which space and time are intimately connected and blended so that each point P in these four dimensions represents a definite place A at a definite time t at which A is to be considered In the course of time a material particle is represented by a succession of these points P. All these points for a given material particle lie on what is called the "world-line" of that particle, and this world-line represents the state of motion (or eventually the state of rest) of the material particle If two objects come into coincidence at an instant their world-lines have a corresponding intersection The things which the physicist deals with ultimately are these intersections of world-lines

In order to deal with them he finds it necessary to introduce certain reference numbers which we may call the coordinates x_1, x_2, x_3, x_4 These numbers change in such a way that their variation along any world-line is continuous and that no two points ever have the same ordered set of four numbers assigned to them This gives us a very general set of coordinates It is clear that coordinates can be set up in an immense variety of ways so as to have these few very general properties One of the first problems in the general theory of relativity is that of the character of the transformation by means of which we can pass from a given choice x_1, x_2, x_3, x_4 of coordinates to a second one ξ_1, ξ_2, ξ_3, ξ_4 It is clear that we must have relations of the form

$$\xi_i = f_i\,(x_1, x_2, x_3, x_4),\ i = 1, 2, 3, 4,$$

where the functions f_i of the variables x_1, x_2, x_3, x_4 are rather general functions of these four arguments and are indeed to a large extent arbitrary Now suppose that the laws of nature are expressed in terms of the coordinates x and also in terms of the coordinates ξ, the question arises as to what relation one ought to expect between these two forms of the law Now there are no coordinates in nature These have been inserted by us for our convenience What is more natural, then, than the demand that we shall formulate our statements of these laws so that they shall have the same form in these two systems of reference, and indeed in all possible systems of reference? This is precisely one of the fundamental basic requirements upon which Einstein insists The corresponding principle he has called the principle of covariance In detailed and precise form, it may be stated somewhat as follows

PRINCIPLE OF COVARIANCE *The laws of nature can be (and are to be) expressed in such mathematical form in terms of the space-and-time coordinates* x_1, x_2, x_3, x_4 *that they shall remain invariant under every transformation of the form*

$$\xi_i = f_i\,(x_1, x_2, x_3, x_4),\ i = 1, 2, 3, 4,$$

where the functions f are subject to the following conditions

1) *They are (apart from exceptional points or regions of fewer than four dimensions) finite and continuous and indefinitely differentiable,*

2) *They are such that the transformation is uniquely reversible, the inverse transformation having the properties demanded for the direct transformation,*

3) *They are such that in both the transformation and its inverse the fourth variable has the character of a time variable while the other three have the character of space variables.*

This principle demands the attainment of an ideal which is admittedly mathematical in its character By means of it alone one could not come to grips with phenomena. One needs some additional hypotheses One of these is to the effect that the restricted theory of relativity is valid in free space, that is, in space free of a gravitational field The other is the celebrated law of the equivalence of gravitational forces and the apparent forces due to acceleration This may be set forth as follows

PRINCIPLE OF EQUIVALENCE *For an indefinitely small region of the world (that is, a region so small that the variation of gravitation in it in both time and space is negligible) there exists a coordinate system* S_0 (X_1, X_2, X_3, X_4) *with respect to which gravitation has no influence either upon the motions of mass particles or upon any other physical phenomena whatsoever.*

Such is the logical basis from which the general theory of relativity proceeds We can not here follow it in its high enterprise of conquest over the laws of nature The road (at present and perhaps for a long time to come) can be followed only by one who is willing to give serious and long-continued attention to the study of certain branches of mathematics In the earlier parts of the argument the reasoning is rather technical and abstruse in character and the general steps are intelligible only to those who have a considerable acquaintance with a certain range of mathematical ideas After a time the exposition comes down, if not to earth, at least to the solar system and cases begin to appear in which it is possible to find means for choosing between the theory of Newton and that of Einstein.

Three crucial phenomena have been brought to light by means of which to test between the two theories We shall now speak briefly of each of these

For a long time astronomers have known that there is a certain forward advance in the perihelion position of the planet Mercury which can not be accounted for on Newton's theory It amounts to about 42 seconds of angular measure per century This is well accounted for by Einstein's theory

Einstein predicted, on the basis of his theory, that a ray of light from a star which is seen apparently close to the edge of the sun would be found to be bent out of a straight path and that the deflection thus caused would turn out to be 1 74 seconds of angular measure, the bending being in such a direction that the star could actually be seen when just behind the edge of the sun. The prediction has been verified with a good degree of precision, observations having been taken at two eclipses of the sun.

A third crucial phenomenon is associated with the vibrations of an atom in a gravitational field Since the periods of an atom furnish a sort of natural clock, it should give an invariant measure of an interval of time. Proceeding from this hypothesis one concludes that an atom vibrates more slowly on the sun than on the earth, due to the influence of the larger gravitational field of the sun Hence the lines of the spectrum should be displaced towards the red For the part of the spectrum usually observed this amounts to about .008 tenth-meters (a tenth-meter $=10^{-10}$ meters) For a long time there was grave doubt whether this phenomenon is actually existent, but the evidence for its existence now (1933) seems to be conclusive

Moreover in recent years it has come to be recognized that the stars known as white dwarfs have masses which are comparable with that of the sun, while their radii are much smaller The companion of Sirius is a star whose radius is about 1/35 of that of our sun Computation shows that the shift in the lines of the spectrum produced by light passing near this star should be about 30 tenth-meters. This matter was put to the test at Mount Wilson Observatory and an actual shift of 32 tenth-meters was found One would conclude then that it is now hardly possible to doubt the actual existence of the spectral shift predicted by the Einstein theory.

Whatever may be the final verdict concerning the validity of the theory of relativity as a whole, it has certainly made a fundamental and permanent contribution to astronomy in developing a modification of Newton's law of gravitation. It has been checked experimentally in three very different ways and is thus established on a rather secure basis. Three such conquests as those just recorded have probably never before been made so nearly simultaneously by a single theory developed from one point of view consistently maintained throughout

SUPPLEMENT II

THE THEORY OF TYPES[1]

BY PAUL WEISS*

It would seem from the interpretation that Whitehead and Russell put on the theory of types, that it is impossible or meaningless to state propositions which have an unrestricted possible range of values, or which, in any sense, are arguments to themselves. Thus on the acceptance of the principle that statements about all propositions are meaningless,[2] it would be illegitimate to say, "all propositions are representable by symbols," "all propositions involve judgment," "all propositions are elementary or not elementary," and if no statement could be made about all the members of a set,[3] it would be impossible to say, "all meanings are limited by a context," "all ideas are psychologically conditioned," "all significant assertions have grammatical structures," etc., all of which are intended to apply to themselves as well. The theory seems also to make ineffective a familiar form of refutation. General propositions are frequently denied because their enunciation or acknowledgment depends on the tacit supposition of the truth of a contradictory or contrary proposition. Such refutations assume that the general proposition should be capable of being an argument of the same type and to the same function as its own arguments, so that according to Whitehead and Russell, they fallaciously refute "by an argument which involves a vicious circle fallacy".[4]

That these limitations on the scope of assertions or on the validity of refutations are rarely heeded is apparent even from a cursory examination of philosophical writings since 1910. Thus Russell, apropos to Bergson's attempt to state a formula for the comic says,[5] "it would seem to be impossible to find any such formula as M. Bergson seeks. Every formula treats what is living as if it were mechanical, and is therefore by his own rules a fitting object of laughter." The characterisation of all formulæ, even though it refers to a totality, seems to Mr. Russell to be of the same type as the formulæ characterised

[1] Chap. II, *Principia Mathematica.*

*[*Reprinted from* MIND *a Quarterly Review of Psychology and Philosophy Vol. XXXVII, N.S., No.* 147, with minor corrections.]

[2] P. 37, *ibid.* (second edition). [3] P. 37, *ibid.* [4] P. 38, *ibid.*

[5] "Prof. Guide to Laughter," *Cambridge Review,* Vol. 32, 1912, and Jourdain's *Philosophy of Mr. B*tr*nd R*ss*ll,* pp. 86-7.

If the theory were without any embarrassments of its own, and were indispensable for the resolution of the so-called paradoxes[1] (which no one seems to believe), there would be nothing to do but to acknowledge the impossibility of cosmic formulations, as well as the inadequacy of philosophic criticisms, and to pass charitably over such remarks as Russell's as mere accidents in a busy life However, the statement of the theory itself involves the following difficulties in connection with (1) its scope, (2) its applicability to propositions made about it, and (3) its description

1 It is either about all propositions or it is not

A If it were about all propositions it would violate the theory of types, and be meaningless or self-contradictory

B. If it were not about all propositions, it would not be universally applicable To state it, its limitations of application would have to be specified One cannot say that there is a different theory of types for each order of the hierarchy, for the proposition about the hierarchy introduces the difficulty over again.

2. Propositions about the theory of types (such as the present ones, as well as those in the *Principia*) are subject to the theory of types, or they are not

A. If they were, the theory would include within its own scope propositions of a higher order, and thus be an argument to what is an argument to it[2]

B. If they were not, there would be an unlimited number of propositions, not subject to the theory, that could be made directly or indirectly about it Among these propositions there might be some which refer to a totality and involve functions which have arguments presupposing the function

3. The statement of the theory of types is either a proposition or a propositional function, neither or both

A. If it were a proposition, it would be either elementary, first order, general, etc, have a definite place in a hierarchy and refer only to those propositions which are of a lower order If it were held to be a proposition of the last order, then the number of orders would have a last term, and there could not be meaningful propositions made about the theory The *Principia* should not be able to say, on that basis, just what the purpose, character and application of the theory is

B Similarly, if it were a propositional function, it would have a definite place in a hierarchy, being derived from a proposition by generalisa-

[1] Paradoxes, though contrary to common opinion, may be and frequently are true Paranoumena, violating principles of logic or reason, if they are not meaningless, are false, and it is only they which are capable of logical analysis and resolution What the *Principia* attempts to do is to solve apparent paranoumena with a real paranoumenon

[2] P. 39, *Principia Mathematica.*

tion. It could not refer to all propositions or propositional functions, but only to those of a lower order

C If it were neither it could not be true or false, nor refer to anything that was true or false It could not apply to propositions, for only propositions or propositional functions, in a logic, refer to propositions

D If both at once, it would be necessarily self-reflexive

a If as function it had itself as value, it would refer to itself But the theory of types denies that a function can have itself as value

b If as function it had something else as value, it would conform to the theory, which insists that functions have something else as values The theory then applies to itself and is self-reflexive, and thus does not apply to itself As, by hypothesis, it is a value of some other function, there must be propositions of a higher order and wider range than the theory of types

It is no wonder that the perpetrators of the theory have not been altogether happy about it[1] What is sound in it—and there is much that is—is best discovered by forgetting their statements altogether, and by endeavouring to analyse the problems it was designed to answer, without recourse to their machinery The result will be an acknowledgment of a theory of types having a limited application, and a formulation of a principle which will permit certain kinds of unrestricted general propositions

To do this we shall deal in detail with two apparent paranoumena dealt with in the *Principia*, where the difficulty is largely *methodological* We shall then treat of Weyl's "heterological-autological" problem, where the difficulty is due to a confusion in *meanings* Those problems which cannot be dealt with under either heading will be those which need a theory of types for their resolution

1 *Epimenides* The proposition "All Cretans are liars" must be false if it applies to Epimenides as well, for it cannot be true, and only as false has it meaning If it were true, it would involve its own falsity. When taken as false, no contradiction, or even paradox, is involved, for the truth would then be "*some* Cretans tell the truth" (The truth could not be "all Cretans tell the truth" for Epimenides must be a liar for that to be true and by that token it must be false). Epimenides himself would be one of the lying Cretans, and one of the lies that the Cretans were wont to make would be "all Cretans are liars". Thus if Epimenides meant to include all his own remarks within the scope of the assertion, he would contradict himself or state a falsehood If it be denied that a contradictory assertion can have meaning, he must be saying something false if he is saying anything significant Had he meant to refer to all other Cretans there is, of course, no difficulty, for he then invokes a kind of theory of types by which he makes a remark not intended to apply to himself All difficulty disappears when it is recognised that the formal implication, "all Cretanic statements are lies" can as a particular statement be taken as one of the values of the terms of this implication Letting $Ep\ !\ p$ represent "Epimenides once asserted p", ϕ represent "Cretanic" and p represent a statement or

proposition, then for "All Cretanic statements are false (or lies)," we have

$$1 \quad \phi p \supset_p \backsim p$$

And as Epimenides is a Cretan, for any assertion he makes we have

$$2 \quad Ep \,!\, p \supset_p \phi p$$

As No 1 is an argument to the above—it being Epimenides' present remark—we get

$$3 \quad Ep \,!\, \{\phi p \supset_p \backsim p\} \supset . \phi\{\phi p \supset_p \backsim p\}$$

No 1, as a Cretanic statement, is an argument to No 1 as a formal implication or principle about Cretanic statements, so that

$$3A \quad \phi\{\phi p \supset_p \backsim p\} \supset \backsim \{\phi p \supset_p \backsim p\}$$

No 3 and No 3A by the syllogism yield

$$3B \quad Ep \,!\, \{\phi p \supset_p \backsim p\} . \supset \backsim \{\phi p \supset_p \backsim p\}$$

so that in this instance Epimenides lied

It is important to note that No 1 states a formal implication, and that No 3, No 3A and No 3B employ No 1 as a particular assertion or specific argument to their functions No 3A is an instance of the implication expressed by No 1, and is this instance because of the particular argument it does have It states the fact that " 'all Cretanic statements are false' is a Cretanic Statement," implies that " 'all Cretanic statements are false' is false" Substitution of another argument would give a different instance, though of course of the same implication The implication contained in its argument does not have instances " 'Some Cretanic statements are false' is a Cretanic statement" or " 'This Cretanic statement is false' is a Cretanic statement" are not instances of " 'All Cretanic statements are false' is a Cretanic statement," but of "P is a Cretanic statement" These three propositions have different subjects, they are different values of the same propositional function That these subjects have relations to one another is of no moment "My wife loves me" and "my mother-in-law is old (or loves me)" are two distinct and logically independent propositions, even though there is a relationship between the two subjects

It is because any considered general proposition is at once an individual fact, and a formal implication or principle, with many possible arguments, that it is capable of being taken as an argument to itself All propositions about words, logic, truth, meaning, ideas, etc , take arguments which fall in these same categories, and in so far as such a general proposition is stated in words, determined by logic, etc , it should, as such a fact, be an argument to itself as a formal implication The principle must be false if this cannot be done, for it is sufficient, in order to overthrow a proposition of this kind, to produce one argument for which it does not hold One may limit the principle by asserting that it holds for "all but ", in which case it is a *restricted* general proposition Nominalism, association of ideas, scepticism, the theory of universal tautology, the denial of logic are defended in propositions which cannot take themselves as arguments, and which as facts are arguments to contradictory principles. Their contradictory principles therefore hold sometimes at least, that these doctrines must be false if they are put forward without restriction, and cannot be universally true, if, in Bradley's words, they "appear".

2 "I am lying"—if it be taken in isolation from all fact—is a meaningless statement. There must be some objective truth that is distorted, and unless it is provided the assertion has no significance. This proposition means either, "I am lying about X", "I always lie," or "I have always lied". The first can be either true or false without giving rise to any problem, except where "all my assertions" is made an argument to X, in which case it is equivalent to either the second or third formulation "I always lie" involves the same situation as with Epimenides, and the proposition is false The supposition of its truth would involve a contradiction; the supposition of its falsity means simply that I sometimes lie and sometimes tell the truth If what is meant is that "I have always lied" that does not involve a contradiction, for what is intended is a restricted proposition, applying to *all but* the present one It can be true because it does not apply to all propositions, if it were false, then sometimes I lied and sometimes I did not In short, there is nothing like a self-reflective universal liar, which is an interesting moral conclusion to derive from a logical analysis Similarly, there cannot be a thorough scepticism held by the sceptic to be valid

Prof Whitehead (to whom I am also indebted for the notation) has pointed out to me that wherever a conjunction of propositions results in a *reductio ad absurdum*, there is no way of determining on logical grounds alone which of the antecedents fails, or is false (though one at least must be) Thus in the case of Epimenides we have

$$4 \quad \underset{(A)}{\{\phi p \supset_p \backsim p\}} \quad \underset{(B)}{\{Ep \,{}^{\prime}\, p \supset_p \phi p\}} \quad \underset{(C)}{Ep \,{}^{\prime}\, \{\phi p \supset_p \backsim p\}} \supset \underset{(D)}{\backsim \{\phi p \supset_p \backsim p\}}$$

It is because B and C are in that case assumed to hold, that we can say that A must fail If the truth of all these antecedents were undetermined, we should have merely the general rule a *reductio ad absurdum* has as a necessary condition the conjunction of one or more false propositions Transposition—

$$4'. \quad \underset{(D)}{\{\phi p \,.\supset_p \backsim p\}} \supset \underset{(A)}{\backsim \{\phi p \supset_p \backsim p\}} \vee \underset{(B)}{\backsim \{Ep \,{}^{\prime}\, p \supset_p \phi p\}} . \vee . \underset{(C)}{\backsim Ep \,{}^{\prime}\, \{\phi p \supset_p \backsim p\}}$$

makes it apparent that to deny the conclusion of a *reductio ad absurdum* is to imply that at least one of the antecedents is false

In connection with the *reductio ad absurdum* involved in the assertions, "I always lie" and "I always doubt," No 4B reduces to the tautologies "If I assert p, p is my assertion,' and "If I doubt, the doubt is mine" In these cases, the only alternatives left are the denial of the fact of the assertion (No. 4C), or the truth of the principle itself (No 4A).

3 Weyl's heterological-autological contradiction[1] is the result of a material fallacy of amphiboly in connection with the employment of adjectives. The simplest form of such a fallacy is due to a failure to distinguish between an adjective as substantive and an adjective as attribute Thus if we treat both the subject and attribute in "large is small" and "small is large" as attributes united by a copula expressing identity (instead of reading it as "large is a small word," "small is a large word") we could say "whatever is small is large, and whatever is large is small" No one, I believe, since the Megarics, has been troubled by this particular confusion

The present problem is the result of a confusion, not between substantive and adjective, but between an adjective which expresses a property, and an adjective which expresses a relation between this property and the substantive. All words can be described in terms of a property—they are long, short, beautiful, melodious, etc , words They can be classified in accordance with these properties, giving us the class of long words, short words, etc They can also be classified as either "autological" or "heterological," depending on whether or not the same word is at once substantive and property-adjective, the terms 'autological" and "heterological" expressing relationships between the substantive and adjective

The autological class is made up of words, each of which expresses a property which it possesses; though all of them have unique properties If "short" be short, and if "melodious" be melodious, they would both be members of the autological class, though in addition, "short" would be a member of the class of short words, and "melodious" would be a member of the class of melodious words

The heterological class is made up of words, each of which expresses a property which it does not possess If "long" be short, and if "fat" be thin, they would both be members of the heterological class, although here also "long" would be a member of the class of short words, and "fat" would be a member of the class of thin words Though when classified according to the relationship of the adjective to the substantive, "short" would be an autological word and "long" a heterological word, they would both be members of that class which was defined in terms of the properties of words—being in this case, members of the class of short words

Now if heterologicality were a property that a word could have, and if the word "heterological" had that property, it would be a member of the autological class, for it would then possess a property that it expressed But it would also be a member of a class of words which had the *property* of heterologicality This class is determined by taking the properties of words, and if it be called

[1] Briefly stated it is all words which express a property they possess are autological, all words which express a property they do not possess are heterological If 'heterological' is heterological it expresses a property it possesses and is thus autological, if it is autological, it expresses a property it does not possess and is therefore heterological. *Das Kontinuum,* p 2

"heterological," must be distinguished from that class which was determined not by properties, but by the relationship between properties and substantives

If there were a property like autologicality and if "heterological" had that property,[1] it would be a member of the heterological class, for it would express a property which it did not possess But it would also be a member of the class of words which possessed autologicality and could thus be classified

Thus if "heterological" had the property of autologicality, it would be in the heterological class owing to the *relation* which held between the property and substantive (or between a property it possessed and the property it expressed), but it would be in the class of autological words, owing to a *property* it possessed If it had the property of heterologicality, it would be in the autological class on the basis of the *relation*, and in the class of heterological words on the basis of *property* classification There is no difficulty in considering something as a member of two distinct classes, owing to the employment of different methods of classification There is no contradiction in saying " 'heterological' expresses the property heterologicality, possesses the property autologicality, and the relation between these properties is heterological, or that it expresses and possesses the property heterologicality and the relation between them is autological " Similarly, Richard's contradiction, Berry's contradiction, and that involving the least indefinable ordinal, are resolvable by recognising that "nameable" and "indefinable" are used in two sharply distinguishable senses They do not require a hierarchy, but a discrimination in the methods of description

When a distinction is made between a class and its membership (the distinction between a number of numbers and a number is a particular case of this), and between a relation of objects and a relation of relations, the requirements for the solution of the other mathematical problems are provided A class is other than its members, and a relation, like all universals, transcends any given instance or totality of instances As they have characters of their own, universals can be described in terms of other universals, which in turn transcend them Arguments are of a different "type" than functions, just so far as they have different logical characteristics, *i e* are different kinds of logical facts. The class which is an argument to a function about classes has, as argument, a different logical import than the function, and its arguments have a different import from it This is true of all functions, restricted and unrestricted alike, for it means simply that they are discriminable from their arguments They can, despite this difference, have characteristics in common with their arguments, and are to that extent unrestricted Thus in the case of "the class of those classes which are identical with themselves," the class of classes can be

[1] 'Heterological,' in fact, has the properties of being long, polysyllabic, etc, and it is questionable whether there are properties like autologicality and heterologicality possessed by words If there be no such properties, "heterological' is a member of the class of long words, polysyllabic words, etc In addition it would be one of the terms related by the heterological relation, which fact would not make it have the *property* of heterologicality

taken simply as a class, without logical embarrassment Yet a class of classes differs from a class, and must therefore be capable of a different characterisation, and thus also be an argument to a function of a different type With some classes, it may not be possible to consider them as arguments to their own functions, without uncovering a contradiction In such cases (*e g* the class of those classes which are not members of themselves, and the relations which are connected by their contradictories), it is the difference between the function and the argument that is of moment That *some* functions cannot take themselves as arguments does not indicate that *all* functions are restricted in scope, but simply that they are *non-restricted* Some classes and functions are restricted and some are not To say that all are restricted because some are is an obvious fallacy

Whenever, as individual, a general proposition is in the class of those objects of which it treats, but cannot be considered as an argument to itself, it is either false or restricted in scope If the second, its range of arguments must be specified Accordingly, we can state as a *necessary* condition for the truth of a general proposition, whose scope is unspecified, that when it has a character, which is one of the characters about which it speaks, it *must* be an argument to itself Thus if Bergson adequately described the comic, his formula should be an object of laughter, and if the theory of types is universal in application, it should be capable of being subject to itself Conformity to this condition indicates that the unrestricted proposition is *possibly* true, not that it is necessarily true To demonstrate that such a proposition was necessarily true, it would be essential to show that the supposition of its falsity assumes its truth. That there is danger in applying this rule can be seen from the consideration of some such proposition as "Everything is made up of language elements". Its denial will be made up of language elements, and would seem to demonstrate that the proposition was necessarily true Supposition of the falsity of a proposition, however, means verbal denial only in so far as the proposition applies to the realm of language If it applies to everything, supposition of its falsity involves the positing of the objects of assertions, not the assertions A necessary unrestricted proposition about everything can be supported only by a demonstration that the supposition of an argument for which it does not hold is self-contradictory If the proposition has to do with grammar, meaning, logic, judgment, etc , the conditions for a necessarily true and unrestricted proposition would be 1 the assertion of it is an argument to it, 2 any possible denial is an argument to it That "any possible denial" rather than "any given denial" is required, is apparent from the consideration of the following propositions "All sentences are made up of eight words," "No sentence is made up of eight words". Each of these contains eight words It is because of the fact that we can formulate propositions such as, "It is false that every proposition must be made up of eight words," that the condition is seen not to have been met

An unrestricted proposition applies to every member of the category, and has some aspect of itself as value. It is in some sense then a determinate in the category which it determines. If the proposition refers to some other category than the one to which it as fact, or some aspect of it as fact, belongs, it is restricted Thus "all men are mortal" is neither man nor mortal, and as condition does not determine itself as fact. Any proposition referring to that statement would be of a different type, and would deal with its truth, falsity, constituents, historical place, logical structure, etc Though the unrestricted propositions have no limitations, the category to which they refer may have Epimenides' remark, for example, referred only to Cretans As his assertion was a determinate in the category, and as his statement of the supposed conditions imposed on the members of that category was not a possible argument to the general proposition, the general proposition was seen to be false or restricted Had he said, "All Cretans tell the truth," he would have stated an unrestricted proposition which was possibly true It could not be said to be necessarily true unless Cretans and lie, against the evidence of history, were actually contradictories

Accordingly, we shall say *All true unrestricted propositions are arguments to themselves, or by transposition, those propositions which are not arguments to themselves are either restricted or false* As this proposition can take itself as argument it is possibly true Unless no proposition is possible which does not conform to it, it cannot be said to be necessarily true I have not been able to demonstrate this and therefore accept it as a definition or "methodological principle of validation" The theory of types, in its most general form, may be stated as *A proposition or function of order n, which cannot be an argument to itself, is, as fact, an argument of a proposition or function of order* $n+1$

In accordance with the scheme of the criticism of the theory of types, we can describe our principle as (1) applying to all propositions, including (2) those which refer to it (3) It is a formal implication with itself as one of its arguments The theory of types, on the other hand, (1) does not apply to all propositions, but only to those which are restricted, (2) *may* apply to those propositions which refer to it, and (3) is a formal implication which cannot take itself as argument

The theory of types cannot be an unrestricted proposition about all restricted propositions As an unrestricted proposition it must take itself as argument, but its arguments are only those propositions which are *not* arguments to themselves It cannot therefore be unrestricted without being restricted Nor can it be a restricted proposition about all restricted propositions for it would then be one of the restricted propositions, and would have to take itself as argument—in which case it would be unrestricted Hence it cannot be restricted without being unrestricted Three possible solutions may be advanced The first is that the theory of types is restricted and does not apply to *all* restricted propositions, but only to *some* of them It is not an argument to itself but to some other proposition about restricted propositions This in turn will have to be restricted and refer only to some propositions, and so on, giving us theories of types of various orders The proposition made about the totality of these orders would be of a still higher order and would in turn presuppose a higher order *ad infinitum* The theory of types thus depends on theories of types of theories of types without end This seems probable on the ground that the theory is based on the recognition that no proposition can be made about all restricted propositions, so that it must by that very fact admit that it cannot apply to all of them Instead, therefore, of the theory of types applying to all propositions, and determining them in various orders, it does not even apply to all of a given class of them. This interpretation would not affect unrestricted propositions, and would merely show that the determination of restricted propositions is subject to determinations without end

The second possibility is suggested by the consideration of a proposition such as "all truths are but partially true" If that were absolutely true, it would contradict itself, and if it were not, could apply only to some truths Considered as referring to the necessary limitations which any finite statement must have, it would take itself as argument in so far as it was finite, thus indicating that it was absolutely true about finite propositions, and yet not absolutely true as regards all truths By pointing out the limitations of a finite statement it indicates that there is an absolute truth in terms of which it is relatively true On this interpretation, any condition which imposes universal limitations is unlimited in terms of what it limits, but limited in turn by some other condition One might hold, therefore, that the theory would be unrestricted as regards restricted propositions, and restricted as regards all propositions, and would point to a higher principle which limits it

The third possibility is to allow for "intensive" propositions which are neither restricted nor unrestricted, being incapable of any arguments. The theory of types could be viewed as such an intensive proposition, and what we have called its arguments, would merely "conform" to it This interpretation means the downfall of a completely extensional logic, and a determination of an extensional logic as subordinate to an intensional one

There are difficulties in each of these interpretations The last seems to me to be best. In any of these cases, however, a restricted proposition which refers to some other than the restricted aspect of the theory would be subject to the theory and the principle we have laid down about unrestricted propositions could still hold Those restricted propositions which refer to the restricted character of the theory would not be an argument to it on the first, would be an argument to it on the second, and would neither be nor not be an argument to it on the third solution

To briefly summarise The theory of types must be limited in application Not all the problems it was designed to answer require it, another principle of greater logical import is desirable, while for the resolution of the problems in which it is itself involved, very drastic remedies are necessary. No matter how the theory fares, the possibility of the methodological principle and the possibility of other solutions for the so-called paradoxes, indicate that it is at least not as significant an instrument as it was originally thought to be.

SUPPLEMENT III

A NON-ARISTOTELIAN SYSTEM AND ITS NECESSITY FOR RIGOUR IN MATHEMATICS AND PHYSICS*

BY ALFRED KORZYBSKI

> We are here dealing with a concrete mathematical problem which is not trivial, but at the same time is solvable, and I cannot imagine that any mathematician can find the courage to elude its honest solution by means of a metaphysical dogma (549) HERMANN WEYL

> I protest against the use of infinite magnitude as something completed, which in mathematics is never permissible Infinity is merely a *façon de parler*, the real meaning being a limit which certain ratios approach indefinitely near, while others are permitted to increase without restrictions (74) K F GAUSS

A very extensive literature shows that the problems of 'infinity' pervade human psycho-logical reactions, starting from the lowest stage of human development up to the present and that without some theory of 'infinity', modern mathematics would be impossible Up to date, no satisfactory theory of infinity, on which all mathematicians could agree, has been produced The results are rather bewildering because what appears to some prominent mathematicians as perfectly sound mathematics is evaluated by other equally prominent scientists as a 'mental' disease (Poincaré), or we find opinions that a large portion of mathematics is devoid of proof and has to be accepted on faith, or that some parts of mathematics must be treated as non-sense (Kronecker, Brouwer, Weyl . ,). 'There are eminent scholars on both sides and the chance of reaching an agreement within a finite period is practically excluded', says Brouwer, and certainly such a state of affairs does not allow us to have any satisfactory modern standards of proof and rigour, the last thing we should expect in mathematics

The majority of those mathematicians who take interest in the soundness of their science seem to believe that the main difficulty centres around the validity of the 'law of excluded third' ('A is B, or not B') of the accepted, sharply two-valued, chrisippian form of *A* 'logic' They disregard the fact that we are born, bred, educated, speak a language, live under conditions, institutions , which still remain desperately *A* or even pre-aristotelian If we attempt to reject one of the two-valued 'laws of thought' or postulates of the *A*-system, but retain *A* or pre-aristotelian *elementalistic* 'psychologies', 'logic', and *s r*, no agreement in 'a finite period' can be expected, and the present mathematical chaos would continue

*Paper presented before the American Mathematical Society at the New Orleans, Louisiana, Meeting of the A A A S December 28, 1931 I continue to use the abbreviations introduced in this book

Among the more important schools we may distinguish roughly:[1]

1) The logistic school represented by Peano, Russell, and Whitehead, who accept the chrisippian, two-valued, restricted form of the *el* 'logic' and so may be called the *chrisippian school*

2) The axiomatic school, represented by Hilbert and his followers, which may be called the *aristotelian school*

3) The 'intuitional' school represented by Brouwer and Weyl who question the 'law of excluded third', and so may be called the *non-chrisippian school*

4) The Polish school of (*a*) 'intuitional' formalism with Łukasiewicz, Tarski, Leśniewski as representatives, which may be called the *non-aristotelian school* Łukasiewicz generalized the A 'logic' to three-valued 'logic' which covers modality Łukasiewicz and Tarski finally produced a general many-valued 'logic' of which the two-valued represents only a limiting case. Leśniewski produced *Protothetic*, a still more general 'logical' system, by introducing variable 'funktors', * (*b*) The *restricted semantic* school represented by Chwistek and his pupils, which is characterized mostly by the semantic approach, and by paying special attention to the *number* of values, establishing the thesis that the older 'freedom from contradictions' depends on one-valued formulations, as discovered by Skarzeński and quoted by Chwistek This school has already produced new foundations (still *elementalistic*) for 'logic' and mathematics, and leads to generalized arithmetics and analysis

5) The average prevalent mathematical technician, who does not realize that he belongs to the numerically large class which may be called the 'christian science' school of mathematics, which proceeds by faith and disregards entirely any problems of the epistemological foundations of their supposed 'scientific' activities

It should be noticed that all existing mathematical schools accept implicitly, at least, A *elementalism* and do not challenge identity, a principle which happens to be invariably false to facts and which therefore should be entirely abolished

The above classification suggests that, in spite of great achievements in the field of mathematical foundations, no school can expect to be convincing or accepted by other schools as long as we all flounder in the A and *el* ambiguities which prevent any possibility of agreement It becomes obvious also that when a $\bar{A}$ and *non-el* system is formulated it will necessitate a new paradox-free foundation for mathematics and so a new school of mathematics will arise which may be called

6) The *general semantic, non-aristotelian, non-elementalistic* school of mathematics It is premature to give the names of the leading pioneers in this field at present

*At present Łukasiewicz and Tarski call their many-valued 'logic' non-chrisippian, but this name does not seem appropriate because these authors generalized both forms of the aristotelian 'logic' to a many-valued 'logic' of which the two-valued becomes only a limiting case Thus it seems that their many-valued 'logic' is better described by the term *non-aristotelian,* yet still *elementalistic* 'logic'

In a $\bar{A}$-system, the 'logical' problems of freedom from contradiction become also semantic problems of *one-valued meanings* made possible only under ∞-valued, $\bar{A}$, *non-el* general semantics, and the recognition of the $\bar{A}$ *multiordinality* of terms , . A $\bar{A}$-system introduces some fundamental innovations, such as completely rejecting identity, elementalism , and becomes based on *m o* structure and order, and so ultimately becomes *non-el* The A, (3+1)-dimensional *el*, (in the main) *intensional* system becomes a four-dimensional, *non-el*, (in the main) *extensional* system In such a system we cannot use the formulations of *elementalistic* 'logics' and 'psychologies', but must have $\bar{A}$, *non-el* general *semantics*, which when generalized become an entirely general discipline applicable to all life, as well as to *generalized* mathematics For the above reasons I shall use the word 'logic', in its *el* sense, with quotation marks, and use the term *general semantics* for a *non-el*, $\bar{A}$ discipline corresponding to the *el*, A or $\bar{A}$ 'logics'.

Investigations show that the primitive man (and the 'mentally' ill) use *one-valued semantics* which have left more or less marked traces in all of us, reflected even in science and mathematics The elimination of these primitive traces clears the foundation for an adult civilization, a theory of sanity, and the elimination of the scientific and mathematical paradoxes

To assume that because a many-valued 'logic' has been produced, all the problems of mathematical infinity, irrational numbers, continuity, mathematical induction, validity of mathematical proof, mathematical existence , have been solved, would be a mistake The aim of the present paper is to analyse some of the fundamental complexities produced by the unconscious operation of the one-valued semantic *identification* concealed in the formulation of the 'law of identity', which have escaped notice until now, and which would make the application of a many-valued 'logic' or ∞-valued semantics and agreement impossible Here, as in the $\bar{E}$ and $\bar{N}$ systems, only the most general formulations help us to *discriminate* between the particular cases, and so to eliminate the undesirable traces of one-valued semantics by building a $\bar{A}$-system, of which the A and pre-A represent only particular cases

Let me recall the 'philosophical grammar' of our language which we solemnly call the 'laws of thought', as given by Jevons [2]

1) The law of identity Whatever is, is
2) The law of contradiction Nothing can both be, and not be
3) The law of excluded third Everything must either be, or not be

These 'laws' have different 'philosophical' interpretations which help very little and for my purpose it is enough to emphasize that (1) The second 'law' represents a negative statement of the first, and the third represents a corollary of the former two, namely, no third possible between two contradictories. (2) The verb 'to be', or 'is', and 'identity' play a most fundamental role in these formulations We should not be surprised to find that the investigation of these terms may give us a long sought solution Such an investigation is very laborious and difficult 'The complete attempt to deal with the term *is* would go to the form and matter of everything in existence, at least, if not to the possible form and matter of all that does not exist, but might. As far as it could be done, it would give the grand Cyclopaedia, and its yearly supplement would be the history of the human race for the time', said Augustus de Morgan in his *Formal Logic*, and this opinion I found fully justified.

So I must be brief, and state but roughly, that in the Indo-european languages the verb 'to be' has at least four entirely different uses. (1) as an auxiliary verb, 'Smith is coming', (2) as the 'is' of predication, 'the apple is red', (3) as the 'is' of 'existence', 'I am', (4) as the 'is' of identity, 'the apple is a fruit'. The fact that four semantically entirely different words should have one sound and spelling appears as a genuine tragedy of the race, the more so since the discrimination between their uses is not always easy.

The researches of the present writer have shown that the problems involved are very complicated and cannot be solved except by a *joint study* of mathematics, mathematical foundations, history of mathematics, 'logic', 'psychology', anthropology, psychiatry, linguistics, epistemology, physics and its history, colloidal chemistry, physiology, and neurology, this study resulting in the discovery of a general semantic mechanism underlying human behaviour, many new interrelations and formulations, culminating in a $\bar{A}$-system. This semantic mechanism appears as a general psychophysiological mechanism based on four-dimensional order, present and abused in all of us, the primitive man, the infant, the 'mentally' ill, and the genius not excluded. It gives us an extremely simple means of training our *s.r.*, which can be applied even in elementary education.

The scientific problems involved are very extensive and can be dealt with only in a large volume. Here I am able to give only a very sketchy summary without empirical data, omitting niceties and technicalities.

(*a*)

Paris ——— Dresden ——— Warsaw
* ——————— * ——————— *

(*b*)

Dresden ——— Paris ——— Warsaw
* ——————— * ——————— *

If we consider an actual territory (*a*) say, Paris, Dresden, Warsaw, and build up a *map* (*b*) in which the order of these cities would be represented as Dresden, Paris, Warsaw, to travel by such a map would be misguiding, wasteful of effort, In case of emergencies, it might be seriously harmful, . We could say that such a map was 'not true' , or that the map had a ***structure not similar*** to the territory, structure to be defined in terms of relations and multi-dimensional order. We should notice that

A) A map may have a structure similar or dissimilar to the structure of the territory. (1)

B) Two similar structures have similar 'logical' characteristics. Thus, if in a correct map, Dresden is given as between Paris and Warsaw, a similar relation is found in the actual territory. (2)

C) A map ***is not*** the territory. (3)

D) An ideal map would contain the map of the map, the map of the map of the map . . , endlessly This characteristic was first discovered by Royce We may call it self-reflexiveness (4)

Languages share with the map the above four characteristics.

A) Languages have structure, thus we may have languages of *elementalistic* structure such as 'space' *and* 'time', 'observer' *and* 'observed', 'body' *and* 'soul', 'senses' *and* 'mind', 'intellect' *and* 'emotions', 'thinking' *and* 'feeling', 'thought' *and* 'intuition' , which allow verbal division or separation Or we may have languages of *non-elementalistic structure* such as, 'space-time', the new quantum languages, 'time-binding', 'different order abstractions', 'semantic reactions' , which do not involve verbal division or separation , also mathematical languages of 'order', 'relation', 'structure', 'function', 'variable', 'invariant', 'difference', 'addition', 'division' , which apply to 'senses' and 'mind', that is, can be 'seen' and 'thought of', (5)

B) If we use languages of a structure non-similar to the world and our nervous system, our verbal predictions are not verified empirically, we cannot be 'rational' or adjusted , . We would have to copy the animals in their wasteful and painful 'trial and error' performances, as we have done all through human history In science we would be handicapped by semantic blockages, lack of creativeness, lack of understanding, lack of vision, disturbed by inconsistencies, paradoxes , (6)

C) Words *are not* the things they represent. (7)

D) Language also has self-reflexive characteristics We use language to speak about language, which fact introduces serious verbal and semantic difficulties, solved by the theory of *multiordinality* (8)

The above unusually simple considerations lead to unexpectedly far-reaching consequences.

A) From (7)—it follows that the objective levels which include the events, ordinary objects, objective actions, processes, immediate feelings, 'instincts', 'ideas', *s r* in general . , represent un-speakable levels, *are not words* (9)

B) From (9)—that the use of the 'is' of *identity*, as applied to objective, un-speakable levels, appears invariably structurally false to facts and must be entirely abandoned. Whatever we might *say* a happening 'is', *it is not* (10)

C) From (10)—*structure* appears as the only possible link between the objective, un-speakable, and the verbal levels (11)

D) From (11)—the only possible 'content of knowledge' becomes exclusively *structural* (12)

E) From (12)—the only aim of 'knowledge' and science appears as the empirical search for, and verbal formulation of, structure. (13)

F) The only method for acquiring 'knowledge' is found in an *empirical* investigation of the potentially unknown structure of the world, ourselves included, only afterwards adjusting the structure of languages so that they would be similar, and so of maximum usefulness, instead of the delusional *reversed order* of ascribing to the world the structure of an inherited primitive language (14)

G) The investigation of the potentially known structure of languages in which we predict and then verify the predictions empirically, appears as an important method for the discovery of the structure of the world. (15)

H) Investigations disclose that all A, *el* languages and disciplines built on them (older 'psychologies', 'logics' , and, based on them, economics, sociology, politics, 'ethics' , reflected in turn in our institutions, systems ,) *are not* structurally similar to the world and our nervous system, as they verbally divide what empirically cannot be divided Under such conditions neither a higher grade civilization, nor general sanity, nor paradox-free science and mathematics are possible In *el* languages, our verbal predictions are not verified empirically, and not being able to foresee we must proceed by animalistic 'trial and error'. (16)

I) Mathematics appears as a very limited but the only language in existence, in the main similar in structure to the world around us *and* the nervous system (17)

J) From the study of mathematics, mathematical physics, and physics, we learn, and will continue to learn, the fundamentals of *m o* structure It is no mystery that all chemistry has become a branch of physics, all physics can be made a branch of geometry, all geometry a part of analysis, and all analysis a part of general semantics The present work shows that the analysis *of all human problems* of daily life or science becomes dependent on *general semantics* which on the verbal levels becomes generalized mathematics Thus mathematics, mathematical physics, and physics become the most important disciplines from which we learn most about *structure*,—the only 'content of knowledge' (18)

K) The older *el* 'psychologies' and 'logics' for their maximum usefulness must be transformed into unified *non-el* psycho-logics and general semantics, possible only after studying all forms of human behaviour, mathematics included (19)

L) The study of mathematics as a form of human behaviour, appears necessary prior to the possibility of formulating any laws of semantics (20)

M) The problem of mathematical foundations do not belong to mathematics but to psycho-logics which would not disregard anthropology, and would not be vitiated by our persistence in the use of structurally inappropriate *el* 'psychologies', 'logics', and an innocence of mathematics (21)

N) The 'intuitional' and the 'intuitional' formalist schools of mathematics must be considered as a legitimate, yet not properly formulated, protest against the older elementalism (22)

O) The general semantic school will represent the *non-el* and $\bar{A}$ school of mathematics (23)

P) The present crisis of mathematics ultimately depends on the meanings and use of a few terms such as 'all', 'there is', 'infinite' , which solution depends on a *non-el* theory of meanings, which ultimately can be solved by transforming what might be called the (3+1)-dimensional *el*, A-system, which divides 'space' *and* 'time', (an attitude which is carried all through the system), into a four-dimensional *non-el*, $\bar{A}$-system (an attitude which is also carried all through the system) (24)

Q) From (8)—it follows that statements about statements represent results of new neurological processes, that their content varies, and that we must *discriminate* and *not identify* these different meanings. In other words, only through consciousness of abstracting which represents the most general s.r. of discrimination, or the elimination of identification, can we assign single values to words which have an essentially many-valued character. Identification confuses these many meanings into one (25)

R) We must differentiate between descriptive and inferential words and phrases, and never use inferential terms as descriptive, without realizing that we are doing so (26)

S) Certain words or phrases used to speak about languages, such as 'all statements', 'proposition about all propositions', lead to self-contradictions. We cannot speak about 'all' propositions without some limitations, if we proceed introducing new propositions. Even St. Paul felt the necessity for limiting the values of 'all'.* We are compelled to introduce some equivalents to the biblical 'illegitimate totalities' or the theory of types of Russell (27)

T) Analysis finds that certain of the most important terms we use, such as, 'yes', 'no', 'true', 'false', 'all', 'fact', 'reality', 'existence', 'definition', 'relation', 'structure', 'order', 'number', 'is', 'has', 'there is', 'variable', 'infinite', 'abstraction', 'property', 'meaning', 'value', 'love', 'hate', 'knowing', 'doubt', . . ., may apply to all verbal levels and in each particular case may have a different content or meanings and so *in general no single content or meaning*. I call such terms *multiordinal terms* (*m.o*). The definition of such terms is always given in other *m.o* terms preserving their fundamental multiordinality. In other words, a *m.o* term represents a many-valued term. If the many values are identified, or disregarded, or confused, we treat a fundamentally many-valued term as one-valued, and we must have every kind of paradox through such an identification. All known paradoxes in mathematics and life can be manufactured by the disregard of this fundamental multiordinality. Vice versa, by formulating the general semantic problem of multiordinality we gain means to discriminate between the many meanings and so assign a single meaning in a given context. A *m.o* term represents a variable in general, and becomes constant or one-valued in a given context, its value being given by that context. Here we find the main importance of the semantic fact established by Skarżeński,** that the 'logical' freedom from contradiction becomes a semantic

*Professor Cassius J. Keyser drew my attention to a passage in the first letter of St. Paul to the Corinthians, Chapter 15, line 27. For he hath put *all* things under his feet. But when he saith *all* things are put under him, it is manifest that he is *excepted*, which did put *all* things under him.' Italics are mine.

**Quoted by Chwistek in his *Neue Grundlagen der Logic und Mathematik*.

problem of one-value But for application we must have a four-dimensional, *non-el*, $\bar{A}$, extensional system, based on structure , and the complete elimination of identity (28)

U) That the disregard of multiordinality, orders of abstractions, may lead to identification and therefore false evaluation resulting in disagreement and maladjustment (29)

V) From (25–29)—it follows that identification or confusion of higher order abstractions must be eliminated Because of (7, 9, 10, 25–29)—*all identification must be eliminated* (30)

W) The elimination of identification on all levels, or a complete and unconscious discrimination between different orders of abstractions, including as a special important case the multiordinality of terms, results in general consciousness of abstracting which in turn, solves the paradoxes of life and mathematics and leads to *generalized* mathematics along the lines suggested by Chwistek (31)

X) The realization of the inherent multiordinality of some of the most important terms we have, gives us an enormous flexibility of language. It makes the number of our words indefinitely great When both the writer and the reader recognize this multiordinality, and look for the meaning in the context and discriminate between the orders of abstractions, indicated by the context, confusion becomes impossible (32)

Y) The test for multiordinality is simple We take any statement and test it to see whether a given term applies to it Then we make a statement about this statement and again test if this term applies to the new higher order statement If it does, the given term must be considered multiordinal, because this procedure may be repeated indefinitely (33)

Z) *The complete elimination of identification* does not allow us to use the term 'is' of identity, and so we must use operational, functional, actional, behaviouristic , languages, requiring new attitudes and new *s r*, impossible without the formulation of a $\bar{A}$-system (34)

Z_1) The *s r* of those who produced the general theory of relativity, the unified field theory, the new quantum mechanics, the new revision of the foundation of mathematics , depend on new $\bar{A}$, *non-el*, and non-identity, operational, actional . , attitudes (35)

Z_2) As the $\bar{A}$-system is based on the general elimination of the 'is' of identity, or on 'is not', it is impossible to reject these premises without producing impossible data, and a theory of agreement 'in a finite period' then becomes a possibility (36)

Z_3) The old 'unknowable' becomes abolished and limited to the simple and natural fact that the objective levels *are not* words. (37)

Observation and experience, scientific and otherwise, show that in nature we find a definite order, which establishes a *natural order,* namely, that the sub-microscopic process, called the event or the scientific object, came first, only later abstracting organisms happened and objects which represent the results of abstracting by amoebas or men, came next In the process of evolution we find object first, label next Descriptions first, inferences next The above *natural order* establishes also a *natural order of evaluation* Proper evaluation becomes the foundation for survival, *non-el s r,* the more so since evaluation requires asymmetrical relations of 'more' or 'less' , impossible to handle properly in an A-system Thus the most important level is represented by the sub-microscopic processes What the organism needs is not the three-dimensional shadow of a four-dimensional event, not the abstraction of low order produced by our nervous systems, called the object, but the sub-microscopic dynamic processes without which the desired end-results would not happen The animal, the primitive, the infant, the ignorant man identify the two, live in a delusional world Similarly the objective levels are more important than the verbal levels, and descriptions are more important than inferences If we *identify* any orders while the natural order is established by the asymmetrical relation of 'more', the semantic process of evaluation is *reversed* and appears pathological in different degrees If $a>b$ and we make them delusionally equal in value (identify), then, in the false-to-fact relation $a=b$ we have either over-evaluated the right-hand side or under-evaluated the left-hand side, in both cases reversing the natural order of evaluation It is important to notice that by basing our *s r* on a *natural order* of evaluation, general semantics become a generalized science of order and values, a very secure guide in life, indispensable for sanity, as experiments have shown, and include also generalized mathematics

Another very serious mechanism of identification is found in language

A) Thus we have only *one name*, say 'apple' for the (*a*) un-speakable, un-eatable event or scientific process, (*b*) the un-speakable but eatable abstraction of low order, the object, (*c*) the un-speakable and un-eatable 'mental' picture, or higher order abstraction, on semantic levels, (*d*) and for a definition on verbal levels (38)

B) The multiordinality of terms was not discovered until 1925 and is still generally unknown It presents a serious difficulty facilitating, perhaps even necessitating, identification unless prevented by special formulations and training. Multiordinal terms sound and look alike on all levels, experience has shown how easy it is to confuse their orders and identify the many values into one. (39)

C) The differentiation between descriptions and inferences, and particularly between descriptive and inferential words as such, is also novel, and was, until the present $\bar{A}$-system was formulated, largely disregarded, which again led to identifications and false evaluations (40)

Investigations show, that in all known primitive peoples and in the 'mentally' ill, we find literal identification of different orders of abstractions, which accounts for these semantic states Even their 'perceptions' are different from those of the so-called 'normal', 'civilized' man, because higher order abstractions are projected and identified with lower order abstractions. They identify or ascribe one value to essentially many-valued different orders of abstractions and so become impervious to contradictions with 'reality' and impervious also to higher order experience

The infant, and the rest of us, identify a great deal because of the reasons given above Investigations show that most of human difficulties, public, private, or *scientific* are due to this $\bar{A}$ *s r*, which accounts for the infantile state of our commercial so-called civilization Identification abolishes the natural order of evaluation, but so does also an unconscious assumption of an 'infinite velocity' of a process The A trilogy involved some fanciful 'infinity' assumptions Thus in the A-system the velocity of nerve currents, which is known to be 126 metres per second in the human nervous system, is at present assumed unconsciously as infinite', made evident by the elementalism of 'intellect' or 'emotions' , as something 'by themselves' and detached In the E-system the length of a line, the space constant, and the natural unit of length were assumed 'infinite' In the N-system the velocity of light, known to be finite, was unconsciously assumed to be 'infinite'. In the $\bar{A}$ trilogy these unjustified or meaningless 'infinities' have been eliminated 'Infinite velocity' of a process has *no meaning* It represents only a play upon symbols Velocity is defined as $v=s/t$ If we assume $t=0$ and write $v=s/0=\infty$, this 'velocity' lacks one of the fundamental factors of its definition, namely, t, and so such an expression ceases to define anything at all and has no meaning, although it may be a symbol for a semantic disturbance But the results of such delusional *s r* are far reaching, no matter how mild they might be in degree In a process propagated with 'infinite' velocity there would be no transition or delay in action, and therefore such a process would *not be ordered* Vice versa, the disregard of order in our observations must introduce some mythological 'infinities' somewhere So we see that the semantic process of identification is intimately connected with 'infinity' assumptions, both *abolishing order* Training in natural order trains *s r* away from delusional evaluation, abolishes pathological identification of different degrees and fanciful 'infinities'

Thus we see that the problems of mathematical 'infinity' are extremely complex and involve many fundamental considerations never analysed before in connection with the semantic process of identification Once these problems are analysed and formulated from a $\bar{A}$, *non-el*, structural point of view the problems of 'excluded third' become secondary in importance, easily managed under the creative freedom of the coveted 'consciousness of abstracting'.

Let me recall for continuity, that the mathematicians recognize at present, two kinds of 'infinities'. One with which we are familiar from our school days, symbolized by ∞, Cantor calls 'potential' infinity and defines as a *variable finite*, the misunderstanding of which introduces paradoxes even in high schools, the other, the 'actual' infinity, which introduces paradoxes in universities All these paradoxes are due, as the present enquiry shows, to fundamental fallacies in connection with semantic processes of identification which we learn at home and in elementary schools

The process of identification of different orders of abstractions may be due to pathological conditions, to ignorance, to 'thoughtlessness', to lack of observation, to unconscious false assumptions, to hastiness, to superficiality, to habits of speech, to the structure of language used, In fact, under the A-system it is practically impossible to avoid it, as we can witness it in such a comparatively advanced field as mathematics. The label 'identification' is applied to the semantic process of wrong evaluation going on inside of our skins on the un-speakable objective levels, when we are not aware of the differences between different orders of abstractions. When making it conscious, we may speak of the confusion of the orders of abstractions. To make such a process conscious, we must train in the differentiation or discrimination between different orders of abstractions, and distinguish the different orders by actually learning how to order them. Such training results in general *consciousness of abstracting* which is not inborn, nor fully acquired, even in university training, but which requires *special* training. Experiments in this field are extremely encouraging, in a number of cases, pathological individuals have become 'normal' and the 'unchangeable' human nature has been actually changed. *Infantile reactions in adults are abolished*, and this training becomes a general and simple method for prevention of future semantic disturbances of false evaluation which must result in maladjustment.

To stop identification we must discriminate or differentiate to the limit between what appears always as four-dimensional, absolute individual stages of processes and situations on all levels, verbal included. Let us follow briefly such an actual performance. If we realize (7)—we accept (11–15)—and on *structural* grounds reject the elementalism of the A trilogy as expressed in its 'psychology', 'logic', the division of 'space' *and* 'time', We accept the non-elementalism of the $\bar{A}$ trilogy as expressed in the new terms in the present work and accept also 'space-time', The difference is very serious in all fields, when carried consistently all through the system. As we actually deal with four-dimensional dynamic processes which must be considered continually different, and with world conditions changing also continually, statements about such structural conditions, in an extensional sense, must be considered as involving variables, generating *propositional functions*, doctrinal or system-functions, funktors, But propositional functions, which involve variables, are neither 'true' nor 'false', but ambiguous, and to have a proposition we must assign a value to the variable by *at least* permanently, in principle, assigning a date to it. We must also introduce, in principle, and as a semantic attitude, numerical subscripts to our words. Thus 'apple' in the A-system represents a *name* attached to an *intensional definition*, and space-time considerations do not enter. The term is applied to a definition which might be considered as one-valued and permanent. Now obviously such a language and s.r. are *structurally non-similar* to the world and our nervous system.

If we try to identify a name for a definition, implying permanence, with the objective level which is made up of absolute individuals, and represents ever-changing processes, we must live in a delusional world in which we should expect every kind of paradoxes and psycho-logical shocks.

In a $\bar{A}$-system, for structural reasons, we must retain the general implications of the term 'apple', so we retain the word We must make our language extensional in principle, and the name 'apple' an *individual name*, by calling it 'apple$_1$', 'apple$_2$', . The combination of letters 'a-p-p-l-e' implying similarities, the subscripts 1, 2 . , implying individual differences, which automatically prevent identification But this is not enough Our 'apple$_1$' represents a name applied to an object *and* a process, its *meaning* becomes only one-valued when we assign to it at least a definite date Thus the objective 'apple$_{1\,(\text{Dec } 1\ 1931)}$' may be a very appetizing affair, and 'apple$_{1\,(\text{Jan } 1,\,1932)}$' an un-edible wet splash It should be noticed that the fundamental difference between the A and $\bar{A}$ systems turns out to be a difference of semantic attitudes The scientific facts are not changed The 'apple' of 'Adam' or our own did not differ in essential characteristics under discussion In both the A and $\bar{A}$ systems we actually deal, in principle, with *many-valued processes* The important problem is to adjust the structure of our verbal processes to the structure of the world, hence a $\bar{A}$-system must be made extensional, *non-el*, four-dimensional , Here once more, as in general semantics, the ascribing of one value (or at least limited to a small range of values in practice), in a given situation (context), eliminates paradoxes and contradictions on the older 'logical' grounds. We should notice that the multiordinal terms must be considered as names for many-valued *s r*, depending upon the order of abstractions, hence the name *multiordinal* Names for happenings on the objective levels apply to many-valued processes but should not be considered multiordinal All the psychologics of the differential calculus, 'space-time', enter here, yet the whole field is covered semantically if we entirely abandon the 'is' of identity Instead of training in 'allness' and 'isness'—'this *is* this', we shall train in non-allness, and non-isness—'this *is not* this', in connection with a special diagram called the Structural Differential.

Experience and experiments show that the above seems essential for sanity It is interesting to notice that mathematics has produced a language similar in structure to the human nervous system Roughly the central part of the brain which we call the thalamus is directly connected with the dynamic world through our 'senses' and with those semantic manifestations which we usually call 'affective', 'emotions' , all of which manifest themselves as dynamic The cortex which gives us the static verbal reactions and definitions, is not connected with the outside world directly but receives all impulses through the thalamus On semantic levels the thalamus can only deal with dynamic material, the cortex with static Obviously for the optimum working of the human nervous system, which represents a cyclic chain, where the lower centres supply the material for the higher centres and the higher centres should influence the lower, we must have means to translate the static into dynamic and the dynamic into static, a method supplied *exclusively* by mathematics

With the above considerations we must discriminate between our semantic capacities for *infinite divisibility* of finites, and for the *generation* of infinite postulated processes which by definition *cannot be exhausted* If we use a three-dimensional A language and apply such an 'all' to such an infinite process then we simply produce a self-contradiction If we apply to such a semantic process a four-dimensional 'all with a date', then we have arrested, for the 'time' being, the process, or taken a static cross section of the infinite process at that date, but then we deal with a finite Once we are constantly conscious of abstracting in different orders, these subtle differences become quite clear and the solutions of the problems of infinity follow a similar path as the older problems of the 'infinitesimal', which also was self-contradictory, unnecessary for mathematics When treated as a *variable finite* it was satisfactory and sufficient, and has proven to be a most creative notion in mathematics In the problems of the irrational, continuity., similar subtle identifications or non-discriminations of *el*, A, three-dimensional terms with $\bar{A}$, *non-el*, four-dimensional terms occur, which once eliminated, clear up not only the paradoxes, but some self-contradictory, often unconscious, postulates of some parts of mathematics

Lack of space does not allow me to go into further details, except to suggest how some subtle *discriminations* may help to eliminate identification In my $\bar{A}$-system the differentiation between orders of abstractions on physiological grounds, the introduction of multiordinality of terms, four-dimensional considerations., as a structural necessity for all languages, makes the theory of types unnecessary

For a better understanding of the present work we must at least differentiate

A) Between numerical experience and mathematics

B) Between languages with content and languages without content

C) Between creative building of verbal schemes which, for the sake of generality, have no content, called pure mathematics, and the application of these schemes to actual problems, with content, called applied mathematics

D) Between the contentless mathematics and the investigation of the foundation of mathematics which represents the investigation of the *s r* of mathematicians and belongs to a future *non-el* psycho-logics with content

E) Between different forms of complex adjustment which we have in common with the primitive man, and even the higher animals, and reasoning which starts with conscious observations, passing to descriptions and inferences,.

F) Between the dynamic process of relating ('thinking') on the unspeakable semantic levels, and the verbal expression of 'relations'

G) Between the use of negative terms, disagreement, contradiction, and self-contradiction. In a $\bar{A}$-system contradictions take the form of self-contradictions.

H) Between *el* 'logics' expressed in terms of 'true', 'false', and modality, and the investigations of one-, two-, three-, and ∞-valued *s r*, which become a *general theory of values*, and which may some day include all human interests

I) Between the inherent circularity of 'human knowledge', which must start with sets of undefined terms, and so start with some knowledge, and circular definitions or explanations which define or explain nothing

This list of suggestions is not exhaustive, and in principle appears as inexhaustible I selected only a few topics of immediate need

We should also notice that because on objective levels we deal structurally with absolutely individual stages of processes and situations and by necessity we speak in higher order abstractions and generalities and use many multiordinal terms (without the use of which no speaking is possible), so *any positive statement* about the objective levels must be only probable in different degrees, which introduces a fundamental and entirely *general $\bar{A}$ principle of uncertainty* Heisenberg's restricted principle in physics appears only as a special case For structural reasons we must preserve determinism but because of (11–15)—the older two-valued determinism must be reformulated into the ∞-valued determinism of the maximum probability The einsteinian introduction of non-elementalism in physics has resulted in the automatic elimination of some semantic blockages in the younger physicists. Some of the semantic results and triumphs of science, besides the new quantum mechanics, can be found in the latest (free from identification of the term 'time' with some objectivity) new entropy of Tolman [3]

To sum up, we find that although the primitive man or the 'mentally' ill may have some reactions of orientation, or capacity for relating, which we have in common with the higher animals, yet these do not involve 'reasoning' in the sense defined before Thus a boxer, football player , does a great deal of reflexive relating and wins his match, but this cannot be considered as reasoning in the strict sense as used in a $\bar{A}$-system If we attempt *to discuss* something with a primitive or 'mentally' ill individual and write down his processes of relating, we would have to conclude that he uses *one-valued semantics of identification* of many values into one, or a semantics of inclusion by which 'everything *is* everything else' The 'law of contradiction', or any 'excluded' 'third', or '*n*-th', practically never appears in our sense, yet it is complicated by the use of positive and *negative* terms, to which any meanings connected with some identifications of higher orders may be ascribed Although his prevailing semantic processes appear as a complete and *literal identification*, yet because of the general orienting and relating capacities of organisms and the character of terms used, it would not be easy or profitable to attempt an *el* formulation of his 'laws of thought' But a semantic formulation, as given above, is very instructive and comparatively simple

Our existing *el* 'logic', besides the two-valued type of formulation, involves many different 'philosophical' elucidations, which instead of clarifying the status of 'logic', in general, tends only to conceal the important issues involved in *non-elementalism* The role that identification plays in a given individual appears always as a deciding factor in his adjustment Unfortunately, at present, the sinister identification is not counteracted but fostered or even induced by the structure of the languages we use, different mythologies , and our whole educational, economic, social , systems

The two-valued A, *el*, three-dimensional 'logic' does not apply to the world of events, to the objective levels , and, for the reasons already explained, does not apply to the study of the foundations of mathematics It applies to a large extent to contentless technical mathematics, including so-called 'formal logic' of that system

Formalism *when free* from identification becomes a unique comparative tool in search for structure, formalism *with* identification of different orders of abstractions, a symptom of semantic disturbances, often of a morbid character It should be realized that we may have one-, two-, three- , many-, and ∞-valued orientations, which with the exception of one-valued, we should utilize when conditions warrant a particular use in a particular case Thus in mathematics, for the sake of having mathematics as a standard of evaluation, *we select* a sharply two-valued orientation by which in the old language '*A* is *B* or not *B*', *to allow sharp* statements that for instance, $1+1=2$ If we would deliberately postulate that $1+1$ may sometimes be equal to 2 and sometimes not equal to 2, we would have forms of representation which would apply perhaps more readily to science and life, *but* mathematics as such would be impossible, and we would be deprived of this sharp tool for evaluation

It is interesting to notice that mathematicians, by the use of two-valued *semantics*, (not 'logic', because an *el* discipline cannot be 'lived through' at all by non-heavily pathological individuals), have produced the most important disciplines Thus we have, for instance, the theory of 'variance' (the theory of function), the theory of invariance, the differential calculus, the 1, 2, 3, 4, and *n*-dimensional systems, and a host of other verbal structures similar not only to the world, but to the human nervous system. These results give us means not only to enlarge our mastery of the external world, but when generalized into a *non-el*, $\bar{A}$-system, give us the means for the mastery of the inner world, leading toward sanity

It is amusing to discover, in the twentieth century, that the quarrels between two lovers, two mathematicians, two nations, two economic systems . ., usually assumed insoluble in a 'finite period' should exhibit one mechanism—the semantic mechanism of identification—the discovery of which makes universal agreement possible, in mathematics and in life

NOTES AND REFERENCES

In the following references the bold face numbers refer to the numbers of the literature in the bibliography, p, or, pp, indicates the page or pages, ff, indicates following pages

In many instances the number of the page is given, but in others, when I refer to a large subject, only the number of a book or paper is indicated, and in such cases the index of the given book should be consulted

In other cases, when no references are given, and yet the serious and educated reader may occasionally feel perplexed, may I not suggest, in this connection, that wide experience, has taught me that we usually forget the structural, not entirely common, subtleties of grammar We also often ascribe to words a very limited, personal, and *habitual* range of meanings, and, so, some purely linguistic difficulties appear as mysterious 'scientific' difficulties, which they are not The reader, on such occasions, will be surprised to find what an enormous amount of knowledge may be found in a mature occasional perusal of a good grammar or dictionary, the neglect of which acts as a psycho-logical blockage to the understanding

CHAPTER II

1—**25.** 2—**212** 3—**591,** pp 93, 94

CHAPTER III

1—The need of International Languages, or a *Universal Language* besides mathematics is becoming increasingly urgent At present there are several such languages, and in many large cities there are organizations, usually called International Auxiliary Language Associations, with addresses listed in telephone directories Any of these organizations will gladly supply information about the whole of the international linguistic movement There is also a number of books written on this subject to be found in the larger public or university libraries Informations about the *Basic English* of Ogden as a *Universal Language,* consisting of the astonishingly small number of 850 words, which do the work of about 20,000 words, may be obtained from the Orthological Institute, 10 King's Parade, Cambridge, England (see also **376, 377**) In my opinion, the possibilities of the Basic for a scientific civilization are unlimited, *provided* the Basic is *revised* from a non-aristotelian, non-identity, point of view

The general and serious defect of all of these languages is, that their authors have, as yet, entirely disregarded the non-aristotelian problems of non-identity, and so of *structure,* without which *general sanity,* or the elimination of *delusional worlds* is *entirely impossible*

CHAPTER IV

1—**590** 2—**579,** Vol II, Part IV, *150 ff , **455-457.** 3—**590.** 4—**457,** p 249

CHAPTER VII

1—**317, 318, 319.** 2—**83**

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1—**92,** pp 50-52 2—**91, 92** 3—**92,** pp 114, 119, 123, 242 4—**564, 560,** pp 16 ff , 28 ff

CHAPTER IX

1—**487.** 2—**304, 7,** Vol II p 461 ff 3—**214.** 4—**7,** Vol II, p 457 5—**214,** p 210 6—**7,** Vol II, p 458 7—**7,** Vol II, p 251 8—**7,** Vol II, p 634 9—**7,** Vol II, p 944 10—**310,** Chap V 11—**364, 365-369, 540** 12—**9, 10, 12, 13, 196, 210, 211, 370** 13—**257, 272, 328, 345** 14—**7,** Vol II, p 917 ff , **273, 313, 416** 15—**7,** Vol II, p 690 16—**7,** Vol II, p 961 ff 17—**7,** Vol II, p 644 ff 18—**7,** Vol II, p 803 ff 19—**7,** Vol II, p 869 ff 20—**363.** 21—**499, 500-504, 127, 128.** 22—**201, 533.** 23—**49.** 24—**7,** Vol II, p 59 ff

CHAPTER X

1—306, p 72 ff 2—306, p 72 ff 3—306, p 72 ff p 193 4—310, p 95 ff 5—309, p 197 6—309, p 202 7—309, pp 204, 211 8—411, p 233 9—221, p 139 10—221, pp 144 145 11—343, 344. 12—559, p 15 13—242.

CHAPTER XI

1—452, 453-457. 2—82, 264, 269. 3—470. 4—176-184.

CHAPTER XII

1—220, p 60 (5th edition) 2—222, pp 3-5 3—411, p 8 4—300, and in a personal letter 5—220, p 334 6—220, p 68 7—411, p 210 8—566

CHAPTER XIII

1—452, 453

CHAPTER XIV

1—264. 2—203, p 39

CHAPTER XV

1—452, 453-457. 2—457, p 22 ff

CHAPTER XVI

1—449.

CHAPTER XVII

1—547, p 102 2—208, p 811 (17th edition) 3—208, p 815 (17th edition)

CHAPTER XVIII

1—455, p 18 2—126, pp 14, 16 3—74, p 467, 468

CHAPTER XIX

1—264. 2—430 3—468, p 133 4—468, p 141 5—264, p 153 ff 6—264, p 153 ff 7—222, p 166 ff 8—289 9—350. 10—318, 319.

CHAPTER XX

1—221, p 277 2—7, Vol II, pp 59-92, 121, 436. 3—216.

CHAPTER XXI

1—394, 395 2—Rudyard Kipling's *Verse* Inclusive edition, 1885-1926 New York 3—501 4—221, p 197 5—552, 553-557. 6—250, p 194

CHAPTER XXII

1—92, p 112 2—221 3—221, pp 86, 87, 91, Chap XIII 4—306, p 48 ff 5—306, pp 37-40 6—306, pp 85 86 7—306, p 61-63 8—304, p 45 9—487, p 393 10—222, pp 213 214 11—222, pp 262, 263 12—394, pp 47, 388 13—278, pp 90 92 14—394, p 99

CHAPTER XXIII

1—394, pp 88-94 2—394, pp 310, 311 3—394, pp 310, 311 4—394, pp 403, 404

CHAPTER XXIV

1—34.

CHAPTER XXV

1—All anthropology gives ample evidence In the present very brief bibliography, consult 172, 173, 200, 298, 299, 331-336, 492.

CHAPTER XXVII

1—579 (1st edition), Vol I, p 40 2—590, No 3, 332 3—97. 4—452, 453-457, 579, and all modern works on 'logic' and the foundations of mathematics 5—579, pp 63, 65 ff, Vol I (1st edition).

CHAPTER XXVIII

1—**247** 2—the whole of primitive and modern mythologies, consult **172, 173, 200, 298, 299, 331-336, 478, 479, 492.** The literature is very large, and cannot be given here Consult also standard treatises on comparative religion, history of religion, and works of psychiatrists who deal with these aspects of mental' ills

CHAPTER XXX

1—**241**, p 37 2—**241**, Fig 2 p 94, Fig 3, p 140 3—**241**, pp 140-142 4—**42**, pp 1, 19, 23, 118-120 5—**42**, p 23 6—**99**, p 897 7—**34**, p 230 8—**34**, p 180 9—**34**, p 233 10—**34**, p 242 11—**411**, p 237 12—**411**, p 10 13—**15, 16, 286.** 14—**298**, Chap IX 15—**181**, Vol IV, p 136, **492.** 16—**70.** 17—**487**, pp 1363, 1334 18—**487**, pp 1327-29, 1363 19—**34**, p 117 20—**34**, p 91 21—**70**

CHAPTER XXXI

1—Science Jan 22, 1932 2—**56.** 3—**105, 290, 329, 413, 467, 521, 532**, the literature on this subject is very large, and I give here only examples, see also **110**

CHAPTER XXXII

1—**72** 2—**381, 510, 585.** 3—**220**, p 368 (5th edition)

CHAPTER XXXIII

1—**547**, p 104 2—**55**, pp 89, 174, 221 3—**411**, p 191

CHAPTER XXXIV

1—**147**, p 79 2—Most of the above presentation follows **45** 3—**204**, Vol II, p 342 4—**45**, pp 260, 261 5—see, however, **472** and **508.** 6—see C Runge, *Vector Analysis* (London, New York), p 178 7—**148**, p 49 8—**147**, pp 86-88 9—**45**, pp 18, 22, 24, 25, 36 10—**411**, pp 24, 50, 91

CHAPTER XXXV

1—**45**, p 132 2—**547**, p 92 3—**204**, Vol I pp 247, 248 4—**45**, pp 158, 159 5—**547**, p 66 6—**547**, p 91 7—**457**, pp 21, 22

CHAPTER XXXVIII

1—**264**, p 327 ff see, however, **348** 2—**314**, p 75 ff 3—**147**, p 58 ff 4—**45**, pp 196, 197, 213

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1—**148**, p 23 2—**45**, p 232 3—**148**, p 136 4—**147**, pp 177, 178 5—**547**, pp 284 285 6—**147**, p 178 7—**147**, p 149

CHAPTER XL

1—**480**, pp 65 66 2—**31**, pp 17-19 3—**480**, pp 467, 468 4—**32**, p 38

CHAPTER XLI

1—**142**, p 5 2—**204**, Vol I, p 79 ff 3—**480**, pp 194-202 4—**32**, p 64, **46**, p 74 5—**123, 39, 522.** 6—**123**, Vol III, p VI 7—**146.** 8—**457**, pp 42-44 9—**32**, later Dirac altered his procedure, see **142.** 10—see last section of appendix 7 in the German, 4th edition of **480**, and also **481.** 11—**32**, p 185, **63, 64, 65** 12—**27**, p 16 ff 13—**32**, pp 148, 149 14—**32**, p 137, **27**, pp 25, 26 15—**27**, pp 64-67 16—**27**, p 38

SUPPLEMENT III

1—for literature, see under respective names 2—**249.**

BIBLIOGRAPHY

A complete bibliography for a *non-aristotelian system* would require many volumes and is, therefore, impossible here. The formulation of a *non-aristotelian system*, with the number of scientific facts known in 1933, turned out to be an extremely laborious process.

A non-aristotelian language and attitude differ considerably from the older languages and attitudes, and so a first non-aristotelian system has no literature which would deal directly with the subject. The statement 'that everything has already been said' is, unfortunately, largely true. This introduces serious complexities, because extremely few men have the genius of a Poincaré, and fully realize that the language used in making a statement plays an overwhelming role as to the consequences which eventually follow. If we even grant that 'every thing was said', I must add, 'but not so', and this *prevented the building* of a non-aristotelian system for more than two thousand years. One of the human tragedies can be found in the fact that *wise epigrams* do not work. It takes *a system* which often expresses similar notions, but they must be expressed in a *unified language* of different structure to make them workable.

In giving this extremely abbreviated, insufficient, and, perhaps, even poorly selected bibliography, I had, in the main, three aims: (1) to acknowledge some of my direct obligations; (2) to give to the future student an outline of the type of literature in existence which has bearing on my subject; and (3) to list such books and articles which give further literature.

I was particularly careful to list as few scientific periodicals as possible, because specialists in a given field do not need them, and laymen do not want them. In a number of instances, I have listed only one or two of the latest papers of an author, which give his previous titles.

Because of the lack of linguistic co-ordination, in most cases, I have had few or no opportunities to refer directly to many authors, although the titles of the books usually suggest the material needed. For further data on a given subject, the reader is referred to the respective indexes. As a rule I had to express many similar notions, but from a different angle, and in a different language.

I have prefaced the books, parts, and chapters with many important quotations, only to show that all modern science requires a fundamental non-aristotelian revision. The attentive reader will discover that, although I am in general sympathy with these quotations, yet, in many instances, I would have to express them differently.

In the case of the two volumes of *Colloidal Chemistry* edited by Alexander, which is a collection of important contributions by different authors, I would have to list about one hundred and twenty more titles, and so I mostly refer to the page without giving the name of the author or the title of his contribution, for which I apologize. In a number of instances I have utilized the material given by *Science Service*, as printed in *Science* and I indicate such references by inserting *SS* before giving the date of the issue of *Science*.

1. ACKERMANN, W. Begrundung des "tertium non datur" mittels der Hilbertschen Theorie der Widerspruchsfreiheit. *Math. Ann.* B. 93, H. ½.
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SCIENCE AND SANITY

AN INTRODUCTION TO NON-ARISTOTELIAN SYSTEMS AND GENERAL SEMANTICS

BY

ALFRED KORZYBSKI

(Author of *Manhood of Humanity*)

SCIENTIFIC OPINIONS ABOUT THE FIRST EDITION, 1933:

1	ANTHROPOLOGY	Professor B Malinowski, University of London
2	BIOLOGY	Doctor C B Bridges, Carnegie Institution
		Professor C M Child, University of Chicago
		Professor H S Jennings, Johns Hopkins University
		Professor R Pearl, Johns Hopkins University
3	BOTANY	Doctor D G Fairchild, United States Department of Agriculture
4	CONDITIONAL REFLEXES	Doctor W H. Gantt, Phipps Psychiatric Institute, Johns Hopkins Hospital
5	EDUCATION	Doctor E L Hardy, President States Teachers' College, San Diego, California
		Williams, President Williams College, Berkeley, California
	GY	Professor W M Wheeler, Harvard University
		B Bridges, D G Fairchild, H S Jennings
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16	SEMANTICS	See B Malinowski

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1 ANTHROPOLOGY.

BRONISLAW MALINOWSKI, Ph D (Cracow), D Sc (London), Professor of Social Anthropology, School of Economics, University of London.

"The functional or relational conception [illegible] matter, mind and, finally, of human culture, seems to be gradually crystallising fr[illegible]n all attempts at scientific synthesis Count Korzybski s work contributes to these [illegible]orts in no mean measure I am perhaps biassed as a countryman, but to me thi[illegible] Polish attempt at synthesis seems to rank as one of the most important I am of [illegible]urse unable to express a competent judgment on its mathematical, scientific—in [illegible]e narrow sense of the word—and philosophical side As regards however sen[illegible]tics and the anthropological issues discussed by Count Korzybski, I am in comp[illegible]e agreement with his approach I should like to add that the approach is so new [illegible]l fundamental that it will take some time for us to become completely familiar with [illegible] For the present I should like to say that I have not yet mastered all the intric[illegible]s of Count Korzybski's system, so my appreciation must naturally be regarded as [illegible]liminary"

2 BIOLO[illegible]

DOCTOR CALVIN B BRIDGES, Biol[illegible]st, internationally known specialist in heredity, Carnegie Institu[illegible]n of Washington, in residence at California Institute of Technolog[illegible] Pasadena

"In several fields of Biological sciences the [illegible]nscious drift of thought has been for some years more and more in the direction [illegible]h Count Korzybski designates as 'non-elementalistic' Thus, the distinction, once [illegible]ht fundamental, between heredity and environment, loses its force, and the or[illegible] [illegible] now redefined as the focus of both internal (hereditary) and external (env[illegible] [illegible]) activities What is considered external rather than internal, changes w[illegible] point of view and the size of the unit (nations, man, gland, cell, nucleus, chrom[illegible] gene) which is made the basis of the formulation The formulation becomes [illegible]l, non-elementalistic, organism-as-a-whole The reformulation of biological [illegible]s made ultimately inevitable, and is greatly hastened and aided in transition by[illegible]ralized point of view established in the non-aristotelian system of Korzybsk[illegible]limination of 'identity' constitutes the first and most fundamental general [illegible] such a non-aristotelian and non-elementalistic reconstruction Following this complete reformulation and its application in science and in life, the psycho-logical and environmental conditions for man would be improved to such an extent that it is not yet possible to foresee the entire result in the new enviro-genetic manifold"

C M. CHILD, Professor of Zoology, University of Chicago

"I think that Count Korzybski has a view point of [illegible]reat interest and that his method of attack on the various problems with which he [illegible]eals cannot fail to be of value."

H. S. JENNINGS, Henry Walters Professor of Zoo[illegible]gy and Director of the Zoological Laboratory, Johns Hopkins Univ[illegible]ity

"The attempt of Count Korzybski to formulate the w[illegible] and its processes, keeping in view as a guiding principle the fact that no two thin[illegible]s are identical, seems to me of the greatest interest and value It is something that [illegible]d to be done, and it has within it the seeds of a much needed intellectual revolution"

RAYMOND PEARL, Professor of Biology, Johns Ho[illegible]kins University.

"I have known and followed Count Korzybski's work f[illegible] many years with the keenest interest In this new book he makes, in my opinion [illegible] contribution to human thought and understanding of the very first rank of import[illegible]ce It states and develops a really new idea The consequences of that idea will, [illegible]n the passage of time be far-reaching and fundamental At long last real hope is offered of measurably freeing man from some of the dreadful consequences of his verbalistic bonds"

See also W M WHEELER

3 BOTANY

DOCTOR DAVID G FAIRCHILD, Botanist, U S Department of Agriculture, Washington, D. C plant explorer and pathologist

"I am much impressed with the profundity of Korzybski's *Science and Sanity* I find it hard to get out of the meshes of the old aristotelianism and wish that I had been able to read this book in my youth, for then I could have acquired the new language of relations

Korzybski's masterly treatise will act as a powerful force in natural selection perhaps, when it brings into common use the non-aristotelian methods, for it will favor in most pursuits those who are capable of conceptual thought and confuse and eliminate those who want quick hand decisions such as are usually blurted out with great show of confidence Of one thing we may be assured once a man grasps the general idea Korzybski is driving at, he cannot fail to look at the world of everyday language from a different standpoint

Korzybski's criticisms are so profound that they change the very foundations upon which we have been used to depend As I look back to my years of travel over the world I can see that I did everything I ought not to have done in the way of bad thinking (we all have done this I suppose, bad thinking must be the common plague of mankind)

These last years among West Africans in Africa and the West Indies have made me realize keenly that primitive man identifies about like the animals do and has no consciousness that he abstracts at all Of course if we are not conscious of abstracting, in other words, if we copy primitives or animals in our nervous reactions, then I suppose any kind of maladjustment can be expected The simple and efficient neuro-psychological non-aristotelian technique which Korzybski formulates in his Structural Differential for the purpose of the elimination of identification holds promises that we may finally outgrow the infantile stage of our civilization I wonder that Educators have not already taken up this pressing problem and made the elimination of identification and the acquiring of consciousness of abstracting, the main aims of all education

These impressions regarding Korzybski's remarkable book come at the close of many years of travel through the world,—in savage countries, in the Orient, in South America and South Africa,—and had they only been a part of my mental training before these travels, the results of my observations could hardly have failed to have been much nearer to actualities

I predict a steady conversion to the point of view of this most interesting and important work"

CONDITIONAL REFLEXES

DOCTOR W. HORSLEY GANTT, Phipps Psychiatric Institute, The Johns Hopkins Hospital. Formerly for five years co-worker with Professor Pavlov in Leningrad

"I have read with great interest Count Korzybski's *Science and Sanity* and feel that it is very important for science as well as general education and progress of human thinking It expresses a point of view and a truth that I have not seen stated previously I was particularly interested in the chapters dealing with conditional reflexes Korzybski discusses the matter with profound and accurate understanding, and the suggestions he makes are most timely and helpful to those who are working in this field Anyone interested in the broader aspects of science I am sure will find in Korzybski's book an original and far-sighted view of the whole modern teaching of the subject"

5 EDUCATION

DOCTOR EDWARD L HARDY, President State Teachers College, San Diego, California

"Count Korzybski's *Science and Sanity* should be read by all persons seriously interested in or concerned with the next necessary steps in the development of educational principles and procedures"

CORA L. WILLIAMS, Mathematician, President Williams Institute, Berkeley, California

"What Einstein has done for the outer realm of our being, Korzybski is doing for our inner realm. It is to be hoped that some understanding person will endow a chair of Non-aristotelian General Semantics for him in the Institute of Advanced Study so that these two lines of research may go on together."

6 ENTOMOLOGY

WILLIAM MORTON WHEELER, Professor of Entomology, Harvard University

"Count Korzybski's work seems to me to be of great interest and value not only to the lay reader but also to the student of science and to the biologist and sociologist in particular for three reasons. First, his views give greater generality to the significance of the organism as a whole, and of structure and creative synthesis, or emergence, which are being increasingly emphasized by biologists, psychologists and sociologists working in the most diverse fields. Second, the sections of his work dealing with the intellectual vices of wishful thinking, verbalism and identification, to which we are all more or less addicted, point the way to the acquirement of mental balance and sanity. And third, his method of attaining this sanity through a non-aristotelian system and a realization of the meaning of the abstractions and symbols which we are constantly using, lays the foundation of a sound and much-needed social, commercial and political ethics."

7 GENETICS

See CALVIN B BRIDGES, D G FAIRCHILD and H. S JENNINGS

8 OPHTHALMOLOGY

WILLIAM H WILMER, M.D., Professor of Ophthalmology, Johns Hopkins University and Ophthalmologist-in-Chief, Johns Hopkins Hospital.

"Count Korzybski's viewpoint is very unique, fascinating, and, I think, very logical. The induction of non-identity would cover a great many ills, mental, moral and physical. For more than a quarter of a century, I have observed the retrogression of a number of great men after a certain time, and I feel that their failure to hold their greatness has been due largely to an egocentricism. What is true of these men whom the world called great for awhile, is equally true of the masses of humanity, who have not attained to greatness. Many of these could probably have been saved by the proper attention to psychophysiology."

9 MATHEMATICS

E. T BELL, Professor of Mathematics, California Institute of Technology

"I think it is obvious that Korzybski is working in a direction of the highest present importance for science and life. This is the more so as some sort of corrective seems to be needed for the well-meaning but ill-considered popular announcements by certain leading scientific men.

A little careful consideration of the recognized fundamentals of scientific and other thinking, such as Korzybski's book aims to set forth clearly, would prevent such really futile pronouncements by prophets of science and make the public more chary in swallowing every transient guess.

Korzybski, among personal contributions of his own concerning the law of identity, has succeeded incidentally in making current the fundamental revolution in mathematical and other basic thinking, which goes under the name of a non-aristotelian logic, and bringing to educated people an account of the most significant advance in abstract thought of the past millennium. The profound modifications of rational mathematical thinking which began about thirty years ago with the work of Brouwer, have, so far as I am aware, escaped the notice of those who undertake

) report science and mathematics t the general public The reader of Korzybski's ook will gain an outlook on these new fields as well as an insight into the author's ontributions to the problem of identity Brouwer challenged one of the laws of ristotle, Korzybski challenges another"

See also P W BRIDGMAN, B F DOSTAL, R J KENNEDY, BERTRAND RUSSELL, M TRAMER, C L WILLIAMS, H B WILLIAMS

10. MATHEMATICAL FOUNDATIONS AND LOGIC

BERTRAND RUSSELL cables from London to the author:

"Your work is impressive and your erudition extraordinary Have not had time for thorough reading but think well of parts read Undoubtedly your theories emand serious consideration"

11 MATHEMATICAL PHYSICS.

B F DOSTAL, Professor of Mathematics, University of Florida

"We still teach classical science on Mondays, Wednesdays and Fridays, and modern science on Tuesdays, Thursdays and Saturdays, as Sir William Bragg truly said Within the bounds of aristotelian system there appears to be no hope of ever finding the requisite unifying principle Mathematicians have been rapidly outgrowing the old forms of so-called logic, but mathematical physicists have in general been slow to appreciate the value of these efforts or to apply these results to their own problems Korzybski's *Science and Sanity* will be of great value to science because it contains the basis of the development of a new and wider, and more unifying form of scientific determinism, without which the outlook for modern science would be gloomy indeed Not only does Korzybski point out a more satisfactory, non-aristotelian non-identity basis for a new science in general, than any hitherto employed, but he goes further in giving several promising suggestions for extensive developments and applications of the results of *modern* science including those of the new wave and *quantum* mechanics His work is bound to become a stimulus to investigators in mathematics, physics, chemistry, biology, 'psychology,' and medicine, and to educators, economists, sociologists, engineers, lawyers, and laymen as well, the majority of which still have a 'philosophy of the universe which takes one form on weekdays another form on Sundays'"

12 NEUROLOGY

C JUDSON HERRICK, Professor of Neurology, University of Chicago

"The disturbances of mental balance and social stability now so prevalent seem to indicate a general failure to adjust our minds to our jobs This results in futile conflict and too often in mental and social derangement The numberless panaceas proposed fail because each attacks a single phase of a very complex situation, and generally a particular symptom rather than the cause of the trouble Count Korzybski has diagnosed a fundamental source of confusion in thinking and in conduct and he presents a plan for radical revamping of our theory and practice that seems worthy of further trial in a wide variety of fields His dynamic definition of *structure* in terms of relations gives promise of important applications in both science and practical affairs, provides a generally useful symbol for experience of all sorts and a technique for recasting traditional ideas and practices more efficiently Adjustments in terms of one dominant motive (or value) are replaced by a broader (many-valued) scheme of motivation which points the way toward personal and social sanity—that I believe is fundamentally correct and practicable"

See also CHILD R S LILLIE, M TRAMER, W M WHEELER, H B WILLIAMS W H WILMER

13 PHYSICS

P. W. BRIDGMAN, Professor of Physics, Harvard University

"Of late the realization has been growing that the ultimate source of a large fraction of the difficulties of society, civilization, and science, is verbal in character Among the few serious attempts to waken full self consciousness of what the situation is, and, having awaked consciousness, to provide *a technique* by which the vicious consequences of verbal habits may be avoided, I believe that of Count Korzybski must be rated as of the very first importance I have been acquainted with his work for a number of years, not only o I believe it to be fundamentally sound, but I have always found his points of view most suggestive and stimulating both in general and technical matters, and I have seen amazed at the breadth of his interests and reading, and the diversity of the fields to which applications are made"

ROY J. KENNEDY, Professor of Physics, University of Washington, Seattle, Washington

"Many of the impasses in which we of this lu ic world are involved are the result of verbal difficulties, and it is precisely these ficulties at which Count Korzybski's technique for the elimination of identity is iefly aimed He has shown a striking versatility in developing this technique wl h he has originated, he discusses the shortcomings of the sciences as facilely as ose of religion Whether or not the reader's sanity is improved by a careful studof the book, he cannot fail to enlarge his capacity for clear thinking Paradoxical lathough *Science and Sanity* deals largely with the unspeakable it is suitable for ission in the most decorous circles"

See also B F DOSTAL

14. PHYSIOLOGY.

RALPH S. LILLIE, Professor of Physiology, U rsity of Chicago

"Count Korzybski's criticism of the present stru re and usages of human society—as failing to keep pace with the advance of l nledge in the physical and biological sciences—is timely and well-founded, and is exessed with clearness, vigor and insight in this interesting book It is certain thath knowledge, if widely diffused and acted upon, would greatly alleviate and pe hs move many of the ills which afflict the modern world The chief obstacle to su press is not the lack of available knowledge, but the anachronistic survival of m mental habits and conceptions which are inconsistent with the facts of natural lity as revealed by science These conceptions are firmly rooted in the general id by language and custom What is needed is a far-reaching revision of conceptsd this book points the way to such a revision Since we are compelled by the citions of existence to think and act in terms of symbols—concepts, words, imagesrmulae—it is all-important that these should conform as closely as possible to thrmanent realities of life and nature How to secure an adequate degree of such crmity and establish it by training and education is one of the most pressing p;ms of the time Count Korzybski describes in detail the nature of verbal, mathei al and scientific symbolisms, and discusses clearly the biological, neurological, other conditions which give them their representative value He shows that mi eptions regarding the nature of language underlie many prevalent confusions fallacies, especially the various fallacies of identification (arising mainly frorbalisms),—as when it is assumed that the application of the same label to diffefacts somehow renders them all alike and justifies the same action toward all S consequences inevitably arise from the failures of discrimination and valuation resulting, and the author makes a special plea for discernment and individua tment in the problems of human personality It is only on this basis that mares of maladjustment can be prevented or corrected These are only a few ls in a book remarkable for its comprehensiveness, scholarship and independenc

HORATIO B WILLIAMS, Dalton Professor of Physic Columbia University, and a Mathematician

"In his *Science and Sanity* Count Korzybski undertakes to brin e attention of his readers the importance of 'consciousness of abstracting,' or thanent and full awareness that (1) the object *is not* the event or the physinical submicroscopic process, (2) that the symbol or label *is not* the objec (3) that an inference *is not* a description Thus he is led to the formulatio non-aristotelian system based on the complete rejection of 'identity' To f training

in non-identity he proposes a si
Differential With its aid, the
ordering which becomes a visu
in non-identity or discrimination
cations, which play such an impor

It has been my privilege to r
and most of it in page proof It
broken down National and inter
Korzybski, by rejecting a principle
principle prevents adjustment, points
perhaps toward a saner solution of our n

This is distinctly not a book for sup
repay the thoughtful reader It should be a
ment, though it may be that, but rather in tl
which refers to 'the joint labors of the author

See also W H GANTT

15. PSYCHIATR.

DOCTOR PHILIP S. GRAVEN, Psychiatrist, V

"I have read *Science and Sanity* through completely and I must admit I have never encountered a work so ri tions It clearly covers a field almost wholly neglected in o From the methodological point of view, therefore the book 'i one endeavoring to carry on sane, clear, scientific work This Medicine and especially Medical Psychology where sane thinking and 'insane' is vital at all time Statements, principles, etc, that attitude toward problems are of most importance These, *Sc* provides in abundance

In addition to the scientists being considerably aided by the use aristotelian principles (aided in carrying on sane, creative, well-temper thinking about their observations and experimental data), there is als group directly affected namely, the mentally disordered By direct clinica tion, I have found the non-aristotelian principles workable in this enormous My observations cover a period of about six years I shall have a great deal t about these observations in contributions to medical and scientific journals

By reading the book carefully, I have derived many benefits personal, cultu professional, scientific The book appeals to me as one that will supply at least a f generations of scientific workers with a means of maintaining a productive, no clogging and obstructive psycho-logical attitude towards investigations needful to human security and advancement And what could be more urgent in this modern age than a means to attain and maintain sanity Korzybski's Theory of Sanity already makes that a possibility"

DOCTOR JOHN A P. MILLET, Psychiatrist, New York City

"It seems to me that Count Korzybski in his book *Science and Sanity* calls attention to one very important difficulty in the thinking process which he so aptly sums up as 'confusion in the orders of abstraction' We have found through our analytical work in dealing with the neuroses that many difficulties in attitude and orientation toward life are derived from unconscious identification, a situation which leads to a real difficulty in individuation and sometimes makes it permanently impossible

From the psychiatric point of view it seems to me that this one point that Korzybski has emphasized gives the chief value to his work I doubt whether analysts, for the most part, have ever considered the problem of identification from exactly this point of view Korzybski's presentation should be interesting to such a group and might well lead to further experimental activities in the field of education designed to offset the dangers of such identifications

A beginning has been made in this field already by Korzybski in his development of the 'Structural Differential' It is too early to say what practical value this Structural Diagram may have but the principles of its development are based on the conception of 'order' in neuro-psycho-logical processes and so provides what might the elimination of false iden-

try, University of Bern, Swit-
m, President Swiss Psychiatric
rologist, formerly a co-worker
ychiatrist

pportunity to follow the researches of
n the formulation of 'time-binding' that
Further results of his researches justi-
r scientific thinking but also our daily lives
comprehensive form to the public, its study
cture of deeper foundations, the beginning of
erent scientific domains, and particularly in
e completed development of his thesis leads to
for a fundamental analysis (revision) of our
zes the trail The means he selects to overcome
h an analysis, and to bring into the foreground the
al thinking above all, are, in my opinion, results of

A. WHITE, Professor of Psychiatry, George Wash-
ersity Medical School; Superintendent Saint Elizabeth's
Washington, D C.

concepts are to me very helpful, particularly his concepts of
inality, degrees of abstractions and finally of non-identity I have
such matters as are included in these concepts are of extreme impor-
as one's mental equipment I am at one with Korzybski in many ways.
am sure that how we think about things is of as much importance as
think about them. I congratulate the author upon the appearance of his
I think he has made a real contribution to the methodology of thinking"

16 SEMANTICS

See B MALINOWSKI